EVALUATING INVERSE TRIG FUNCTIONS

THE RANGES OF FOUR INVERSE TRIG FUNCTIONS

FUNCTION	RANGE	QUAD'S
$y = \sin^{-1}(x)$	$-\pi/2 \le y \le \pi/2$	IV (−), I (+)
$y = \cos^{-1}(x)$	$0 \le y \le \pi$	I (+), II (−)
$y = \tan^{-1}(x)$	$-\pi/2 < y < \pi/2$	IV (−), I (+)
$y = \sec^{-1}(x)$	$0 \le y < \pi/2$ OR $\pi \le y < 3\pi/2$	I (+), III (−)

EACH RANGE COVERS TWO QUADRANTS. THE (+) or (−) indicates which QUADRANT CONTAINS THE TRIGFUN$^{-1}(x)$ ANGLE WHEN x has the given sign $\pm$.

GENERAL DEFINITION OF TRIGFUN$^{-1}(x)$:

$y = \text{TRIGFUN}^{-1}(x) \iff \text{TRIGFUN}(y) = x$ AND y is in the RANGE.

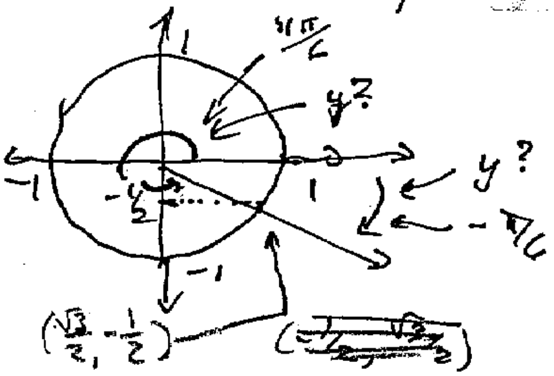
EXAMPLE 1: DETERMINE $\sin^{-1}(-1/2)$.

SOLUTION: Write $y = \sin^{-1}(-1/2)$. Here $x = -1/2$.

So, $\sin(y) = -1/2$ and $-\pi/2 \le y \le \pi/2$.

On the unit circle, the QUADRANT $\underline{IV}$ angle y such that $\sin(y) = -1/2$

is either $y = 11\pi/6$ or $y = -\pi/6$



By $-\pi/2 \le -\pi/6 \le \pi/2$

whereas $\pi/2 < 11\pi/6$.

So, $\boxed{\sin^{-1}(-1/2) = -\pi/6}$

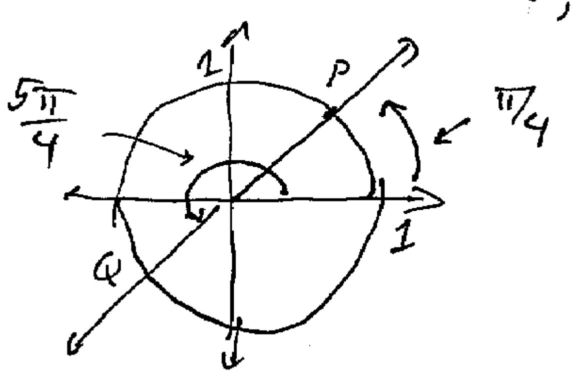
EXAMPLE 2: DETERMINE $\tan^{-1}(1)$.

Solution: Write $y = \tan^{-1}(1)$. Here $x = 1$.

Then, $\tan(y) = 1$ and $-\pi/2 < y < \pi/2$.

On the unit circle, Two angles y with $\tan(y) = 1$ are

$y = \pi/4$ and $y = 5\pi/4$.



But only $y = \pi/4$ is in the range of $y = \tan^{-1}(x)$.

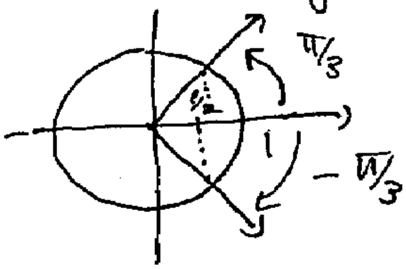
$Q = (-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$, $P = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

So, $\boxed{\tan^{-1}(1) = \pi/4}$

EXAMPLE 3: Determine $\cos^{-1}(1/2)$.

