Absence of the Vortex Solution in Gor'kov's Formalism

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Abstract

It is shown that the Abrikosov's vortex solution or its corresponding two-particle pair potential is not the solution of the self-consistency equation in Gor'kov's formalism. Since the self-consistency equation leads to a superposition of different types of off-diagonal long-range order (ODLRO) instead of one type of ODLRO only, it may not handle the vortex problem appropriately. A possible resolution is suggested.

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I. INTRODUCTION

Recent STM experiments[1−3] show that the microscopic structures of the vortex state in type II and high T_c superconductors are much more complicated than expected. In particular, the vortex core and the lattice structure depend strongly on the microscopic details of the samples. For instance, using $NbSe_2$ samples Hess and his collaborators[1,2] found unexpected zero-bias peaks and sixfold star-shaped structures in the vortex core states. These experiments revived much theoretical interest.[4−6] However, a satisfactory quantitative explanation is still not available.[7] Maggio-Aprile et al.[3] also reported that the vortices in $YBa_2Cu_3O_{7-\delta}$ arrange in an oblique lattice, which remains unexplained. On theoretical side, Thouless et al.[8] noted that there is still a lot of controversy in the dynamics of vortices.

It seems that there is some fundamental problem in our theoretical understanding of the vortex state in superconductors. In this paper, we point out that Gor'kov's Green's function formalism or the Bogoliubov-de Gennes equations may not handle the vortex problem properly, since the self-consistency equation gives rise to an incoherent superposition of different types of off-diagonal long-range-order (ODLRO) instead of a coherent vortex state. The same problem was already found in Gor'kov and Galitskii's (GG)[9] solution for the d-wave pairing BCS theory. GG allowed a superposition of several distinct types of the off-diagonal long-range-order, which was proven to be invalid.[10−14] As Anderson noted,[13] this problem occurs in many cases even in discussing flux quantization. In fact, it also hindered correct understanding of the effects of magnetic impurities[15,16] and weak localization[17−20] on superconductivity. Balian, Nosanow, and Werthamer[11] showed that this problem is caused by the difficulty in the method of Green's functions for a manybody system. It is inspiring to remind their statement: *Thus the Green's function method as usually formulated is not a complete dynamical description of the system, and requires in addition, some criterion to distinguish these extraneous solutions from the correct one*. The criterion may be provided by the physical constraint of the Anomalous Green's function, which is nothing but the pairing constraint.[17,18]

The vortex problem may be very complicated; so we use the following simplifications which are not crucial in our discussion though.

(i) We consider only the lowest Landau Level in the presence of the very high magnetic fields, i.e.,

$$\omega_c > \omega_D, \tag{1}$$

where ω_c and ω_D are the cyclotron frequency and the Debye frequency, respectively.

(ii) The Zeeman splitting is disregarded.

(iii) We use the Landau gauge where

$$A_x = 0, A_y = Bx. \tag{2}$$

The z-axis motion will be suppressed.

(iv) We consider the self-consistency equation near T_c.

II. LANDAU LEVELS AND ABRIKOSOV'S VORTEX SOLUTION

We consider a rectangular sample with sides L_x and L_y. For the Hamiltonian

$$H = \frac{p_y^2}{2m} + \frac{(p_x - \frac{eBx}{c})^2}{2m}, \tag{3}$$

the eigenfunctions are given by

$$\phi_n(x, y) = N_o e^{iqny} exp[-(x - qn\ell^2)^2/2\ell^2], \tag{4}$$

where

$$\ell = \sqrt{\hbar c/eB}, \tag{5}$$

and

$$q = \frac{2\pi}{L_y}. \tag{6}$$

Here N_o is a normalization constant and we have used the periodic boundary condition along the y-direction. If the x dimensions of the system are confined to $-L_x/2 < x < L_x/2$, n is determined by the condition

$$-\frac{L_x}{2} < nq\ell^2 < \frac{L_x}{2}. \tag{7}$$

(We neglect the effect of boundary on N_o.)

The famous Abrikosov's vortex solution is[21]

$$\Psi(x,y) = \sum_{n=-\infty}^{\infty} C_n e^{iqny} exp[-(x - qn\ell^2)^2/2\ell^2], \tag{8}$$

with the periodicity conditions, $C_{n+1} = C_n$ for a square lattice and $C_{n+2} = C_n$ for a triangular lattice,[22] respectively. The C_n and ℓ are adjusted to minimize the free energy in the Ginzburg-Landau theory. Note that the solution is a linear combination of one-particle eigenfunctions.

