SUGGESTED WORDING OF THE REQUIRED JUSTIFICATION

for the Test for Divergence

Whenever you apply the TEST FOR DIVERGENCE to conclude that the series $\sum_{n=1}^{\infty} a_n$ is **Divergent,** you must, with regard to the SEQUENCE OF TERMS $a_1, a_2, a_3, a_4, a_5, \ldots$

(and **not** with regard to the SEQUENCE OF PARTIAL SUMS $s_1, s_2, s_3, s_4, s_5, \ldots$),

first show that the limit $\lim_{n \to \infty} a_n \neq 0$, which is also the case if $\lim_{n \to \infty} a_n$ does not exist.

Then, you must write a justification as clear and complete as the following:

"Since $\lim_{n \to \infty} a_n \neq 0$, the series $\sum_{n=1}^{\infty} a_n$ is DIVERGENT,

, by the TEST FOR DIVERGENCE."

} **Suggested Wording of the REQUIRED JUSTIFICATION**

For example: Consider the series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \left(\frac{4n^3 - 7}{n - 5n^3} \right)$. We show that $\lim_{n \to \infty} a_n \neq 0$.

$$\lim_{n \to \infty} \left(\frac{4n^3 - 7}{n - 5n^3} \right) = \lim_{n \to \infty} \left(\frac{n^3 \left(4 - 7/n^3 \right)}{n^3 \left(1/n^2 - 5 \right)} \right) = \lim_{n \to \infty} \left(\frac{\left(4 - 7/n^3 \right)}{\left(1/n^2 - 5 \right)} \right) = -\frac{4}{5} \neq 0 .$$

"Since $\lim_{n \to \infty} \left(\frac{4n^3 - 7}{n - 5n^3} \right) \neq 0$, ,

the series $\sum_{n=1}^{\infty} \left(\frac{4n^3 - 7}{n - 5n^3} \right)$ is DIVERGENT,

by the TEST FOR DIVERGENCE."

} **Suggested Wording of the REQUIRED JUSTIFICATION**

Note: If $\lim_{n \to \infty} a_n = 0$,

then the series $\sum_{n=1}^{\infty} a_n$ might be convergent or this series $\sum_{n=1}^{\infty} a_n$ might be divergent.