EXPLAINING THE

ALTERNATING SERIES LEVEL-OF-ERROR FORMULA

$|s - s_n| \leq b_{n+1}$ (NOTE: $|s-s_n|$ = The ERROR IN $s_n \approx s$)

GIVEN: $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n+1} b_n$ is an alternating series with $b_n = |a_n|$ for all $n \geq 1$.

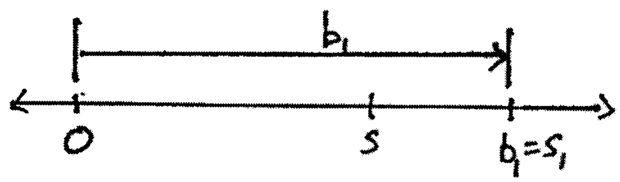
$\sum_{n=1}^{\infty} a_n$ is convergent and $b_1 > b_2 > b_3 > b_4 > \cdots$

$s = \sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n+1} b_n$ is the sum of the series.

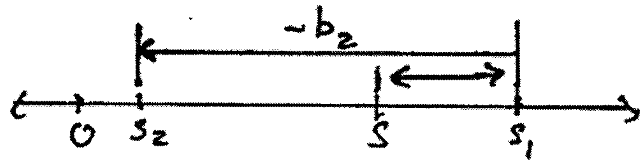
$s_1, s_2, s_3, s_4, \ldots, s_n, \ldots$ is the sequence of partial sums of the series.

$$s = \sum_{n=1}^{\infty} (-1)^{n+1} b_n = \overbrace{b_1 - b_2 + b_3}^{s_3} \underbrace{- b_4}_{} + b_5 - b_6 + b_7 - \cdots$$

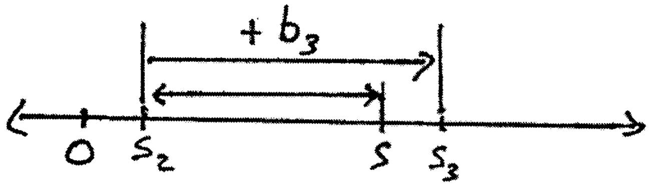
$$\underbrace{b_1 - b_2 + b_3 - b_4}_{s_4}$$



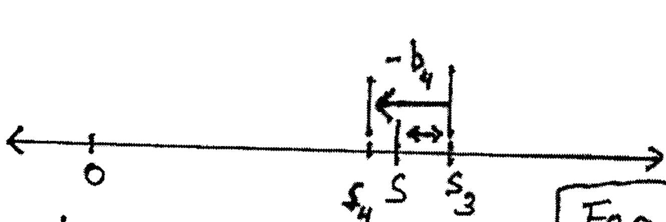
Note: $s_2 < s < s_3$ and $s_4 < s < s_3$



$|s - s_1| \leq b_2$ { The error in $s_1 \approx s$ is $|s - s_1|$



$|s - s_2| \leq b_3$ { The error in $s_2 \approx s$ is $|s - s_2|$



$|s - s_3| \leq b_4$ { The error in $s_3 \approx s$ is $|s - s_3|$.

etc. . . .

$\boxed{\text{For all } n \geq 1, \ |s - s_n| \leq b_{n+1}}$