

## Algebra Prelim part A

August 2015

*Directions:* You have 90 minutes. Solve two of the three problems. Clearly mark which ones you want graded.

- A1.** Suppose  $G$  is a group acting transitively on a set  $X$  (which may be infinite) and that  $H$  is a finite index subgroup of  $G$ . For  $x \in X$  write  $H_x$  and  $G_x$  for its stabilizers in  $H$  and  $G$ .
- (a) Show that  $H$  has finitely many orbits on  $X$ .
  - (b) Show that if  $H$  is transitive, with  $H_x = G_x$ , then  $H$  is all of  $G$ .
  - (c) Show that if  $H$  is normal, then  $[G_x : H_x]$  is independent of  $x$ .
- A2.** Suppose  $G$  is a finite group, such that for every nontrivial proper subgroup is cyclic of prime order. Show that  $G$  is solvable.
- A3.** Determine the maximal ideals of the rings
- (a)  $\mathbf{R} \times \mathbf{R}$
  - (b)  $\mathbf{R}[x]/(x^2)$
  - (c)  $\mathbf{R}[x]/(x^2 - 3x + 2)$
  - (d)  $\mathbf{R}[x]/(x^2 + x + 1)$