

**Lee:** You have begun a mentoring program in Institute for Advanced Study(IAS) in Princeton for women mathematicians in 1994. Could you say something about the motivation, environment and current status?

**Uhlenbeck:** Women were not an issue when I was a student. Nobody was interested in questions about women. Later on Association for Women in Mathematics (AWM) was formed and there was a lot of discussion about woman. There were many programs which had political slants which were not helpful for working mathematicians. It was clear that if you wanted to be a mathematician, you paid attention to the mathematics. I think everybody in my generation and Chu-Lien's generation did believe that the situation was getting better. Hence we didn't do anything to help ourselves. There certainly was legal discrimination against women when I first found a job, but then the law was changed. Most mathematicians believed that, once the legal barriers were removed, women and minorities would walk through the gates in great numbers and whatever problem there was would be solved.

However, in early 80's and early 90's, it just wasn't happening. I mean women were actually having a difficult time. Chu-Lien Terng and I, as well as others like Jean Taylor and noticed that the numbers of women had stopped increasing. We hadn't had anything to do with programs since we hadn't thought it would be necessary. When we realized it was necessary, many of us became involved. We tried to think of what would be good for women. The Woman's program at IAS in Princeton started as part of the Park City Mathematics Institute. I started this with a group of mathematicians in response to a prospect put out by the National Scientific Foundation. I thought it would be good for women as well as good for mathematics. We would all go and meet other women in the field. The Park City Summer School was started in 1991. Then after few years, we noticed there were not many women coming to Park City, so the Park City Women's program started to encourage more women to apply. Chu-Lien Terng and I ran the program together, which was fun for both of us as we could also work together on mathematics. It was a 10-day program, and now it is two-week. The program is now not part of Park City anymore. It is funded by IAS and Princeton University as well as the NSF and private foundations. We give opportunities for women at all levels, from undergraduates to research mathematicians. I think it is a very good program.

**Lee:** Do you think the situation is now improving?

**Uhlenbeck:** I think things have been improved. I still meet female mathematicians

who struggle a lot. We struggle a lot to fit into math departments in a subtle way. I don't think there is any overt discrimination, but I think there are still difficulties. It is very difficult to raise a family and have a professional career. I think it is difficult for both for men and women, but it is more difficult for women. Many professions are hard for woman. I know many women who play classical music. It is a hard life too. A career in mathematics is not a particular exception.

In the past, when mathematicians were young, they had enough money. Childcare is a big issue and is very expensive now. When I was young, childcare was not so expensive and our income was relatively higher. Two assistant professors in a family had plenty of money to hire childcare and afford medical care. Now in the U.S., although not so much Europe, the young academic women struggle with not enough money.

**Lee:** You have some special topic every year in Park City. The topics change a lot among science and not particularly in math because the diverse backgrounds of professors, speaker, and students join the program.

**Uhlenbeck:** It started as a program in geometry. We moved around and now we change the topic every year. We try to have a topic connected with Princeton and IAS research. We pick the topic in which we have good woman speakers.

I also have a bias towards applied programs, because the opportunities in applied math are much greater. It is simply a fact that more jobs are available for mathematicians in applied areas. I am not the only person who has noticed that women are expected to be more social. Women are supposed to be able to talk to people and to be good at interacting with people. This should make women better in applied math and in talking to scientists. I hope this gives an advantage to women in applied mathematics.

**Liu:** Scientifically, applied math has much more possibilities.

**Uhlenbeck:** Yes. It is weird that it is not crowded in applied math. Pure math is very crowded. When I studied in geometry, there were only few of us. I was very excited. Now I find it is very crowded. Lots of people are studying in geometry and there are too many papers to read.

**Liu:** When you said it was not crowded, actually you feel lonely. However, now you

feel more excited.

