THE LEVEL OF ERROR in $S_n \approx S$

FOR CONVERGENT ALTERNATING SERIES

(When the ALTERNATING SERIES TEST APPLIES)

Suppose $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} (-1)^k b_k$ ($b_k = |a_k|$) is Convergent

and $\sum_{k=1}^{\infty} a_k = S$. Then $\left(\begin{matrix}\text{The ERROR} \\ \text{in } S_n \approx S\end{matrix}\right) \leq |a_{n+1}| = b_{n+1}$ for all $n \geq 1$.

THUS,

If $S = \sum_{k=1}^{\infty} a_k = \underbrace{a_1 + a_2 + a_3 + \cdots + a_n}_{= S_n} + a_{n+1} + a_{n+2} + \cdots$

THEN $\left(\begin{matrix}\text{The ERROR} \\ \text{in } S_n \approx S\end{matrix}\right) \leq |a_{n+1}| = b_{n+1}$

Problem: The series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ is Convergent with sum S.

Determine the Level of ERROR in using S_{19} to approximate the sum S.

Solution: Since the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ is a Convergent alternating series (for which the Alternating Series Test applies),

$\left(\begin{matrix}\text{The ERROR} \\ \text{in } S_{19} \approx S\end{matrix}\right) \leq |a_{20}| = b_{20} = \frac{1}{20} = 0.05 = \text{LEVEL OF ERROR}.$