Math 408C

Practice Exam 6
(ONLY NEW MATERIAL)

Name: ______________________

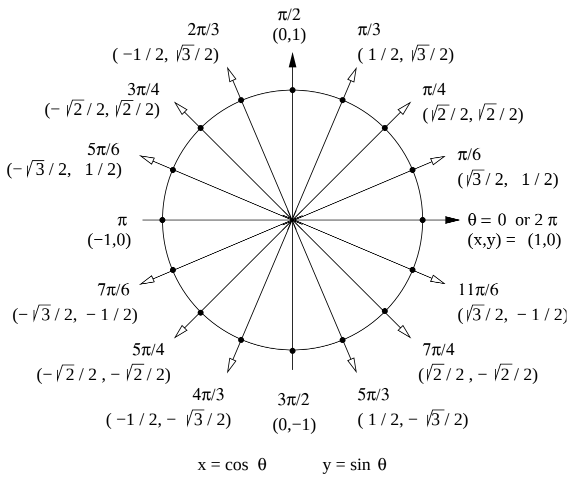
Tuesday Class Time (Circle 1): Morning Afternoon

Directions: Indicate the correct answer by filling in the appropriate space as shown.

1. Select the choice B.

(A) (●) (C) (D) (E)

Make sure that your choice is clearly indicated. If you mark more than one answer, no answer, or an incorrect answer no credit will be given. All problems are equally weighted.



$\tan\theta = \sin\theta/\cos\theta$, $\cot\theta = \cos\theta/\sin\theta$,

$\sec\theta = 1/\cos\theta$, $\csc\theta = 1/\sin\theta$, $\cot\theta = 1/\tan\theta$,

$\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, $1 + \cot^2\theta = \csc^2\theta$,

$$\sin(A \pm B) = \sin A\,\cos B \pm \cos A\,\sin B,$$

$$\cos(A \pm B) = \cos A\,\cos B \mp \sin A\,\sin B,$$

$$\sqrt{2} \doteq 1.414, \quad \sqrt{3} \doteq 1.732, \quad \pi \doteq 3.142$$

$$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$\ln(pq) = \ln(p) + \ln(q)$	$\ln(p/q) = \ln(p) - \ln(q)$	$\ln(p^r) = r\ln(p)$	$\ln(e) = 1$	$\ln(1) = 0$
$e^p e^q = e^{p+q}$	$e^p/e^q = e^{p-q}$	$(e^p)^r = e^{rp}$	$e^1 = e$	$e^0 = 1$
$\ln(e^p) = p$	$e^{\ln p} = p$	$\log_a x = (\ln x)/(\ln a)$	$a^x = e^{x\ln a}$	$e \doteq 2.718$

$\sin(\sin^{-1}x) = x$	$\cos(\cos^{-1}x) = x$	$\tan(\tan^{-1}x) = x$	$\csc(\csc^{-1}x) = x$	$\sec(\sec^{-1}x) = x$	$\cot(\cot^{-1}x) = x$
$\sin^{-1}(\sin\theta) = \theta$	$\cos^{-1}(\cos\theta) = \theta$	$\tan^{-1}(\tan\theta) = \theta$	$\csc^{-1}(\csc\theta) = \theta$	$\sec^{-1}(\sec\theta) = \theta$	$\cot^{-1}(\cot\theta) = \theta$

$\lim[cf] = c\lim f$ $\quad$ $\lim[f \pm g] = \lim f \pm \lim g$ $\quad$ $\lim[fg] = \lim f \lim g$ $\quad$ $\lim\left[\frac{f}{g}\right] = \frac{\lim f}{\lim g}$

$[\sin x]' = \cos x$	$[\cos x]' = -\sin x$	$[\tan x]' = \sec^2 x$	$[\csc x]' = -\csc x\cot x$	$[\sec x]' = \sec x\tan x$	$[\cot x]' = -\csc^2 x$
$[c]' = 0$	$[x^p]' = px^{p-1}$	$[e^x]' = e^x$	$[cf]' = cf'$	$[f \pm g]' = f' \pm g'$	$[fg]' = f'g + fg'$

$$\left[\frac{f}{g}\right]' = \frac{gf' - fg'}{g^2} \qquad [g^p]' = pg^{p-1}\,g' \qquad [f(g)]' = f'(g)\,g'$$

$[\sin^{-1}x]' = \frac{1}{\sqrt{1-x^2}}$	$[\cos^{-1}x]' = \frac{-1}{\sqrt{1-x^2}}$	$[\tan^{-1}x]' = \frac{1}{1+x^2}$	$[\csc^{-1}x]' = \frac{-1}{x\sqrt{x^2-1}}$	$[\sec^{-1}x]' = \frac{1}{x\sqrt{x^2-1}}$	$[\cot^{-1}x]' = \frac{-1}{1+x^2}$
	$[\ln x]' = \frac{1}{x}$	$[\ln\lvert x\rvert]' = \frac{1}{x}$	$[\log_a x]' = \frac{1}{(\ln a)\,x}$	$[a^x]' = (\ln a)\,a^x$	

