

TRIG DERIVATIVES (See p. 152)

~~DERIVATIVES OF TRIG FUNCTIONS~~

MEMORIZE THESE!

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x = (\sec x)^2$$

$$\frac{d}{dx}(\sec x) = (\sec x)(\tan x)$$

$$\frac{d}{dx}(\csc x) = (-\csc x)(\cot x)$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x = -(\csc x)^2$$

Which one has the minus sign? (2)

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

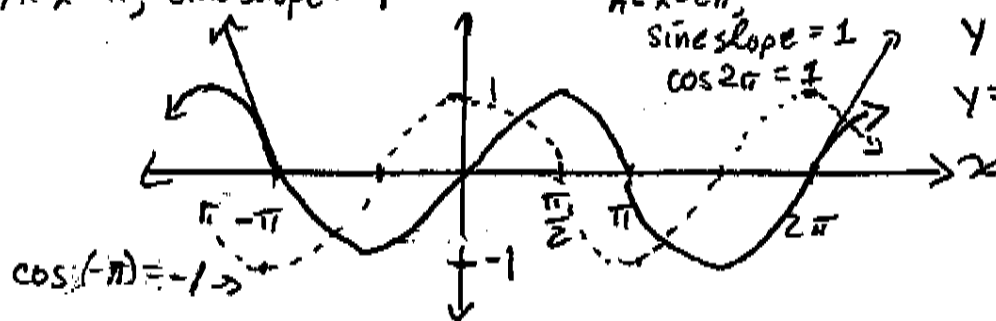
You will mix these two up often. LEARN THEM RIGHT, RIGHT NOW!

The Sine slope vs. Cosine x

(THEY ARE EQUAL: $\frac{d}{dx}(\sin x) = \cos x$)

At $x = -\pi$, Sine slope = -1

At $x = 2\pi$, Sine slope = 1
 $\cos 2\pi = 1$

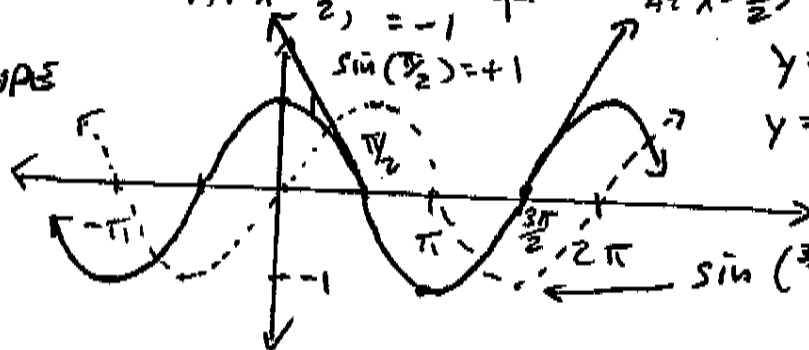


$y = \sin x$ —
 $y = \cos x$ - - -

THE COSINE SLOPE VS. SINE X

At $x = \frac{\pi}{2}$, cosine slope = -1
 $\sin(\frac{\pi}{2}) = +1$

At $x = \frac{3\pi}{2}$, cosine slope = 1



$y = \cos x$ —
 $y = \sin x$ - - -

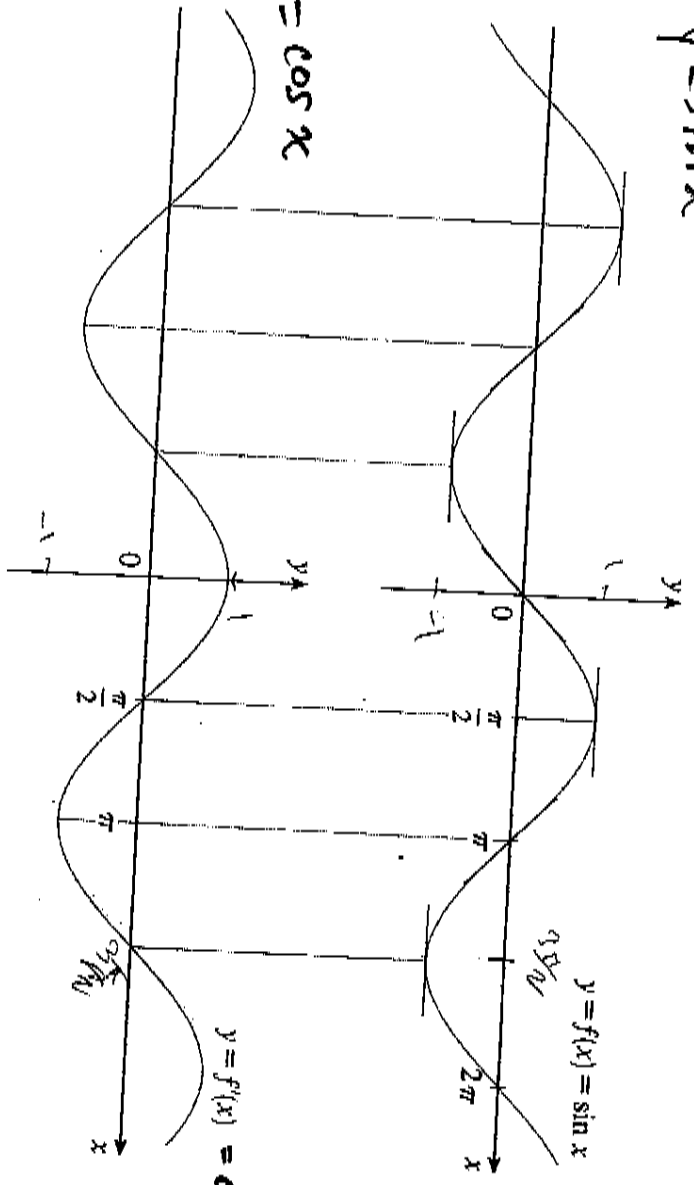
(THEY ARE OPPOSITE:

$$\frac{d}{dx}(\cos x) = -\sin x)$$

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SECTION 3.4 DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

$y = \sin x$



$y = \cos x$

FIGURE 1

Let's try to confirm our guess that if $f(x) = \sin x$, then $f'(x) = \cos x$. From the definition of a derivative, we have

~~$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(x+h) + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$~~

formula for sine

To recall

$$\frac{d}{dx}(\tan x) \quad \text{and} \quad \frac{d}{dx}(\sec x)$$

use the identities:

$$\tan x = \frac{\sin x}{\cos x} \quad \text{and} \quad \sec x = \frac{1}{\cos x}$$

together with the QUOTIENT FORMULA:

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) =$$

$$= \frac{\frac{d}{dx}(\sin x) \cdot \cos x - \sin x \cdot \frac{d}{dx}(\cos x)}{(\cos x)^2}$$

$$= \frac{(\cos x)(\cos x) - \sin x(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{(\cos x)^2} = \frac{1}{(\cos x)^2}$$

$$= \left(\frac{1}{\cos x}\right)^2 = (\sec x)^2 = \underline{\underline{\sec^2 x}}$$

DERIVE

$$\frac{d}{dx} (\sec x) = \sec x \tan x$$

for yourself using
the identities and the
Quotient Formula