Algebraic Topology Prelim Exam
January 12, 2022

Problem 1. Suppose $f : \mathbb{R}P^2 \to \mathbb{R}P^2$ is not surjective. Then

(a) the induced map $f_* : \pi_1(\mathbb{R}P^2) \to \pi_1(\mathbb{R}P^2)$ is trivial (Hint: what is $\mathbb{R}P^2 - \{\text{point}\}$?);

(b) f is homotopic to a constant map. (You may use the conclusion of (a), whether you prove it or not.)

Problem 2. In this problem, all surfaces are closed, connected and orientable.

(a) Suppose S is a surface of genus 2. Show that for every $g > 1$, the surface of genus g occurs as a covering space of S.

(b) Now suppose S has genus 3. For which values of g does the surface of genus g occur as a covering space of S?

Problem 3. Compute $\pi_1(X)$, where X is obtained from two tori $S^1 \times S^1$, by identifying $\{\text{point}\} \times S^1$ in the first torus with the diagonal of the second torus.