M 408 K Spring 2002

Important Decimal Approximations and Useful Trig Identities

Decimal Approximations:

$0 = 0.000$

$\frac{1}{2} = 0.500$

$\frac{\sqrt{3}}{3} = 0.577 = \frac{1}{\sqrt{3}} = \left(\frac{1}{2}\right) \Big/ \left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}}{2} = 0.707$; $\frac{\sqrt{3}}{2} = 0.866$

$1 = 1.000$

$\frac{2\sqrt{3}}{3} = 1.155 = 1 \Big/ \frac{\sqrt{3}}{2}$

$\sqrt{2} = 1.414 = 1 \Big/ \frac{\sqrt{2}}{2}$

$\sqrt{3} = 1.732 = \frac{\sqrt{3}}{1} = \left(\frac{\sqrt{3}}{2}\right) \Big/ \left(\frac{1}{2}\right)$

0	$= 0.000$	$=$	$0\,\frac{\pi}{12}$
$\frac{\pi}{6}$	$= 0.524$	$=$	$2\,\frac{\pi}{12}$
$\frac{\pi}{4}$	$= 0.785$	$=$	$3\,\frac{\pi}{12}$
$\frac{\pi}{3}$	$= 1.047$	$=$	$4\,\frac{\pi}{12}$
$\frac{\pi}{2}$	$= 1.571$	$=$	$6\,\frac{\pi}{12}$
$\frac{2\pi}{3}$	$= 2.094$	$=$	$8\,\frac{\pi}{12}$
$\frac{3\pi}{4}$	$= 2.356$	$=$	$9\,\frac{\pi}{12}$
$\frac{5\pi}{6}$	$= 2.618$	$=$	$10\,\frac{\pi}{12}$

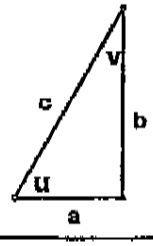
Trigonometric Identities:

When u + v = pi / 2 = 90 degrees,

sin (u) = b / c = cos (v) = cos (pi/2 - u),

tan (u) = b / a = cot (v) = cot (pi/2 - u),

sec (u) = c / a = csc (v) = csc (pi/2 - u).



