Chaos and fractals in Otolaryngology

AG Bibas MSc, FRCS (Otol) [1], *R Carretero-González PhD* [2,*], *VG Kolokotronis PhD* [3]

1 Otolaryngology department, Medway Meridian Hospital, Kent
2 Centre for Nonlinear Dynamics and its Applications, University College London
3 National Observatory of Athens, Greece

Address for correspondence:

A. Bibas
16 Galleon Close
Rochester, Kent
ME1 3PF

[*] E-mail: R.Carretero@ucl.c.uk, URL: http://www.ucl.ac.uk/~ucesrca/ric.html

Introduction

Aeons of scientific thought have been dominated by the Newtonian philosophy that physical systems in the cosmos are predictable. Given the exact knowledge of a system initial condition and the physical laws that govern it, it is possible to predict its long-term behaviour. However, there are systems whose nature defies any practical attempt to predict their behaviour. Such an example is the weather system, which even with the latest technology, it is difficult to forecast accurately beyond two or three days. However, what is responsible for this unpredictability? Is it a mere lack of adequate model/data or is it an intrinsic property of the system? This dichotomy and its causes is exactly what chaos theory attempts to investigate and illuminate.

The concept of *chaos* (or deterministic chaos) emerged in between early 60's and early 70's in theoretical and applied mathematics and it is now engulfed in the theory of *non-linear dynamics*. It subsequently developed and was refined, hugely aided by the advent of powerful computers, in as diverse disciplines as meteorology and molecular biology, chemistry and ecology, economics and medicine, fluid dynamics and humanities, particle physics and cosmology, to name but a few (Gleick, 1997). The term chaotic is commonly used to describe a system that, although governed by a handful of non-linear equations, behaves in an apparently random manner. However, to be more precise, chaos refers to the practical inability to predict a deterministic system rather than to complete disorganisation implied by the common every day use of the word.

The difference between chaos and random lies on the concept of determinism. Random processes cannot be predicted by any means, they are not deterministic (future evolution completely uncorrelated to past behaviour). On the other hand, it is possible, in principle, to predict a chaotic system because it is deterministic: the future evolution is completely determined by the past in an unequivocal way. Although chaotic systems are deterministic, it is not possible to predict their future behaviour in practice due to the sensitive dependence on initial conditions.

One of the main and crucial characteristics of chaos is that infinitesimal changes or inaccuracies on the specification of the initial conditions may give rise to unexpected behaviour. This paradigm is known as *sensitive dependence on initial conditions* and it was first suggested by the prominent mathematician and astronomer Henri Poincaré as early as the beginning of the 1900's (Poincaré, 1913). This property is best illustrated by E. N. Lorenz (1963) in the context of meteorology as the 'Butterfly effect': a butterfly stirring the air today in Athens can induce a cascade of unpredictable changes that may affect the weather next month in London (Becker and Dorfler 1989, Gleick, 1997). Sensitive dependence on initial conditions means that the system balances on the boundary between a plethora of possibilities where a slight *perturbation* (i.e. disturbance) forces the system into a different direction. Because of this property, precise predictions become unreliable and the long-term behaviour is simply uncomputable. This is indeed the case with weather forecast and so many other physical phenomena. It is worth mentioning that chaotic behaviour is not unique to very complicated (high-dimensional) phenomena like weather dynamics. One might think that the unpredictability of the weather is a sole consequence of the complex interaction of all the parts that constitute the system (i.e. air molecules). However, even a very simplistic weather model consisting of 3 coupled first order ordinary differential equations possesses the hallmark of chaos (Lorenz, 1963). Moreover, a system as simple as a pendulum where its rod has been replaced by a spring can exhibit chaotic dynamics for sufficiently large energy (Carretero-González et al. 1994).