III. SELF-CONSISTENCY EQUATION

In terms of the Lowest Landau Level wavefunctions, the normal state Green's function G is written as

$$G_\omega(1,2) = \sum_n \frac{\phi_n(x_1,y_1)\phi_n^*(x_2,y_2)}{i\omega - \epsilon_o}, \tag{9}$$

where $\epsilon_o = \frac{1}{2}\hbar\omega_c - \mu$.[23] μ is the Fermi energy and ω are the Matsubara frequencies. The Gor'kov's self-consistency equation is then given by[24]

$$\Delta(x_1,y_1) = VT\int \sum_\omega G_\omega^\uparrow(1,2)G_{-\omega}^\downarrow(1,2)\Delta(x_2,y_2)dx_2dy_2. \tag{10}$$

According to the Gor'kov's microscopic derivation of the Ginzburg-Landau equations,[24] the order parameter $\Psi(\mathbf{r})$ is proportional to the (two-body) pair potential $\Delta(\mathbf{r})$:

$$\Psi(\mathbf{r}) = \Delta(\mathbf{r})\sqrt{7\zeta(3)N}4\pi T_c, \tag{11}$$

where ζ is Riemann's zeta function and N is the electron number density in the normal metal. Consequently, the Abrikosov's vortex solution Eq. (8) should be the solution of the above self-consistency equation. Unfortunately, this is not the case. For example, if we substitute the pair potential

$$\Delta(x,y) \sim C_1 e^{iqy} exp[-(x-q\ell^2)^2/2\ell^2], \tag{12}$$

into the self-consistency equation, we obtain the different form of the pair potential

$$\Delta(x,y) \sim C_1 e^{iqy} exp[-(x-q\ell^2/2)^2/\ell^2]. \tag{13}$$

In other words, the self-consistency condition is not satisfied. This evidence cast serious doubt on the Gor'kov's microscopic derivation of the Ginzburg-Landau equations. In fact, this difficulty is anticipated since the Abrikosov's vortex solution is a linear combination of one-particle eigenfunctions while the pair potential consists of the multiplication of two one-particle eigenfunctions.

IV. ABSENCE OF THE VORTEX SOLUTION

Another possible form of the pair potential is

$$\Delta(x,y) = \sum_n \Delta_n e^{iqny} exp[-(x-qn\ell^2/2)^2/\ell^2]. \tag{14}$$

Now we show that this pair potential is not the solution of the self-consistency equation, either. First, we consider the constant pair potential in the y-direction, i.e.,

$$\Delta(x,y) \sim \Delta_0 exp(-x^2/\ell^2). \tag{15}$$

Upon substitution of this simple solution into Eq. (10), we find pairing between $\phi_n \uparrow$ and $\phi_{-n} \downarrow$ and the transition temperature T_c is determined by

$$1 = VT_c \frac{N_o^2}{\sqrt{2}} \sum_\omega \sum_n \frac{e^{-2n^2q^2\ell^2}}{\omega^2+\epsilon_o^2}. \tag{16}$$

We could have obtained the same equation from the BCS theory with the pairing matrix elements,

$$V_{nn'} = V \int \phi_n^*(r)\phi_{-n}^*(r)\phi_{-n'}(r)\phi_{n'}(r)dr$$
$$= V\frac{N_o^2}{\sqrt{2}}e^{-(n^2+n'^2)q^2\ell^2}. \tag{17}$$

The manybody ground state is, then

$$\tilde{\phi}_{BCS} = \prod_n [u_n + v_n(\phi_n \uparrow, \phi_{-n} \downarrow)]|0> \tag{18}$$

Second, we consider the pair potential for $n = 1$ in Eq. (14),

$$\Delta(x,y) \sim \Delta_1 e^{iqy} exp[-(x - q\ell^2/2)^2/\ell^2]. \tag{19}$$

It is straightforward to show that this pair potential leads to pairing between $\phi_n \uparrow$ and $\phi_{-n+1} \downarrow$ and the resulting transition temperature T_c' is determined by

$$1 = VT_c'\frac{N_o^2}{\sqrt{2}}\sum_\omega\sum_n \frac{e^{-2(n-1/2)^2q^2\ell^2}}{\omega^2+\epsilon_o^2}. \tag{20}$$