SOLUTION: $y = \cos^{-1}(1/2)$

$\Rightarrow \cos(y) = \frac{1}{2}$ and $0 \le y \le \pi$.



Two angles y with $\cos(y) = 1/2$ are $y = \pi/3$ and $y = -\pi/3$.

Onely $y = +\pi/3$ is in the range.

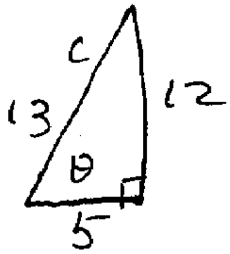
So, $\boxed{\cos^{-1}(1/2) = \pi/3}$

ALSO, since $x = 1/2 > 0$, we know that $y = \cos^{-1}(1/2)$ is in QUADRAN I so $y = \pi/3$.

EXAMPLE 4. Determine $\cos(\tan^{-1}(\frac{12}{5}))$ and simplify $y = \cos(\tan^{-1}(x))$.

solutin: Draw a Right TRIANGLE:

Put $\theta = \tan^{-1}(12/5)$ in bottom angle



$\tan\theta = \frac{12}{5} = \frac{OPP}{ADJ}$: Assign values: OPP = 12, ADJ = 5

$c^2 = 5^2 + 12^2$

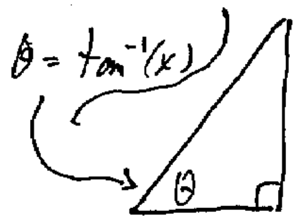
$c = \sqrt{169} = 13$

$$\boxed{\cos\theta = \cos(\tan^{-1}(\tfrac{12}{5})) = 5/13}$$

Example 4 (continued):

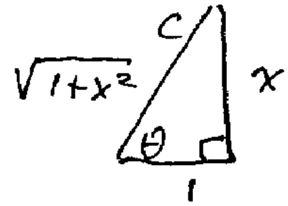
To simplify $y = \cos(\tan^{-1}(x))$,

Draw a Right Triangle with $\theta = \tan^{-1}(x)$ in the bottom angle and write $\frac{x}{1}$ for x.



$\tan\theta = x$ since $\theta = \tan^{-1}(x)$.

so $\tan\theta = \frac{x}{1} = \frac{OPP}{ADJ}$.



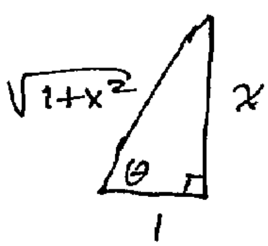
Solve for length of third side: $c^2 = 1^2 + x^2$

$c^2 = 1 + x^2 \Rightarrow c = \sqrt{1+x^2}$

So, $y = \cos(\tan^{-1}(x)) = \frac{ADJ}{HYP} = \frac{1}{\sqrt{1+x^2}}$

$\therefore \boxed{\cos(\tan^{-1}(x)) = \frac{1}{\sqrt{1+x^2}}}$

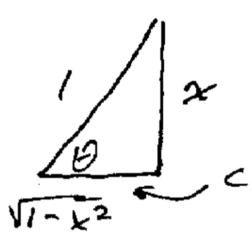
Example 5: Simplify $\sec(\tan^{-1}(x))$



$\theta = \tan^{-1}(x) \Rightarrow \tan\theta = \frac{x}{1} = \frac{OPP}{ADJ}$

$\boxed{\sec\theta = \sec(\tan^{-1}(x)) = \frac{\sqrt{1+x^2}}{1} = \sqrt{1+x^2}}$

Example 6: Simplify $\cos(\sin^{-1}(x))$.



$\theta = \sin^{-1}(x) \Rightarrow \sin\theta = \frac{x}{1} = \frac{OPP}{HYP}$

$1^2 = x^2 + c^2 \Rightarrow c^2 = 1 - x^2 \Rightarrow c = \sqrt{1-x^2}$

$\boxed{\cos(\sin^{-1}(x)) = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}}$