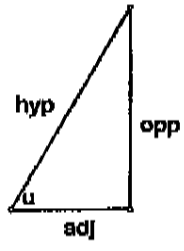
SOH-CAH-TOA

sin (u) = opp / hyp

tan (u) = opp / adj

sec (u) = hyp / adj

cos (u) = adj / hyp



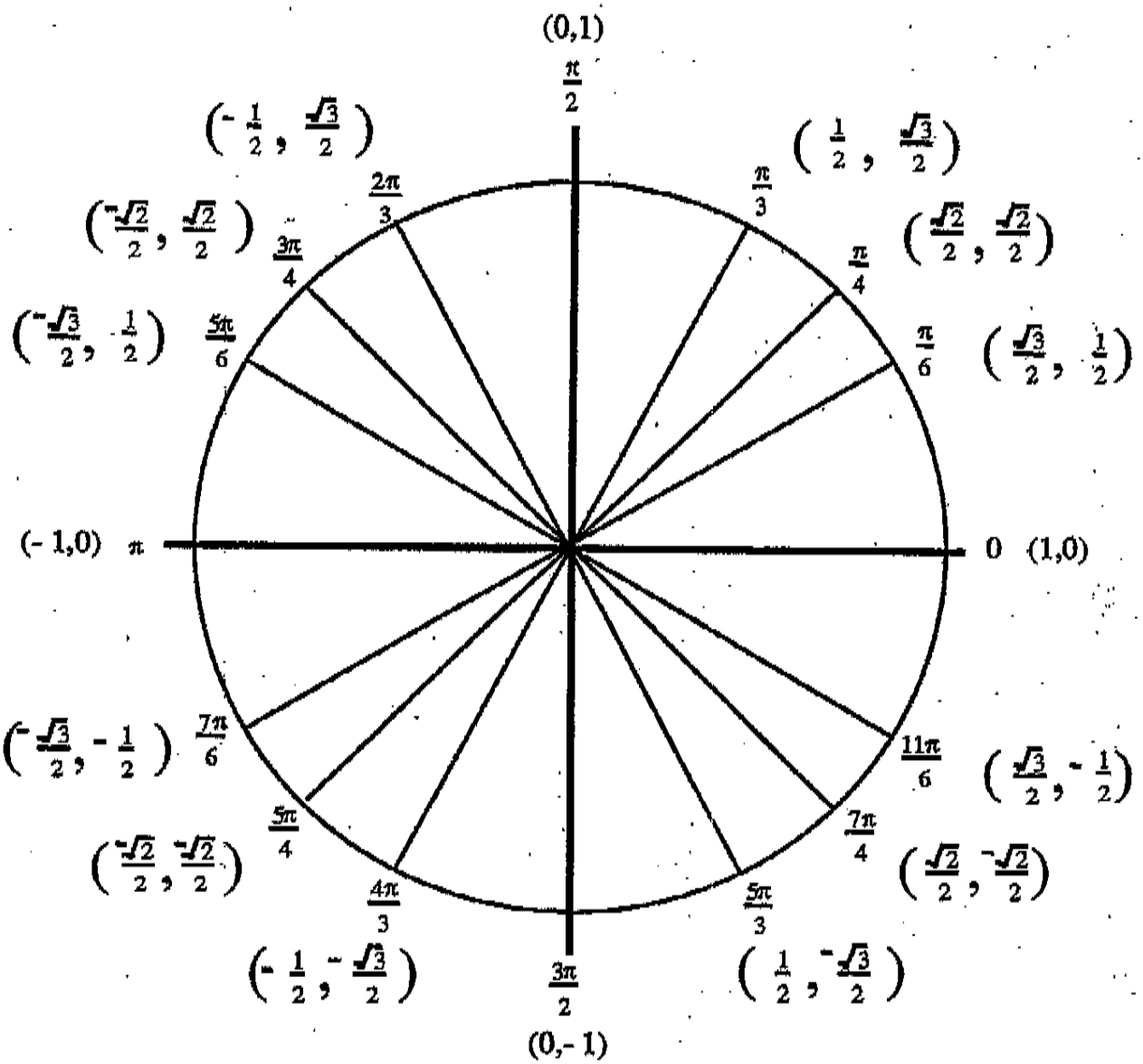
sin (- x) = - sin (x)

tan (- x) = - tan (x)

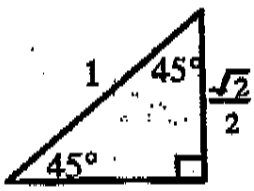
cos (- x) = cos (x)

sec (- x) = sec (x)

