Algebra Prelim part B

January 9, 2017

Directions: You have 90 minutes.

B1. For each prime power p^n, there exists exactly one field of that order, up to isomorphism.

B2. Let $\mathbf{F}$ be a field.

(a) Show that the fraction field of the power series ring $\mathbf{F}[[x]]$ is the Laurent series ring

$$\mathbf{F}((x)) = \left\{ \sum_{i=N}^{\infty} a_i x^i : N \in \mathbf{Z}, a_i \in \mathbf{F} \right\}$$

(b) Show that the fraction field of $\mathbf{Z}[[x]]$ is strictly smaller than $\mathbf{Q}((x))$.

B3. Prove that the following polynomials are irreducible over the indicated fields, and determine their Galois groups:

(a) $f(X) = X^3 + X + t$ over the ground field $\mathbf{C}(t)$.

(b) $g(X) = X^5 - 4X + 2$ over the ground field $\mathbf{Q}$.