**Uhlenbeck:** I don't know. It is not so exciting. I used to know all the women in math.

**Liu:** But you are one of the main people to make it exciting.

**Uhlenbeck:** Thanks. When I look back my career, I really think it was exciting. I did all sorts of different things. I studied geodesics in a Lorentz manifold, hard estimates for regularity of PDE, then minimal surfaces and gauge theory. I tried mathematical conformal field theory, but I think math physics is too hard. This led to my interest in integrable systems. Now I also try nonlinear waves, but it is getting harder. Nonlinear waves are very hard. But all in all, it is exciting.

**Lee:** What leads to these changes?

**Uhlenbeck:** I try to look around and see what I am interested in and what I can understand at least a little about. I have just done what I am interested in. In the beginning of my career, I didn't really work with women much. Now I have some coauthors, Andrea Nahmod whom I work with on nonlinear waves, and Chu-Lien Terng whom I work with on integrable systems.

**Liu:** Perhaps the key word is interest. Different people like different things. Can you say what is interesting for you?

**Uhlenbeck:** I guess it is the inner connection between different kinds of ideas in math. I mean the connection between different mathematics. In minimal surfaces, you have topology and algebraic aspects, or hard analysis, where I spent most of my time. They all mix together. Gauge theory is particularly like that. It has every thing. It has algebraic topology, hard analysis, deep topology. I find it is very interesting. In fact, I always wanted to understand integrable systems. It took years before I really understood the relationship between the algebra and the analysis. Now I sort of understand it. However, I am not sure how to get much deeper into the algebraic aspects. In nonlinear waves, my feeling is that right machinery is not yet developed. I hope mathematicians get good ideas to further our understanding. For example, an important problem is to understand singularities. We don't really have the correct way to describe or imagine what the singularities are. I am talking to an expert here. I think it is a field that is wide open. But I am 63 years old and I don't know if I have many ideas. I feel that it is a field that has potential.

**Li:** You are pointing out the direction of the main issue. It is a very important area. You have done lots of the bubble phenomenon. That is you meant by what kinds of singularities.

**Uhlenbeck:** In fact, I remember when I was thinking about it. I talked to people who had been classically working on minimal surfaces. To solve the bubbling, you look very closely at the Riemann mapping problem. Also something else that appears in nonlinear waves is different scales. This is not solved at all. You can imagine nonlinear waves at different scales, but we do not have the tool to match it very well. You do something in a large scale, but how this interacts with the small scale of model singularities very difficult. It is difficult to do the numerical also. I was very naive when I went to study this. I thought, "well, when you learn a little bit of computer analysis and geometric insight, hard technical PDE should give you something". I discovered that computer analysis needs ideas even more than pure mathematics. If you just stick something in the computer, it doesn't necessarily give you anything useful. This was a revelation to me. I had wrong idea about what people did in the numerical arena. Actually, you need to have ideas.

**Liu:** A former colleague of mine, Babuska, said, "even if you find 90% in your computation is correct, the question is to find out which 90% is correct." Computation is a field that many undergraduates or graduates in Taiwan don't sufficiently appreciate.

**Uhlenbeck:** If you learn computation, especially modeling, one of the problems is how to take a problem and actually formulate it and then do computation. If you do that, you are very much interdisciplinary. Those people are actually lost to the mathematics community. It happens in biology as well. Mathematicians that study biology are in great demand by biology departments. They tend to disappear from the sight of pure mathematicians. I think it is not good that we lose these mathematicians from math departments due the opportunity and money in biology and computation.

**Liu:** Many mathematics departments try to keep it pure.

**Uhlenbeck:** Actually that causes much more problems. Mathematics departments used to try to keep their mathematics pure, but I don't know any pure mathematics department is doing that nowadays. Maybe there are more problems in Taiwan than in the U.S. We actually know we have to keep contact with applied mathematics. It is

really in some sense a great opportunity for our students.