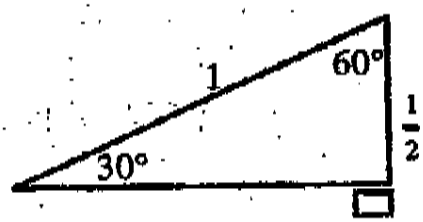
THE UNIT CIRCLE



SPECIAL ANGLE TRIANGLES



$\frac{\sqrt{2}}{2}$



$\frac{\sqrt{3}}{2}$

The (±) Question for Trigonometric Functions

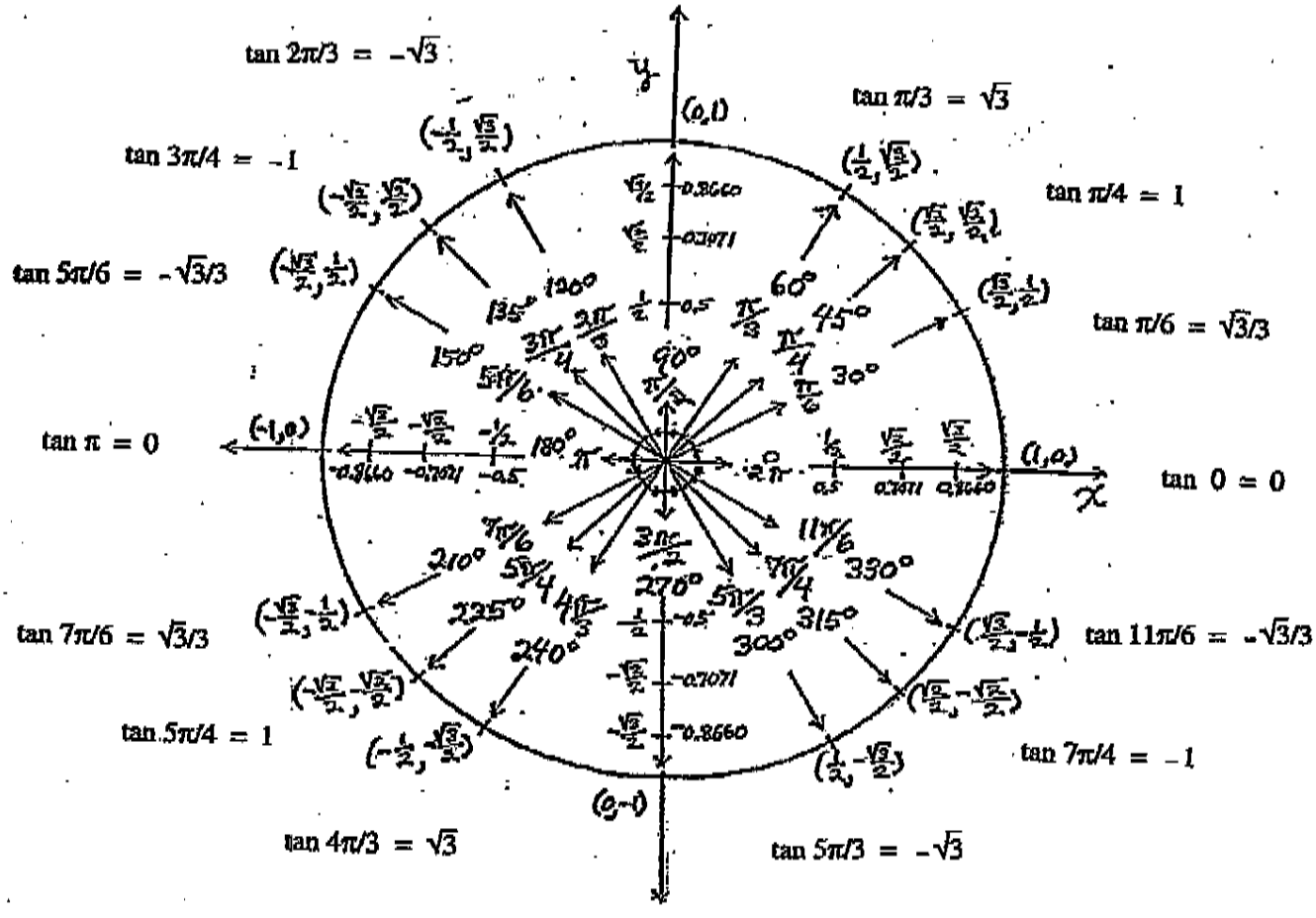
If θ is in Quad II (−,+),

then cos(θ) = (−)
sin(θ) = (+)
tan(θ) = (−)

If θ is in Quad I (+,+),

then cos(θ) = (+)
sin(θ) = (+)
tan(θ) = (+)

tan π/2 is Not Def'd



tan 3π/2 is Not Def'd

If θ is in Quad III (−,−),

then cos(θ) = (−)
sin(θ) = (−)
tan(θ) = (+)

If θ is in Quad IV (+,−),

then cos(θ) = (+)
sin(θ) = (−)
tan(θ) = (−)

TRIGONOMETRIC IDENTITIES

Fundamental Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Negative Angle Identities

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Cofunction Identities IF $A + B = \frac{\pi}{2}$

$$\sin A = \cos B$$

$$\sec A = \csc B$$

$$\tan A = \cot B$$

Addition and Subtraction Identities

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Double Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half-Angle Identities