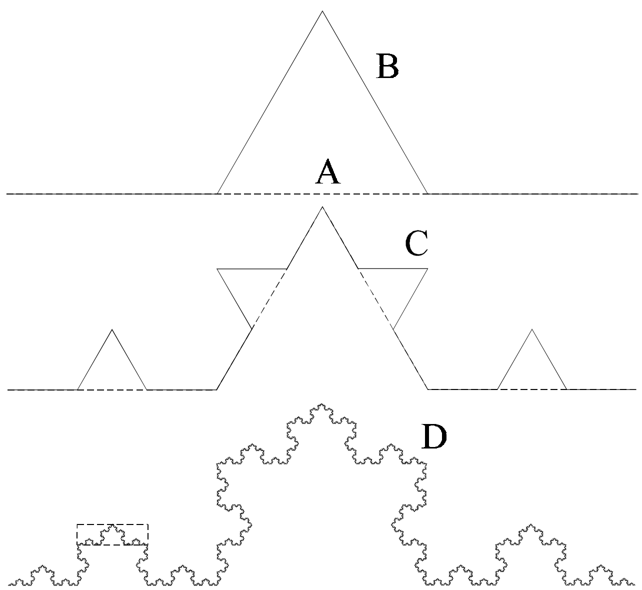
The formalism

The chaos breakthrough has introduced its own language and a variety of elegant and useful tools. Fractals and fractal (correlation) dimension comprise two of its major constituents.

Fractal (a word introduced by Mandelbrot in 1975) is a non-Euclidean geometric concept related to, yet not synonymous to chaos. It is the geometric correlate of a chaotic process. It refers to peculiar structures with no smoothed boundaries, whose geometry cannot be delineated by the common mathematical forms. The main property of a fractal is that its geometric structure is *self-similar* across different scales. A fractal would look qualitatively the same under different magnifications (i.e. it is *scale-invariant*). It may not be identical in detail but the structure will have the same topological properties. The branching of major arteries to smaller arteries to arterioles and capillaries is a fractal structure. Similar structures can be found everywhere in the body: in the branching of the bronchial tree, the billiary duct in the liver, the urinary collecting system, the neuronal networks in the brain. The fractal concept can also be applied to time-series where the self-similarity is evident on statistical grounds. From microcosmos to macrocosmos recent applications of chaos formalism on cosmology have indicated that structures on very large scales may indeed be of fractal nature (Coles, 1998; Khokhlof, 1998).

Because Euclidean measures like length and area are not meaningful concepts for fractals, the term *fractal dimension* was introduced. Fractal dimension is a useful measure of qualities that could not be described by standard geometry like the degree of roughness or irregularity of an object. In order to exemplify this concept and stress the impossibility to use Euclidean measures in some cases, consider the problem of measuring the coastline of, let us say, Britain. However simple this may seem, this task is far from obvious. After careful observation it becomes clear that the measure of the length of the coastline depends on the scale at which one is observing it. The finer the observation scale is, the more details one encounters, so that the perimeter of smaller and smaller rocks has to be taken into account and thus increasing the overall measure of the length of the coastline. This problem is a never-ending dilemma since no mater how refined the scale of observation is, it is possible to still be missing some structure (unless we reach the atomic level where even the meaning of length loses its sense). In cases like this, the concept of fractal geometry becomes the only alternative to accurately describe properties such as length an area.

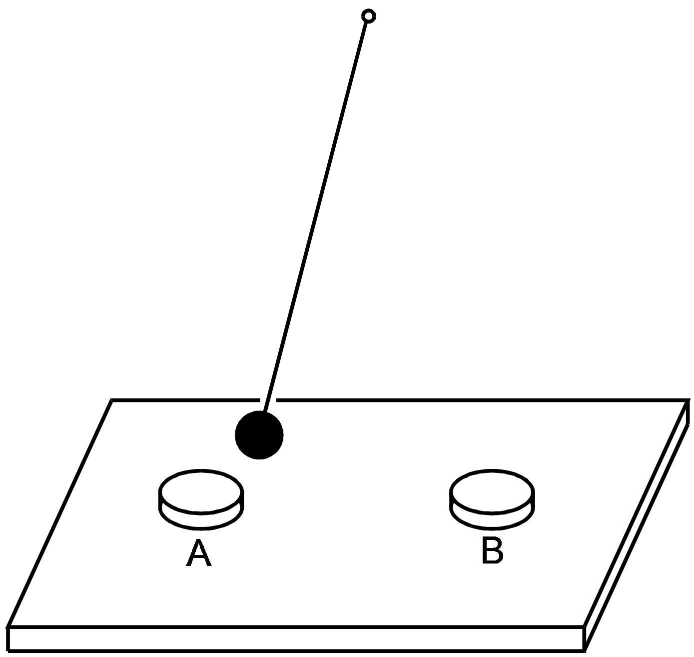
Following the idea of measuring a coastline, let us work out a simple geometric example where one is able to compute exactly some of its fractal properties. The classical example is provided by the iterative construction of the *Koch curve*, designed by Helge von Koch in 1904. Consider first a line of unit length (Fig. 1.A), and raise an equilateral triangle from its middle third (Fig. 1.B) corresponding to step one. Then, for the successive steps apply repetitively the process of raising an equilateral triangle from the middle third of *every* segment of the previous step. After applying this iterative scheme ad infinitum we end up with the Koch curve depicted, as far as our graphic capabilities allow, in Fig. 1.D. Let us measure the Euclidean length of this curve. In order to achieve this, we have to count how many segments are at each step and multiply this by their length (all segments at the same iteration step have same length). Denote by s_j the number of segments at the j-th step and denote by d_j their length, then the total length of the object at the j-th step is $l_j = s_j \times d_j$. For the initial object (unitary segment) $s_0 = 1$ and $d_0 = 1$, for step one $s_1 = 4$ and $d_1 = 1/3$ and for the step two $s_2 = 16$ and $d_2 = 1/9$. By following carefully the construction it is straightforward to see that two consecutive steps are related through $s_{j+1} = 4\ s_{\mathrm{j}}$ (since each segment is replaced by 4 segments) and $d_{j+1} = d_j/3$ (since we are using thirds of segments). The general formulae $s_j = 4^{\,j}$ and $d^{\,j} = (1/3)^{\,j}$ follow and the length of the whole curve at step j is given by $l_j = (4/3)^{\,j}$. Therefore, the Koch curve has infinite Euclidean length ($l_\infty = (4/3)^\infty = \infty$) and thus it is not a good indicator of its geometrical structure (many objects with quite different constructions have infinite length). However, the fractal dimension of the Koch curve is a well defined quantity and happens to be D_f=log(4)/log(3) = 1.26… (Schroeder, 1991).