**Huang:** We have a meeting for female in science every week in Stanford. A woman from biology who studied math in Stanford mentioned that many of her friends went to have different jobs, like NASA, JPMorgan, industries, and so on, after they got their Ph.D. degree in math. Probably there are more job opportunities in the U.S. for mathematicians who quit math.

**Uhlenbeck:** Actually they do not quit math. There are these great opportunities because people want mathematicians. They do not necessarily want the subjects that mathematicians study, but they want the kind of thinking that mathematicians have. People don't waste money, so they have found by experience that mathematical thinking is quite useful.

They are not doing pure math research, but pure math research is a just a small part of the ambient whole. It is not our only job to produce pure mathematics. Many mathematicians think that way. But in fact, some of my best students are completely doing different things. In fact, some of my best students are teaching. I now have two female students and one male student. The male student is sure that he wants to teach. He wants to do a little bit research but actually more teaching. I encourage that. In fact, I discourage pure math students who don't like teaching. If you don't like teaching, being a pure mathematician in a university would be very unexciting, because your job is to interact with undergraduates, graduates, and post-docs. There are of course pure mathematicians who do nothing but only research, but they are very unusual.

**Liu:** Actually mathematicians are not productive if they don't teach.

**Uhlenbeck:** I have to say, when you become older, you need to sort of manage your time more careful, especially if you are women. It is easy to run out of time just doing teaching and research.

**Lee:** I don't have enough time.

**Uhlenbeck:** I know you don't have enough time, but you have enough energy. I don't have energy to do all things I want anymore. It is better to have a good relation with many other departments. Encourage your students to take courses in the computer science, biology and so on. Computer science particularly offers lots of courses that

pure mathematicians who are doing things like number theory and algebra might take.

**Liu:** When you recall your earlier career, there were so many exciting things and people around you. Can you say something about them?

**Uhlenbeck:** I remember it very well the year in IAS, 1979 or 1980. S.-T. Yau organized the seminar. When I think back, I was one of the older people there. I was 38 and Yau is 5 years younger. Leon Simon was nearly my age as was Jean Pierre Bourguignon. Some other young kids, like Peter Li and Robert Bryant had just got their Ph.D's. It was my first time to spend so much time with other mathematicians. It was really very exciting. I was very isolated before then, but this is the year I started to work with Rick Schoen. I still list S.T. Yau as one of my favorite people. He was like a general marshalling his forces and taking his troops into mathematical conquest. He is a very good general because he has had fantastically good ideas and has led many mathematicians. I credit him on giving me my real start in the mathematical community. He is still giving students that start. That was a very exciting time.

**Liu:** Before 1979, you were already very accomplished.

**Uhlenbeck:** Yes, but I did all by myself. I had couple students whom I cooperated with, like Jonathan Sacks who was my post-doctor in University of Illinois at Urbana-Champaign. I worked a little bit with Lesley Sibney and Robert Sibner.

I have a story that I need to tell you. I still remember in the 1971 or 1972 summer I met Lesley Sibner in Trieste. I was in the audience. This woman got up dressed in a purple leather pant suit and gave a lecture. She lectured on nonlinear Hodge theory which was very much in the style and the way I was thinking. It was an amazing experience for me. I always remember that when I met her, I was very excited. She was a sophisticated New Yorker and I thought myself as a country girl. We actually become quite good friends.

I was a student for one year in New York University before I got married and moved to Brandeis where I got my Ph.D. I had Cathleen Morawetz for second semester complex variables. I was not very impressed by her at that time. She looked tired and she worked hard at lecturing. Many years later, when I was more or less the age she had at that time, I looked back and thought I could succeed if she could succeed. At that time, she had four teenage children. She was lecturing, raising 4 children and succeeding. I always think that the role model is very important. It is important that

you have pictures of all different types of people that you admire. Cathleen still comes to the Courant Institute everyday. I had a nice talk with her last time I went to Courant.

