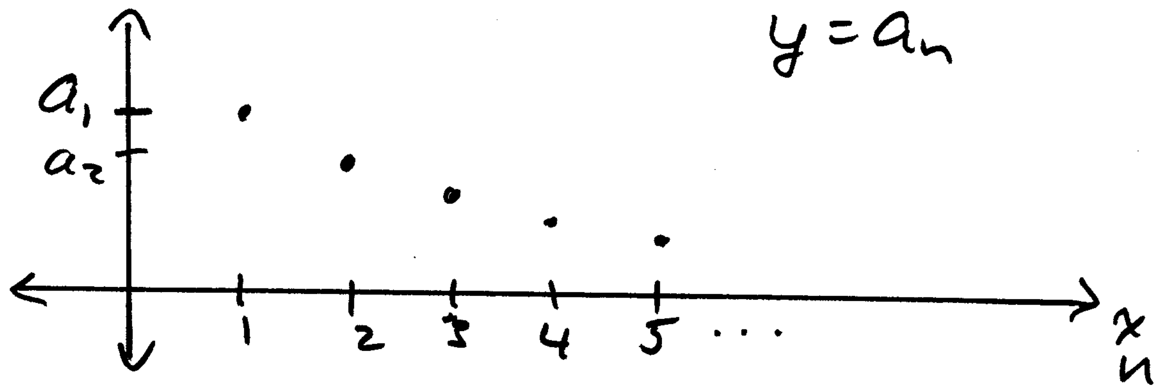
WHY THE INTEGRAL TEST WORKS
(when it applies)

Given $\sum_{n=1}^{\infty} a_n$ and function $f(x)$ is such that $f(n) = a_n$ for $n = 1, 2, 3, \ldots$

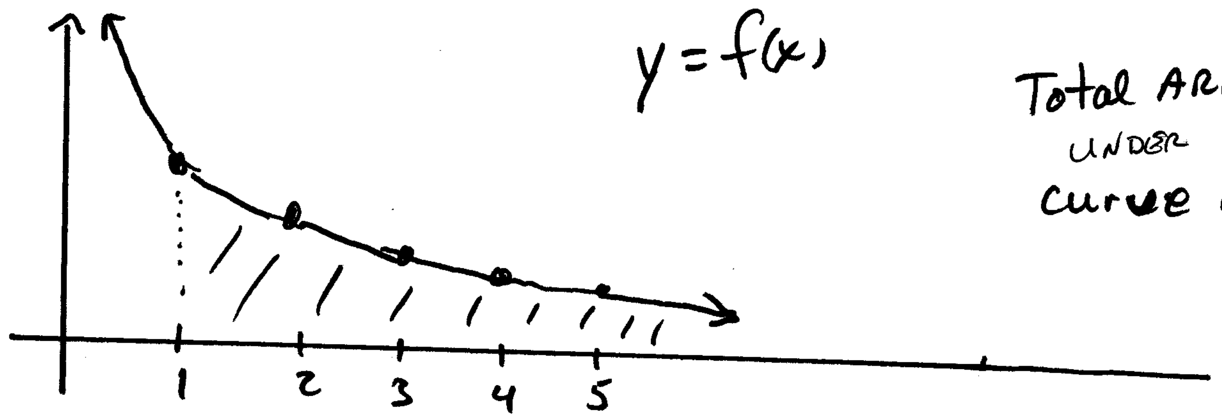
and such that the Integral Test Applies

[Conditions ①, ②, ③, ④ are met]:

The graph of the underlying sequence $\{a_n\}$



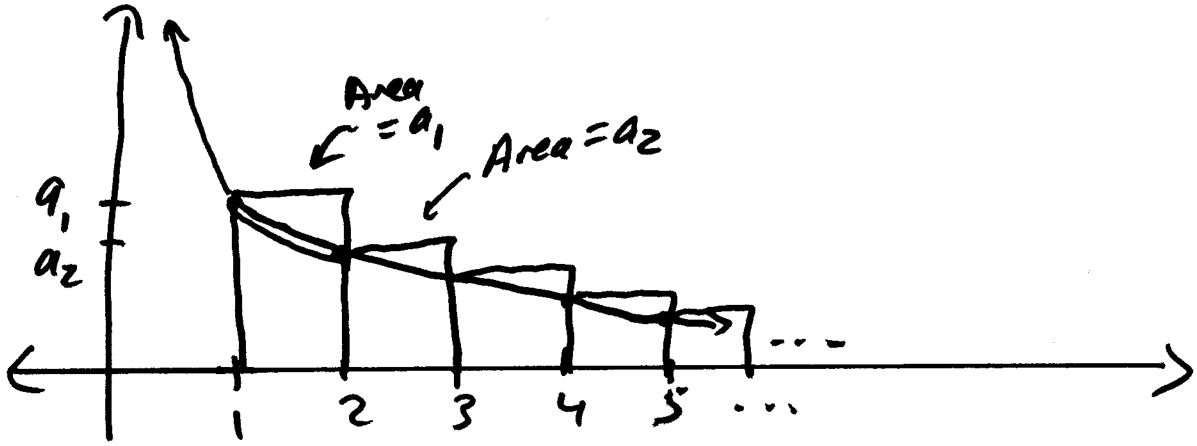
The graph of $y = f(x)$



Total AREA under the curve is $\int_1^{\infty} f(x)\,dx$.

If $\int_1^\infty f(x)\,dx$ Diverges $(=\infty)$, then $\sum_{n=1}^\infty a_n$ Diverges

Because $\sum_{n=1}^\infty a_n$ Dominates $\int_1^\infty f(x)\,dx$.



If $\int_1^\infty f(x)\,dx$ Converges $(<\infty)$, then $\sum_{n=1}^\infty a_n$ Converges

Because $\sum_{n=2}^\infty a_n$ is Dominated by $\int_1^\infty f(x)\,dx$.

!!! $\rightarrow$