As we already mentioned, fractals posses the self-similarity property (i.e. look qualitatively the same under scale magnifications). This property is depicted in Fig. 1.D where the region contained in the small rectangle is an *exact* copy of the whole Koch curve. Perhaps it is important to recall that smaller portions of typical fractal objects need not to be *exact* copies of the whole object. Indeed it suffices that every smaller portion has the same topological properties (fractal dimension for instance) than the whole object. Thus, for a ``perfect'' fractal one may measure its fractal dimension using solely a smaller region. Nevertheless, it has to be noticed that ``perfect'' fractals do only exist as mathematical oddities, and that in the real world no object is fractal since it has finite resolution. Therefore, in practice, a fractal is referred as an object whose geometry is self-similar under a significant range of magnifications. The actual fractal dimension algorithms are based in this idea and they only check for rescaling over some finite range of amplifications.

In essence, the fractal dimension is the efficiency of a structure to take up space. The larger the fractal dimension, the more the object fills up the space. The Koch curve for example has dimension 1 in Euclidean geometry, but a fractal dimension between 1 and 2 since it fills up more space than a simple line (for it has infinite Euclidean length), but it does not cover a complete two-dimensional area. Likewise, the fractal dimension of the small intestine's surface (with its numerous microvilli) is between 2 and 3. For cosmology, this number is still debatable and may depend on various geometrical properties of the sample used, which signifies the passage from fractality to homogeneity in the universe. However, disordant findings put it between 2 and 3 (Guzzo, 1998; Pietronero et al., 1998; Scaramella et al., 1999). Concluding the fractal dimension can be seen as an index of the space filling property of a fractal object.

The connection between fractals and chaos is a subtle matter; we thus use the following example as an illustration. Consider a pendulum constituted by a metal ball and a string. The pendulum is then put to oscillate on top of two fixed powerful magnets separated by some distance (see Fig. 2). After giving to the pendulum an initial position, it will oscillate around the two magnets in a chaotic fashion and after some transient behaviour it will come to a stop (because of friction) near one of the magnets. For different initial positions of the pendulum, the outcome of the final state varies between stopping near the magnet A or the magnet B. If one would plot the initial position of the pendulum using two different colours depending on the outcome; so that if the pendulum stops near magnet A we plot its initial position with a red dot, whereas if the pendulum stops near the magnet B we plot its initial position with a blue dot. After trying all possible initial positions, the final picture, often referred as *basins of attraction*, is a striking pattern of intermingled red and blue islands/dots filling the plane. The structure of these red and blue patterns is indeed fractal. Since the pendulum system is chaotic, it is impossible to predict in practice its outcome for some regions of the space of initial conditions. This is simple to explain in terms of the fractal structure of the red and blue dots. Because of the fractal nature of the basins of attraction, no matter how precise we are in the determination of the initial condition there is always, at any scale, red and blue islands and dots intermingled together. Thus any inaccuracy, however small, in the determination of the initial conditions will have fatal consequences in the final outcome. Generally speaking, dissipative (friction) systems whose behaviour is chaotic typically posses fractal basins of attraction.



