IMPORTANT LIMITS

$$\lim_{n\to\infty} \left(\frac{n+1}{n}\right) = 1 \quad \text{since } \left(\frac{n+1}{n}\right) = \left(1+\frac{1}{n}\right) \to 1+0$$

$$\lim_{n\to\infty} \left(\frac{n}{n+1}\right) = 1 \quad \text{since } \left(\frac{n}{n+1}\right) = 1\Big/\left(\frac{n+1}{n}\right) = 1\Big/\left(1+\frac{1}{n}\right)$$

$$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e \quad \text{since } \left(1+\frac{1}{n}\right)^n = e^{n \ln(1+\frac{1}{n})}$$

$$\lim_{x\to\infty} x \ln\left(1+\frac{1}{x}\right) = 1$$

by L'HOSPITAL'S Rule

IN THE FOLLOWING, k is a positive constant. AS A RESULT, $\ln(k)$ is constant also.

$$\lim_{n\to\infty} \sqrt[n]{k} = 1 \quad \text{since } \sqrt[n]{k} = k^{\frac{1}{n}} = e^{\frac{1}{n}\ln(k)} \to e^0 = 1$$

$$\lim_{n\to\infty} \sqrt[n]{n} = \lim_{n\to\infty} \left(n^{\frac{1}{n}}\right) = 1 \quad \text{since } n^{(\frac{1}{n})} = e^{(\frac{1}{n}\ln(n))}$$

and by L'HOSPITAL'S RULE,

$$\lim_{x\to\infty} \frac{1}{x}\ln(x) = 0$$

$$\lim_{n\to\infty} \sqrt[n]{n^k} = 1$$

since

$$\sqrt[n]{n^k} = \left(n^k\right)^{\frac{1}{n}} = \left(n^{\frac{1}{n}}\right)^k = \sqrt[n]{n}^{\,k} \to 1^k = 1$$