**Liu:** Most of the work was done by yourself before 1979. How do you find problems?

**Uhlenbeck:** I knew the problems because I worked my thesis on the calculus of variations. I knew what you had to do to get the existence. I knew about all the problems to get regularity. These were all very classical. The modern theory didn't really help you solve classical problems. I worked my thesis in global analysis and global analysis was the first of all the grand theories which did nonlinear analysis on manifolds. Basically, what mathematicians do now when they go to nonlinear problems in geometry is global analysis. The problem is that global analysis didn't solve any of the classic problems. It was just big machinery. It is actually very hard for students to learn. You know what it is like. It is like the nonlinear heat flow in geometry. There is no paper for the nonlinear heat flow used in geometry. It is a classical theory, but is not very interesting. It is very standard and isn't written anywhere. People still refer to Eells and Sampson's paper in 1967 for analysis on manifolds. My thesis advisor Palais taught me the basic analysis on manifolds, but there is no place to go from there. Therefore people who got Ph.D. in global analysis all do different things. I knew about the problems and the abstract set-up. I just didn't know how to solve the classical problems. I started a little bit by working on geodesics, minimal surfaces and regularity theory.

In fact my hardest paper is a paper in 1977 called "regularity for a class of non-linear elliptic systems". It is my hardest paper. I have a story about that. I worked on the paper and I sweated blood. I had met Jurgen Moser and I read his papers. Moser was very nice to me. He was very friendly and helpful. I read what they called Moser iteration, but I never talked to anybody about that. At some point, in the middle 70's, I don't remember why I went to Stanford. Somehow I remember I came into S.T. Yau's office and Rick Schoen was there. They asked me about this paper. I was completely surprised. I thought no one had ever paid any attention to it. No one had ever asked me about it. I was just amazed that someone knew about it. The paper was motivated by the question of regularity for  $p$ -harmonic mappings.

**Liu:** In some sense, when you said that you knew the problems, it means the problems you were facing and you turned the crisis into success.

**Uhlenbeck:** Yes. In fact, I still remember Guisti regularity theory. It just came out when I was a graduate student. Palais was a very good thesis advisor. He was that kind of person that I went to ask about heat equations and he gave me one hour lecture what everything I needed to know about heat equations for the next 15 years. (I only recently needed to know more about the heat equations.) In any case, he gave me a couple of papers by Guisti and one of his students. I find that to read a paper really takes months. It is not just something you go and read. You need to really have you time and think about it. I don't read many papers but I learn a lot from every paper I read. I learned an awful lot form that those papers. But now I know less well what paper to read, it is not so clear.

**Liu:** It is true that everybody knows how good Uhlenbeck is, but it is also important to enter a field at a right moment.

**Uhlenbeck:** It is very important. In fact, right now I don't know good advice to give to young people about which fields to study. I feel less certain. I look and see what kinds of things are going on in geometry. It is very algebraic now with a big overlap with algebraic geometry. It is algebraic geometry. In fact that is why I don't continue work in gauge theory and the geometry of physics any more. The geometry in physics is very algebraic. In fact, I read a little about it, but I am too old to learn it. That is clear to me because I understand the same problem in integrable systems. There is a gap between hard analysis and things we call loop group actions. To formulate it in a proper way analytically works only when you can do the scattering theory. If you can't do scattering theory, you need to do the very algebraic part.

**Huang:** How about the general relativity?

**Uhlenbeck:** I tell you my story about the general relativity. When I got my Ph.D. from Palais in Brandise, he sent me to UC Berkeley to learn the general relativity. I have two quarters on special and general relativity and quantum field theory from Abe Taub and one quarter from Ray Sacks. Taub was a wonderful person. I was an instructor in UC Berkeley then, but you should always sit in courses even after you get your Ph.D. Many mathematicians were sitting there. There was a young assistant professor who worked on compact groups. The graduate students were actually somewhat discouraged by us. It was during the Vietnam war. There was demonstration. Classes were cancelled. It was not talked about too much, but it was a very exciting time although student activities were not mathematical. In any case, I discovered that to do general relativity you have to put the Navier-Stokes equation on



the right hand side for cosmology. I decided it was too hard. The problem is to understand cosmological general relativity you need to understand Navier-Stokes equation and the fluid dynamical systems.

