THE COMPARISON TEST AND THE LIMIT COMPARISON TEST

BOTH TESTS ARE TESTS FOR CONVERGENCE OR DIVERGENCE OF SERIES WITH ALL TERMS POSITIVE, $\sum_{n=1}^{\infty} a_n$ where $a_n > 0$ for all n.

BOTH TESTS INVOLVE A SECOND SERIES $\sum_{n=1}^{\infty} b_n$ for which the CONVERGENCE OR DIVERGENCE STATUS IS ALREADY KNOWN! [It may be that the roles of $\sum a_n$ and $\sum b_n$ are reversed!]

THE COMPARISON TEST

Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ have all terms positive.

Suppose N is some positive integer.

① If $a_n \leq b_n$ for all $n \geq N$,
and if $\sum_{n=1}^{\infty} b_n$ is convergent,
then $\sum_{n=1}^{\infty} a_n$ is convergent, too.

> So, a series dominated by a convergent series is a convergent series.

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② If $a_n \leq b_n$ for all $n \geq N$,
and if $\sum_{n=1}^{\infty} a_n$ is divergent,
then $\sum_{n=1}^{\infty} b_n$ is divergent, too.

> So, a series which dominates a divergent series is a divergent series.

The Limit Comparison Test

Suppose that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms.

If $\lim_{n \to \infty} \left(\frac{a_n}{b_n} \right) = C$ AND

C is a finite non-zero number,

then either both series converge

OR

both series diverge.

If $C = 0$ or $C = \infty$, then the test fails and some other test must be used to identify convergence or divergence.

THE LAST SENTENCE ABOVE SHOWS HOW THE LIMIT COMPARISON TEST CAN FAIL.

THE COMPARISON TEST CAN FAIL TOO!

Suppose $a_n \leq b_n$ for all $n \geq N$,

Then if $\sum_{n=1}^{\infty} a_n$ is convergent OR

if $\sum_{n=1}^{\infty} b_n$ is divergent, Then The test fails!