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Product Identities

$$\sin A\cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$$

$$\cos A\sin B = \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$$

$$\cos A\cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$$

$$\sin A\sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$

Sum Identities

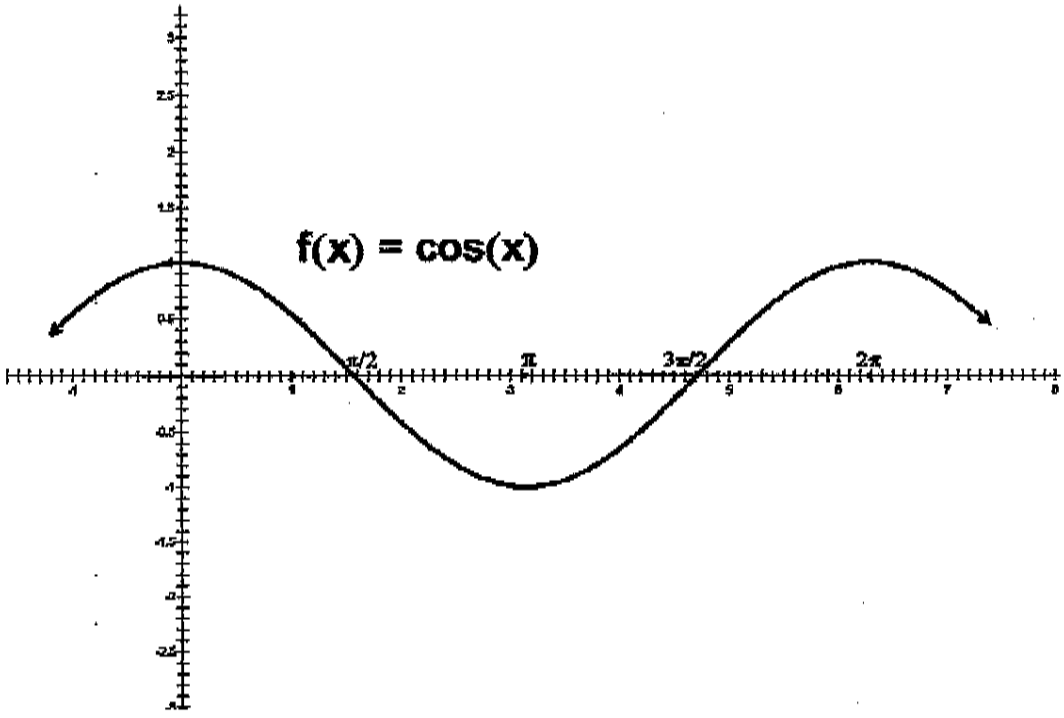
$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

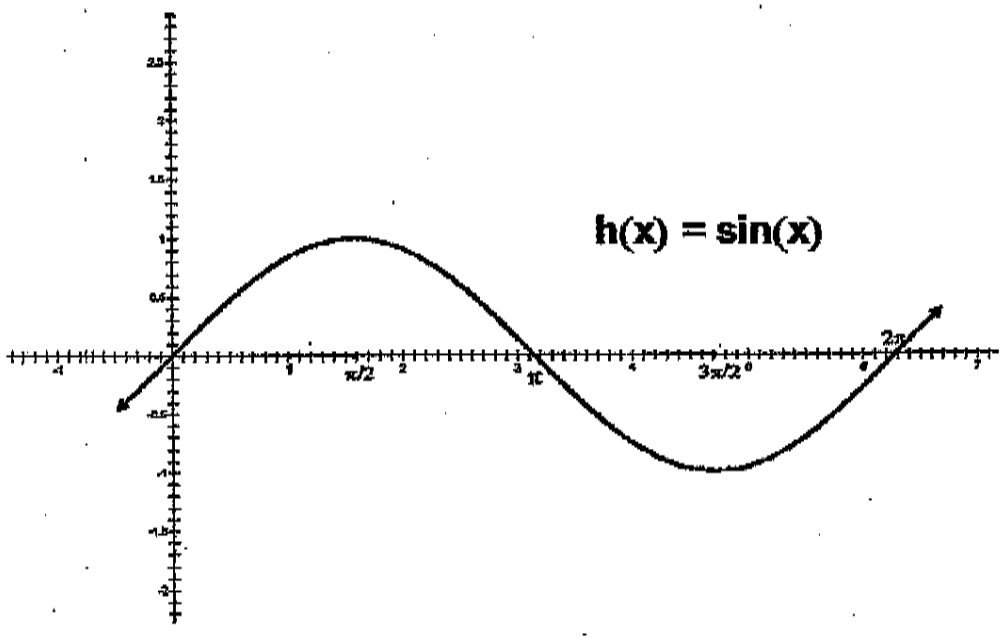
$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

