ALGEBRA PRELIMINARY EXAM: PART II

Problem 1

Let p and q be distinct primes. Set $\mathbb{F}_p := \mathbb{Z}/p\mathbb{Z}$.

a) Describe extensions of $\mathbb{F}_p$ of degree q (state the main results; no proofs required).

b) Compute the number of irreducible polynomials in $\mathbb{F}_p[x]$ of degree q.

Problem 2

Let F be the splitting field of $(x^2-2)(x^2-3)$ over $\mathbb{Q}$.

a) Determine the degree of $F/\mathbb{Q}$.

b) Determine the Galois group $\mathrm{Gal}(F/\mathbb{Q})$ as an abstract group.

c) Prove that $F/\mathbb{Q}$ is a simple extension.

d) Find an element $\alpha \in F$ such that $F = \mathbb{Q}(\alpha)$.

Problem 3

Let E be the splitting field of $x^3 - 5$ over $\mathbb{Q}$.

a) Determine the Galois group $\mathrm{Gal}(E/\mathbb{Q})$ as an abstract group.

b) Prove that $x^2 - 3$ is irreducible in $E[x]$.
Hint: Use the Fundamental Theorem of Galois theory.

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