About Geometric Phases Serving as Memory in Quantum Computing

Spacetime Memory: Phase-Locked Geometric Phases

BERND BINDER[1]

Date: 29.8.-1.9.2002

Spacetime memory is defined with a holonomic approach to information processing, where multi-state stability is introduced by a non-linear phase-locked loop. Geometric phases serve as the carrier of physical information and geometric memory (of orientation) given by a path integral measure of curvature that is periodically refreshed. Regarding the resulting spin-orbit coupling and gauge field, the geometric nature of spacetime memory suggests to assign intrinsic computational properties to the electromagnetic field.

PACS 02.10.De, 03.65.Bz, 03.65.Vf, 03.67.Lx

Recently, geometric phases [1] are getting considerable attention in quantum computing [2]. In this paper we shall evaluate basic couplings of non-abelian geometric phases (holonomies) and try to find out what is necessary to setup, couple, and process spacetime memory. Geometric phases are subject of concepts in differential-geometry and topology [3] associated with *non-abelian* groups, i.e. $U(N)$ [4]. The first successful implementations via NMR have been reported [5], so it is likely that a quantum computer will be operated in the near future by the non-abelian Berry connection of a quantum computational bundle. To reveal the role of geometric phases, one has to discuss the necessities in information

- storage, transfer, and processing.

In [2] it has been shown how quantum information can be encoded in an eigenspace of a degenerate Hamiltonian H such, that one can in principle achieve the full quantum computational power by using holonomies only. For an introduction to geometric phases see i.e. [6].

Information storage

What characterizes memory? A memory has a time scale much longer than the time scale of information processing. This is a basic requirement, since computation is useless if the input is forgotten while waiting for the output. These two timescales are natural requirements of geometric phases: in a physical situation the long time scale is given by the path of a vector signal on a curved manifold, i.e. the orbital period, the short time scale defines the vector signal, i.e. it's spinning period. The information has to be 'imprinted' in a spacetime structure, where the topology is implemented by the proper spacetime manifold of the signal path. The information coded in the geometric phase is given by a path integral measure of curvature that modulates the vector, on S^2 by a typical conic precession. In the holonomic approach, information is encoded in a degenerate eigenspace of a parametric family of Hamiltonians and manipulated by the associated holonomic gates. Important for the realization of a memory unit, the non-adiabatic generalization of [7] defines a geometric phase factor for any cyclic evolution of a quantum system. Consider a T-periodic cyclic vector $|\,\psi(\tau)\,\rangle$ that evolves on a closed path $\mathcal{C}$ according to

$$|\,\psi(T)\,\rangle = e^{i\varphi(T)}\,|\,\psi(0)\,\rangle, \tag{1}$$

where the total phase $\varphi(T)$ acquired by the cyclic vector can naturally be decomposed into a geometric $\varphi_g(T)$ and dynamical phase $\varphi_d(T)$

$$\varphi(T) = \varphi_g(T) + \varphi_d(T). \tag{2}$$

The dynamical phase for one loop $t \in [0;T]$ is with the Schrödinger equation given by

$$\varphi_d(T) = -\frac{1}{\hbar}\int_0^T \langle\,\psi(\tau)\,|H(\tau)|\,\psi(\tau)\,\rangle d\tau. \tag{3}$$

The Berry phase or geometric phase depends not on the explicit time dependence of the trajectory and is for one loop given by

$$\varphi_g(T) = i\oint_{\mathcal{C}} \langle\,\psi(\tau)\,|d|\,\psi(\tau)\,\rangle. \tag{4}$$

The 'parallel transported' spin vector will come back after every loop with a directional change $\varphi_g(T)$ equal to the curvature enclosed by the path $\mathcal{C}$. On the unit sphere the curvature increment is proportional to the area increment that can be a spherical triangle with area given by

$$d\Omega := [1-\cos\theta(\tau)]d\varphi(\tau), \tag{5}$$

[1] *email: binder@quanics.com, Weildorferstr.22, 88682 Salem-Neufrach, Germany*

the total area enclosed by the closed orbit (loop) is equal to

$$\Omega = \oint_{\mathcal{C}} d\Omega := \int_0^T d\tau[1 - \cos\theta(\tau)]\dot{\varphi}(\tau). \tag{6}$$