A further connection between fractals and chaos is given by the so-called *strange attractors*. These attractors are produced by taking an imaginary plane crossing through the orbit of a chaotic system. Every time the orbit crosses this imaginary plane we plot the exact point of piercing and we continue to fill-in this plane by more and more piercings as the orbit progresses. This process, called a *Poincaré surface of section,* produces a representation of the attractor of the orbit. For similar reasons as the explained above with the pendulum example, the resulting set of piercing points is fractal due to the chaotic nature of the orbit. Therefore, attractors corresponding to chaotic orbits are fractal and thus called strange attractors.

Finally, we would like to end this section by clarifying the connection between non-linearity and chaos. Many times the terms non-linear and chaotic are wrongly used as synonyms. It is a fact that linear processes do not behave in a chaotic manner. It is also true that non-linearity is a requirement for the occurrence of chaos. However, non-linearity is not a *sufficient* requirement for chaos: there exist some non-linear systems that do not display chaos whatsoever. For those systems, even though a closed solution (analytical solution) cannot be found in terms of known functions (the only way to track their behaviour is through computer simulations), there is a lack of sensitive dependence on initial conditions and thus long-term predictions are feasible.

Applications in Biomedical sciences

In the early 1980's, scientists began to apply chaos theory to physiological systems. As described above, many organs and systems proved to be fractal in nature. Intuitively it seems logical that chaos would be more apparent in pathological or disease states. However, careful analysis sometimes demonstrates the opposite: an organ or system in a healthy state may operate in a chaotic fashion, while exhibiting an increasingly periodic behaviour and loss of variability in a pathological state. For example, careful analysis of a normal heart rate will demonstrate a considerable fluctuation even at rest. On the other hand, it has been discovered that the pattern of heart rate of patients with severe heart disease became less variable than normal anywhere from minutes to months before sudden cardiac death (Goldberger et al., 1990).

Fractal techniques have already been applied to a variety of disciplines in medicine. Growth of tumours might be influenced by the fractal structure of their tissues of origin (Pansera, 1994). Tumour boundaries and chromatin texture have been studied by fractal analysis and this may prove useful in discriminating between benign and malignant cells (Cross and Cotton, 1994; Cross, 1997). Publications on the application of chaos and fractals are increasingly seen in a variety of disciplines including ophthalmology, neuropathology, and urology (Cross, 1997). Perhaps the most exciting prospects for the application of chaos theory in medicine are those related to the development of new therapeutic interventions (Goldberger, 1996).

The signature of chaos in Otolaryngology

Although the possible applications of chaos have been explored in other specialities (Ref) this is not the case with Otolaryngology, where very few studies have been published on the possible applications of this fascinating theory. What follows is a brief outline of areas where its significance has been explored.

It has been suggested that the auditory system exhibits chaotic behaviour. It is known that the inner and outer hair cells function in a non-linear way. In the auditory nerve the auditory potential spikes appear in pairs and clusters with fractal properties (Teich, 1989). The significance of this is still obscure but it is suggested that it may create an efficient way of sampling auditory information and enhancing signal processing. Based on the above observations, Gstoettner et al. (1996) used fractally coded electrical signals for frequency, intensity, and temporal discrimination tests in cochlear implant patients. Their results showed that fractally coded information is possible in humans, but whether this is a more efficient way of coding strategy remains to be seen.

The fractal dimension of nystagmus has been suggested as a parameter to quantify the irregular pattern of the nystagmus movement (Aasen, 1993). It has been calculated that the fractal dimension (or correlation dimension when used for time series) of nystagmus ranges from 3.3 to 7.7 in normal individuals. Hypothetically, vestibular pathology can increase or decrease the correlation dimension of the system. Techniques for quantifying irregular nystagmus patterns for diagnostic purposes are being developed. A preliminary study on calorically induced nystagmus used the correlation dimension to distinguish between various vertigo types with statistically significant results (Nordahl et al., in press). Similar promising results have been observed in the fractal analysis of optokinetic nystagmus, where a statistically significant lower correlation dimension was found in a group of vertigo patients when compared to normal subjects (Aasen et al., 1996).