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{\sinh x}{\cosh x} \qquad \operatorname{csch} x = \frac{1}{\sinh x} \qquad \operatorname{sech} x = \frac{1}{\cosh x} \qquad \coth x = \frac{1}{\tanh x}$$

$$\sinh(-x) = -\sinh(x) \qquad \cosh(-x) = \cosh(x) \qquad \cosh^2 x - \sinh^2 x = 1 \qquad 1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\sinh(A \pm B) = \sinh A \cosh B \pm \cosh A \sinh B \qquad \cosh(A \pm B) = \cosh A \cosh B \pm \sinh A \sinh B$$

$$[\sinh x]' = \cosh x \qquad [\cosh x]' = \sinh x \qquad [\tanh x]' = \operatorname{sech}^2 x$$

$$[\operatorname{csch} x]' = -\operatorname{csch} x \coth x \qquad [\operatorname{sech} x]' = -\operatorname{sech} x \tanh x \qquad [\coth x]' = -\operatorname{csch}^2 x$$

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right) \qquad \cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right) \qquad \tanh^{-1} x = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$$[\sinh^{-1} x]' = \tfrac{1}{\sqrt{x^2+1}} \qquad [\cosh^{-1} x]' = \tfrac{1}{\sqrt{x^2-1}} \qquad [\tanh^{-1} x]' = \tfrac{1}{1-x^2}$$

$$\int_a^b f(x)\,dx = F(b) - F(a), \quad F'(x) = f(x). \qquad \frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x). \qquad \frac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] = f(g(x))\,g'(x).$$

$$\int_a^b 0\,dx = 0. \qquad \int_a^b 1\,dx = b - a. \qquad \int_a^a f(x)\,dx = 0. \qquad \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx. \qquad \int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx.$$

$$\int_a^b f(x) + g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx. \qquad \int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx.$$

If $f(x) \geq g(x)$ on $[a,b]$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$. If $m \leq f(x) \leq M$ on $[a,b]$, then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$.

$$\sum_{i=1}^n c = nc. \qquad \sum_{i=1}^n ic = \frac{n(n+1)c}{2}. \qquad \sum_{i=1}^n i^2 c = \frac{n(n+1)(2n+1)c}{6}. \qquad \sum_{i=1}^n i^3 c = \frac{n^2(n+1)^2 c}{4}.$$

$$\int_{x=a}^{x=b} f(g(x))\,g'(x)\,dx = \int_{u=g(a)}^{u=g(b)} f(u)\,du, \quad u = g(x).$$

$$\text{Area}(\Omega) = \int_a^b [h(x) - g(x)]\,dx. \qquad \text{Vol}(S) = \int_a^b A(x)\,dx \quad \text{(general)}.$$

$$\text{Work} = \int_a^b f(x)\,dx. \qquad \text{Hooke's Law: } f(x) = kx.$$

$$f_{\text{avg}} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

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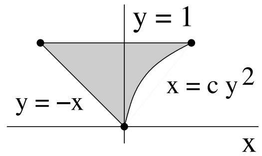
1. If Ω is the shaded region, for what value of the constant $c > 0$ would we have Area$(\Omega) = 4$?



   (A) 17/3.
   (B) 23/2.
   (C) 13/3.
   (D) 21/2.
   (E) 19/3.

2. Find the area between $y = \cos(x)$ and $y = \cos(2x)$ when $x \in [0, \pi]$
   (A) $\frac{3\sqrt{2}}{4}$.
   (B) $\frac{3\sqrt{3}}{8}$.
   (C) $\frac{3\sqrt{3}}{2}$.
   (D) $\frac{3\sqrt{2}}{8}$.
   (E) $\frac{4\sqrt{3}}{9}$.

3. Let Ω be the region in the first quadrant bounded by $y = 2 - x$, $y = x^2$ and $x = 0$. Then the volume of the solid S generated by rotating Ω around the x-axis is:
   (A) $\frac{16\pi}{11}$.
   (B) $\frac{23\pi}{7}$.
   (C) $\frac{32\pi}{15}$.
   (D) $\frac{26\pi}{9}$.
   (E) $\frac{16\pi}{5}$.

4. The region bounded by $x = 1 + y^2$, $x = 0$, $y = 0$ and $y = 1$ is rotated about the axis $y = -2$. The volume of the resulting solid is:
   (A) $\frac{37\pi}{6}$.
   (B) $\frac{32\pi}{3}$.
   (C) $\frac{28\pi}{3}$.
   (D) $\frac{26\pi}{3}$.
   (E) $\frac{41\pi}{6}$.

5. A force of 30N is required to maintain a string stretched from its natural length of 0.12m to 0.15m. How much work is done stretching the spring from 0.12m to 0.20m?

   (A) 2.6 J.

   (B) 3.2 J.

   (C) 4.6 J.

   (D) 3.4 J.

   (E) 2.4 J.

6. If the temperature in a freezer over a two-hour period is $T(t) = \frac{80}{2t+3}$ °F where $2 \leq t \leq 4$, then the average temperature over this period is:

   (A) $20 \ln \dfrac{11}{7}$ °F.

   (B) $40 \ln \dfrac{12}{5}$ °F.

   (C) $80 \ln \dfrac{21}{4}$ °F.

   (D) $60 \ln \dfrac{9}{2}$ °F.

   (E) $80 \ln \dfrac{16}{5}$ °F.