I worked on positive mass theorem when I was an undergraduate student. My thesis advisor told me about that. There was a way to go about the problem using the calculus of variations. I don't remember it very well because it didn't go to anywhere. At UC Berkeley I wrote a paper on geodesics on Lorentz manifold. I actually studied quite a bit about complete geodesics and things like that. In fact in my second job at the University of Illinois in Champaign-Urbana, I spent time in meeting the papers of Penrose and Hawking. This was before the field was popular. I was one of the only people studying Lorentz geometry at that time. Later when I became involved with minimal surfaces, I found these harder harder. Several years later I met other mathematicians who had read these papers, but I was never inspired to take it up again.

**Liu:** You just follow your intuition and do whatever you want to do.

**Uhlenbeck:** As I told you, probably the change to do nonlinear wave equations was not particularly a good part. The motivation was good. I didn't know enough about it. It took me a long time to basic material. I still don't really know the papers to read. There are quite few papers and there are many difficult papers. The most brilliant person in this field is Terry Tao who I don't know very well. He does many different things and he is very young. We will see where he can go. He is a Chinese-Australian. He does number theory, harmonic analysis, PDE. If I pick a young person whose work I understand anything about, I think you should watch him. He is a really brilliant young man. Probably there are very brilliant people in algebraic geometry, but I mean people who do mathematics in anything I understand.

**Liu:** It is nice to know that in every generation there are brilliant people. It is also nice to know there are still possibilities for those young people.

**Uhlenbeck:** It is right. I am thinking that. It is a little disappointing that there are few young women who are particularly brilliant. However Mellisa Liu, for example, was a student of Yau and has done lot of things in general relativity, Yang-Mills, quantum cohomology, mirror symmetry. She is younger than Terry Tao and is doing very well. Probably she feels more pressure because she is a young Taiwanese female mathematician. Francis Kirwan who was a student of Atiyah and pretty senior is also

very famous. She got a lectureship in a British system which was good for her. She got a lectureship. She raised her children and didn't need to worry about the tenure problem. In fact, the British system has some advantages because the job you got is permanent. There are certainly other bright young women coming along.

**Lee:** I am also told that you had a very strong intuition to know where to go in research. Is it from your person characteristic?

**Uhlenbeck:** You need to spend lots of time thinking about the problem. When I was young I had a great difficulty in sleeping, so I spent lots of time thinking about mathematical problems instead of sleeping. It is very difficult to build up intuition. Somehow you need to spend time with the problems and I am not sure how to teach that.

**Liu:** Probably we could go through your childhood.

**Uhlenbeck:** My parents both went to colleges and they were the first generation of people in their families to go to college. They graduated from college 1932 which was in the Great Depression. My parents were as a consequence very conservative about money. My father was an engineer and my mother was an artist who taught art in a high school. They had a very difficult time. My father got his first job in the mines in Utah with a bachelor's degree from MIT. My parents didn't tell me much but I know there was family help for my father to pay for MIT. My mother studied in an art school, but she realized that she could not get paid work as an artist and so switched to a teacher in high school. I was influenced quite a lot by my parents. Parents of many other people in my age had real difficulties at the Great Depression. Life was actually very hard at that time.

I was born in 1942 in Cleveland, Ohio when my parents had enough money to have children as the Great Depression was finally over.. My parents are from New Jersey and New York. In 1945 after World War II we moved back to New Jersey and I grew up there. I was fortunate. I was given background not so in mathematics, but in art, literature and music through my family. Those things are particularly valuable for me as I am in my 60's. I am not so excited and involved in mathematics, but I have other things to do in my life.