Fractal analytic procedures have also been applied to the analysis of vocal signals. It appears that the fractal dimension of the period and amplitude of vocal signals has a much smaller relative variance than the more usual indices of vocal variability

(Baken, 1990). The studies of specific pathologies by chaotic analysis are promising. Different underlying disorders may cause different changes in the fractal dimension of the vocal signals. Non-linear dynamics methodology has been used to study vocal folds vibrations and model vocal cord palsies (Steinecke and Herzel, 1995). Also it is hypothesised that fractally coded signals can be applied to the generation of synthetic acoustic signals making them more 'natural' and acceptable to the listeners (Baken, 1990).

Conclusions

We have attempted a brief review of some of the basic properties and applications of chaos and fractals in medical sciences, while employing the minimal use of mathematics. From the smallest to the largest metric scale, as we have exemplified, the universal signature of chaotic behaviour becomes apparent. The centre of focus is then shifted from general systems to specific medical analysis within the mathematical and chaos framework.

As already pointed out, there are but a few studies on the possible applications of Chaos theory to otolaryngological practice, but future research looks promising. The present paper has been designed to be descriptive, speculative and thought provoking. Chaos is not a new 'theory of everything', but offers a new, and hopefully useful, perspective for the interpretation and analysis of our cosmos. It still remains to be seen whether this initial enthusiasm will prove justifiable in the long term.

References

1. Aasen T (1993) Chaos theory applied to the caloric response of the vestibular system. Computers and Biomedical Research 26: 556-567.
2. Aasen T, Kugiumtzis D, Nordahl SHG (1997) Procedure for estimating the correlation dimension of optokinetic nystagmus signals. Computers and Biomedical Research 30: 95-116.
3. Baken RJ (1990) Irregularity of vocal period and amplitude: A first approach to the fractal analysis of voice. Journal of Voice 4(3): 185-197.

4. Becker K-H, Dorfler M. Dynamical systems and Fractals: Computer experiments in Pascal. Cambridge University Press, Cambridge, 1989.
5. Carretero-González R, Núñez-Yépez HN, Salas-Brito AL. Regular and chaotic behavior in an extensible pendulum. Eur J Phys 15: 139-148.
6. Coles P (1998) An unprincipled universe? Nature 391: 120-121.
7. Cross SS (1997) Fractals in pathology. Journal of Pathology 182:1-8.
8. Cross SS, Cotton DWK (1994) Chaos and antichaos in pathology. Human pathology 25(7): 630-637.
9. Gleick J. Chaos. Minerva, London, 1997.
10. Goldberger AL (1996) Non-linear dynamics for clinicians: chaos theory, fractals, and complexity at the bedside. Lancet 347: 1312-1314.
11. Goldberger AL, Rigney DR, West BJ (1990) Chaos and Fractals in human physiology. Sci Am 35-41.
12. Gstoettner W, Baumgartner W, Hamzavi J, Felix D, Svozil K, Meyer R, Ehrenberger K (1996) Auditory fractal random signals: Experimental data and clinical application. Acta Otolaryngol (Stockh) 116: 222-223.
13. Guzzo L (1998) Is the universe homogenous (on large scales)? New Astronomy 2: 517-532.
14. Khokhlof DL (1999) Fractal universe. http://xxx.soton.ac.uk/abs/astro-ph/9901207.
15. Lorenz EN (1963) Deterministic non-periodic flow. J Atmos. Sci 20: 130-141.
16. Nordahl SHG, Aasen T, Molvaer O (in press) Fractal analysis of ENG-signals. Correlation dimension analysis. Acta Otolaryngol (Stockh).
17. Pansera F (1994) Fractals and cancer. Medical Hypotheses 42: 400.
18. Pietronero, L, Montuori M, Sylos-Labini F (1997) On the fractal structure of the visible universe. http://xxx.soton.ac.uk/abs/astro-ph/9611197.
19. Poincaré, H (1913) *The foundation of science: science and method*, English translation, 1946, The Science Press, Lancaster, PA, p.397.
20. Ruelle D (1993) Chance and Chaos. Penguin, London.
21. Scaramella R (1998) The ESO slice project (ESP) Galaxy redshift survey. V. Evidence for a D=3 sample dimensionality. Astronomy and Astrophysics 334: 404-408.
22. Schroeder M (1991) Fractals, chaos, power laws. Minutes from an infinite paradise. W.H. Freeman and Company.

23. Steinecke I, Herzel, H (1995) Bifurcations in an asymmetric vocal-fold model. J Acoust Soc Am 97(3): 1874-1884.
24. Teich MC (1989) Fractal character of the auditory neural spike train. IEEE Transactions on Biomedical Engineering.