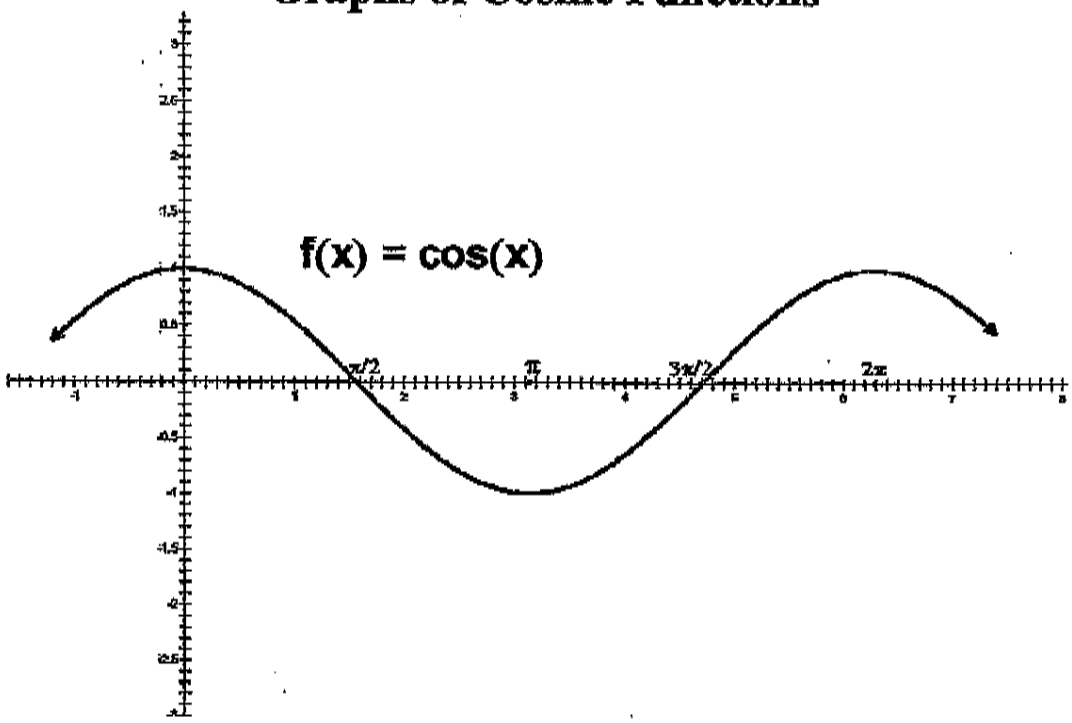
$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

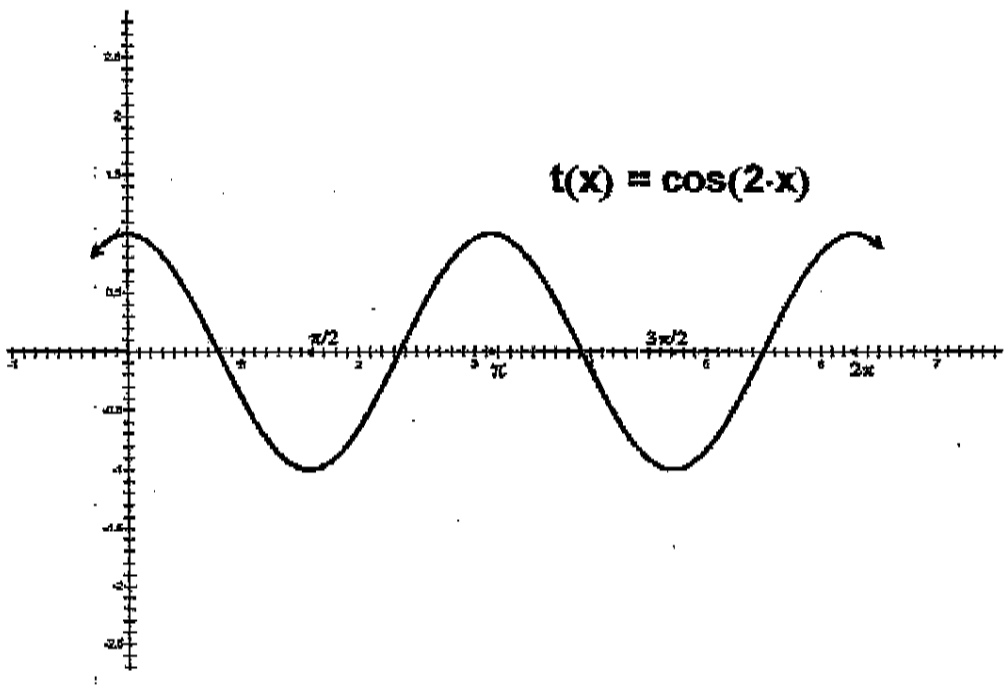
Graphs of the Sine and the Cosine Functions