In the BCS theory, the corresponding pairing matrix elements are

$$V_{nn'} = V \int \phi_n^*(r)\phi_{-n+1}^*(r)\phi_{-n'+1}(r)\phi_{n'}(r)dr$$
$$= V\frac{N_o^2}{\sqrt{2}}e^{-(n-1/2)^2q^2\ell^2}e^{(n'-1/2)^2q^2\ell^2}. \tag{21}$$

Note that the transition temperatures may be extremely small due to the exponential factors and T_c and T_c' are different. The ground state is now

$$\tilde{\phi}_{BCS}' = \prod_n [u_n + v_n(\phi_n \uparrow, \phi_{-n+1} \downarrow)]|0> . \tag{22}$$

If we combine the two solutions, we obtain the pair potential

$$\Delta(x,y) \sim \Delta_0 exp(-x^2/\ell^2) + \Delta_1 e^{iqy} exp[-(x - q\ell^2/2)^2/\ell^2]. \tag{23}$$

Inserting Eq. (23) into Eq. (10) one finds

$$\Delta_0 exp(-x_1^2/\ell^2) = VT \int \sum_\omega G_\omega^\uparrow(1,2) G_{-\omega}^\downarrow(1,2) \Delta_0 exp(-x_2^2/\ell^2) dx_2 dy_2, \tag{24}$$

and

$$\begin{aligned} \Delta_1 e^{iqy_1} exp[-(x_1 - q\ell^2/2)^2/\ell^2] = VT \int \sum_\omega G_\omega^\uparrow(1,2) G_{-\omega}^\downarrow(1,2) \\ \times \Delta_1 e^{iqy_2} exp[-(x_2 - q\ell^2/2)^2/\ell^2] dx_2 dy_2, \end{aligned} \tag{25}$$

since the two solutions are linearly independent. Notice that we can not find the temperature at which the two different types of the condensation occur simultaneously due to the difference in the pairing matrix elements as shown above. Nevertheless, as Galitskii[25] suggested, it is tempting to write the resulting manybody state as a combination of the above ground states, that is,

$$\tilde{\phi}_{BCS} + \tilde{\phi}'_{BCS} = \prod_n [u_n + v_n(\phi_n \uparrow, \phi_{-n} \downarrow)]|0> + \prod_n [u_n + v_n(\phi_n \uparrow, \phi_{-n+1} \downarrow)]|0> . \tag{26}$$

This combination is just the so-called incoherent superposition of different types of off-diagonal long-range-order(ODLRO),[9−14] which does not correspond to any real physical state. Hone[10] actually demonstrated the impossibility of constructing a complete hierarchy of Green's functions in such a case.

For the pair potential

$$\Delta(x,y) \sim \Delta_2 e^{i2qy} exp[-(x - q\ell^2)^2/\ell^2], \tag{27}$$

one finds the T_c equation

$$1 = VT_c \frac{N_o^2}{\sqrt{2}} \sum_\omega \sum_n \frac{e^{-2(n-1)^2 q^2 \ell^2}}{\omega^2 + \epsilon_o^2}. \tag{28}$$

When the electrons are confined in the x-direction as assumed here, the T_c is different from those of the previous cases. Whereas for an infinite system or the system with the periodic boundary condition in the x-direction, the T_c may be the same as that for the pair potential corresponding to $n = 0$ in Eq. (14). We have then T_c for $\Delta_0, \Delta_2, \Delta_4, \cdots$ and $T_c'(\neq T_c)$ for $\Delta_1, \Delta_3, \Delta_5, \cdots$. It is interesting to note that the Abrikosov solution with $C_{n+2} = C_n$ has a lower energy than that with $C_{n+1} = C_n$.

Thus, the self-consistency equation does not allow the Abrikosov's vortex solution or the corresponding two-particle pair potential Eq. (14). It is obvious that adding different types of off-diagonal long-range-order (ODLRO) in Gor'kov's formalism does not lead to a coherent vortex state. On the other hand, previous workers first sum over the eigenstate ϕ_n in the one-particle Green's function and then consider the self-consistency condition, which fails to take into account the two-particle correlations correctly.

V. DISCUSSION

To describe the vortex state, we need to devise a coherent manybody ground state which may be closely related to the Abrikosov's vortex solution. Feynman's vortex solution of the superfluid He-4 may be a good starting point. More details will be published elsewhere.[26] Gor'kov's formalism may also be generalized to obtain a coherent superposition of the different types of the pairing.[17,18]

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