The First Task in Applying the (Direct) Comparison Test and Limit Comparison Test

and SUGGESTED WORDINGS of the Required Justifications for Each Test:

The FIRST TASK for both:

When $\sum_{n=1}^{\infty} d_n$ is the series whose convergence or divergence is known

and is to be used in either the comparison test or the limit comparison test, the divergence or convergence of the series $\sum_{n=1}^{\infty} d_n$ must be explicitly established in the solution of the problem, and this must be done before concluding whether or not the given series that is compared with it is C or D.

That means that there will be a required justification to write regarding the convergence or divergence of the series

$$\sum_{n=1}^{\infty} d_n$$

before writing the required justification for applying the (Direct) Comparison Test or the Limit Comparison Test to the given series.

SUGGESTED WORDINGS OF THE REQUIRED JUSTIFICATIONS

When Applying the (Direct) Comparison Test:

Here, $\sum_{n=1}^{\infty} c_n$ will represent the given series to analyze. We will use the notation $\sum_{n=1}^{\infty} d_n$ to represent the series chosen because its convergence or divergence is known and it can be compared, term-by-term, favorably with the given series, $\sum_{n=1}^{\infty} c_n$.

When applying the (Direct) Comparison Test to conclude that the series $\sum_{n=1}^{\infty} c_n$ is **Convergent,**

YOU MUST WRITE A JUSTIFICATION as clear and complete as the following:

"Because (1) $c_n \leq d_n$ for all n, and (2) the series $\sum_{n=1}^{\infty} d_n$ is convergent, the series $\sum_{n=1}^{\infty} c_n$ is also convergent, by the (Direct) Comparison Test." } **SUGGESTED WORDING**

When applying the (Direct) Comparison Test to conclude that the series $\sum_{n=1}^{\infty} c_n$ is **Divergent,**

YOU MUST WRITE A JUSTIFICATION as clear and complete as the following:

"Because (1) $d_n \leq c_n$ for all n, and (2) the series $\sum_{n=1}^{\infty} d_n$ is divergent, the series $\sum_{n=1}^{\infty} c_n$ is also divergent, by the (Direct) Comparison Test." } **SUGGESTED WORDING**

SUGGESTED WORDINGS OF THE REQUIRED JUSTIFICATIONS

When Applying the Limit Comparison Test:

Here, $\sum_{n=1}^{\infty} c_n$ will represent the given series to analyze. We will use the notation $\sum_{n=1}^{\infty} d_n$ to represent the series chosen because its convergence or divergence is known and it can be compared favorably with the series $\sum_{n=1}^{\infty} c_n$ using either one of the two limits: $\lim_{n \to \infty} \left(c_n / d_n \right)$ or $\lim_{n \to \infty} \left(d_n / c_n \right)$.

When applying the Limit Comparison Test to conclude that the series $\sum_{n=1}^{\infty} c_n$ is **Convergent,**

YOU MUST WRITE A JUSTIFICATION as clear and complete as the following:

"Because the series $\sum_{n=1}^{\infty} d_n$ is convergent and $\lim_{n \to \infty} \left(c_n / d_n \right) = \text{L}$ and L is a finite non-zero number, the series $\sum_{n=1}^{\infty} c_n$ is also convergent, by the Limit Comparison Test." } **SUGGESTED WORDING**

(Similarly, if $\lim_{n \to \infty} \left(d_n / c_n \right) = \text{L}$ and L is a finite non-zero number.)

When applying the Limit Comparison Test to conclude that the series $\sum_{n=1}^{\infty} c_n$ is **Divergent,**

YOU MUST WRITE A JUSTIFICATION as clear and complete as the following:

"Because the series $\sum_{n=1}^{\infty} d_n$ is divergent and $\lim_{n \to \infty} \left(c_n / d_n \right) = \text{L}$ and L is a finite non-zero number, the series $\sum_{n=1}^{\infty} c_n$ is also divergent, by the Limit Comparison Test." } **SUGGESTED WORDING**

(Similarly, if $\lim_{n \to \infty} \left(d_n / c_n \right) = \text{L}$ and L is a finite non-zero number.)