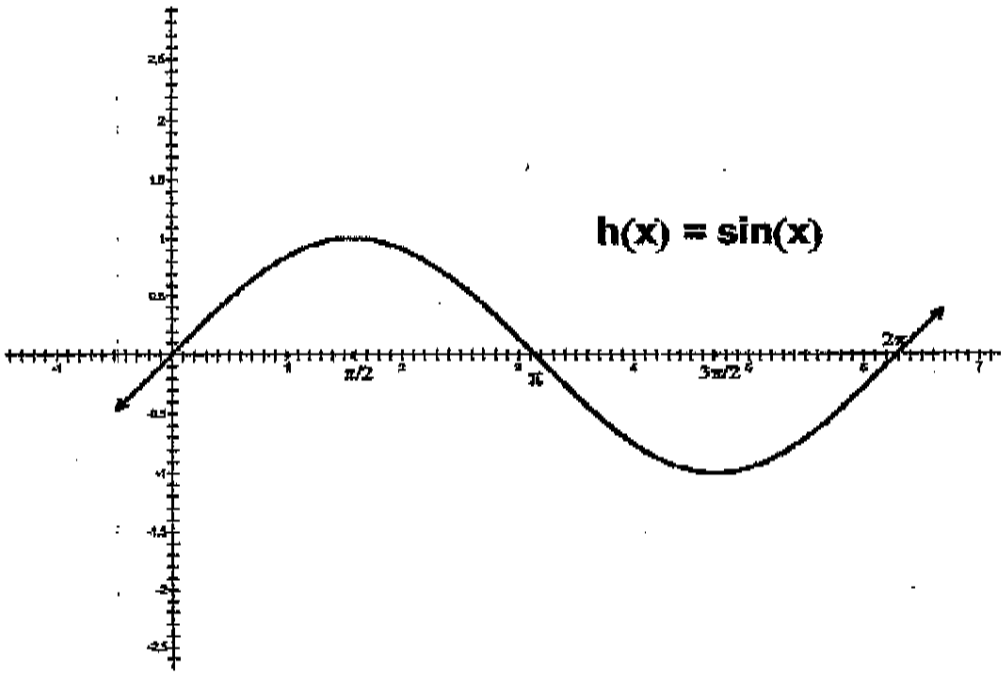


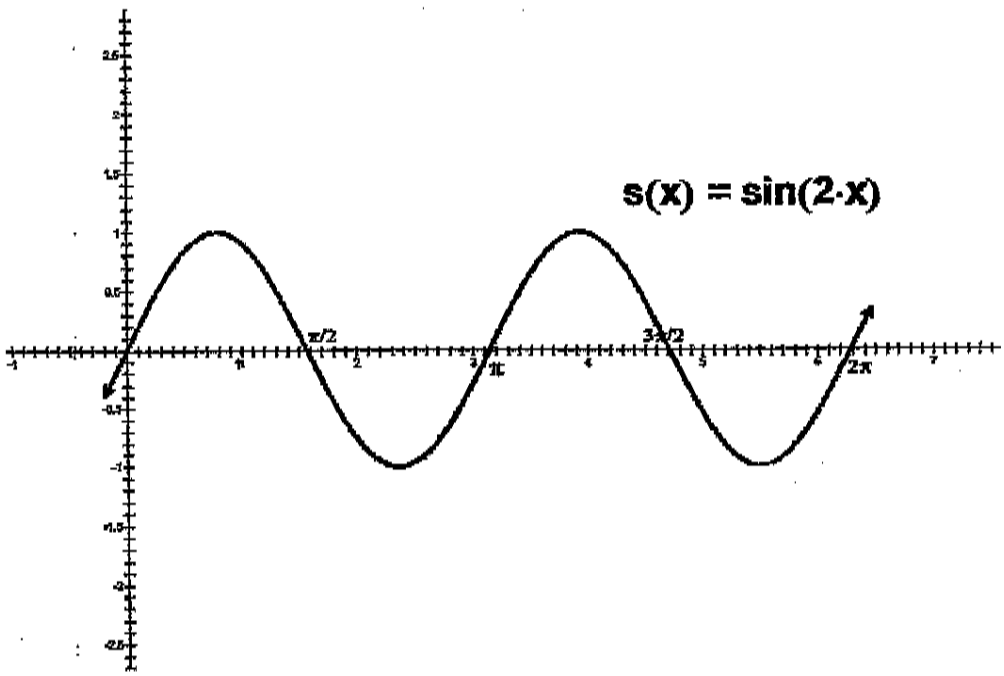
Graphs of Cosine Functions





Graphs of Sine Functions





Graphs of Sine Functions