Analysis Prelim Spring 2019

Ex 1: Let T(x) from R^n to R^n be a Lipschitz transformation:
$| T(x) - T(y)| < C |x-y|$
Show that if A is a set of measure zero, also T(A) is of measure zero

Ex 2: Show that C_0 is dense in L^1 (R^n)

Ex 3: Find an uncountable family of measurable functions F(X), such that for any F $||F||_{L^\infty} = 1$ and for any two of the functions $||F_b - F_a||_{L^\infty}$ is bigger or equal to 1

Ex4: Let the sequence of measurable functions f_k (x) converge in measure to zero in B_1 (R_n) and satisfy $||f_k||_{L^2}$ less or equal than M for all k
Show that f_k converges to zero in L^1

Ex 5: Let H be a monotone function of f(x), a non negative measurable function∞
Write

$$\int H(f(x))\, dx$$

in terms of $g(\lambda) = |\{f>\lambda\}|$