The Berry phase $\varphi_g(T) = J\Omega$ and the total phase are proportional to spin J. In the standard case of precession on the sphere

$$\varphi_g(T) = 2\pi J(1 - \cos\theta), \quad \varphi(T) = 2\pi J, \tag{7}$$

where θ is the vertex cone semiangle, $\varphi_d(T) = 2\pi J \cos\theta$. For the two level system, the geometric phase is equal to half of the solid angle subtended by the area in the Bloch sphere enclosed by the closed evolution loop of the eigenstate. With n parameters $\lambda_\mu(t)$, $\mu = 1, 2, ..., n$ that span a closed curve $\mathcal{C}$ in the T-periodic parameter space $\lambda_\mu(0) = \lambda_\mu(T)$, the Berry phase may be represented in terms of the 'gauge potential' A with connection matrix

$$(A_\mu)^{\alpha\beta} := \langle\psi^\alpha(\lambda)|\, \partial/\partial\lambda^\mu \,|\psi^\beta(\lambda)\rangle \tag{8}$$

where $A = \sum_\mu A_\mu \, d\lambda_\mu$, and

$$\varphi_g(T) = \oint_{\mathcal{C}} A = \int_{\mathcal{S}_\mathcal{C}} F, \quad F = dA. \tag{9}$$

A is the non-abelian gauge potential that can be regarded as a winding number density and allows for parallel transport of vectors over $\mathcal{S}_\mathcal{C}$, an arbitrary surface in the parameter space bounded by the contour $\mathcal{C}$. For more details regarding monopoles and Wilson loops on the lattice in non-abelian gauge theories, see e.g. [8].

Information transfer

What characterizes information channels? Our geometric memory will have quantum nature: it is periodically refreshed or regenerated with quantum memory loss and information transfer given by the correspondent phase-frequency modulation. Degeneracy plays a crucial role in quantum computing and allows to transfer the phase states between energetically equivalent sub-systems. The dimension d of the manifold $U(N)$ reaches its minimum for $d = 1$, in this extreme case $N = 1$ denotes the maximally degenerate case [2]. In this case (the gauge group $U(1)$ has the topology of a circle on which the homotopy classes of closed curves are labelled by their winding or loop numbers) the wave-function in the M-fold degenerate case transforms as

$$\psi \to e^{\pm iM\Lambda}\psi, \tag{10}$$

where one unit corresponds to the phase subinterval $[0, 2\pi/M]$. Information transfer of M quantum information units per unit cycle (adiabatic loop) at dynamical phase evolution frequency ω_M could be realized by a coupling energy $\triangle E$ sponsored by a carrier with energy E where

$$\triangle E = E - M\hbar\omega_M, \tag{11}$$

where $\triangle E \ll M\hbar\omega_M$. In electromagnetism M is the (magnetic) quantum number quantizing charge taking integral values $\pm0, \pm1, \pm2, ...$ [9]. It is quite often that the relevant systems provide for the required discrete symmetries and large degenerate eigenspaces, i.e. rotational invariance, see eq.(10).

Information processing

What characterizes information processing? Computation requires that memory is multi-stable and coupled to the quantum state transfer, where the time evolution of a quantum sub-system can be controlled by the state of another sub-system. The computational dynamics is obtained by switching on and off by a set of gate Hamiltonians that generate a small set of basic paths given by unitary transformation on the quantum state-space. Multi-state stability can be introduced by non-linear behavior. The well known example of a simple flip-flop-type feedback process (a bi-state or half spin configuration) can be realized with a geometric phase that is driven by it's own precession dynamics. Let the precession cone vertex semiangle θ of eq.(7) realize a bi-stable flip-flop configuration characterized by two states:

- $M_+ > 0$, $0 < \theta_+ < \pi$
- $M_- < 0$, $\pi < \theta_- < 2\pi$

that can be stabilized by the iteration

$$\theta_{\pm,i+1} = \frac{\pi \cos\theta_{\pm,i}}{M_\pm}, \tag{12}$$

and converges after a few steps to a special $\theta_\pm$-value. A fast convergence is crucial for the performance of the spacetime computer. The coupling can be interpreted as a navigational iteration on the closed path, where one iteration step requires to exchange one bit of information between the orbital system partners controlling each other in a center-of mass system. The bit is the sign of phase that gets lost in the cosine-function on S^2 independent of the resulting coupling shift. Therefore, a virtual coupling bit-stream can be assigned to the closed orbital path on S^2. The choice of the form eq.(12) is adjusted to the energy transfer relation eq.(11) with $M \to M_\pm$ and $\theta \to \theta_\pm$. This has the following background: the phase evolution can be divided into the two parts of geometric and dynamic phase evolution, where the geometric evolution can be assigned to a precession frequency ω_p with ratio adjusted to the sponsored energy

$$\frac{\omega_p}{\gamma\omega} = \frac{\triangle E}{E} = \frac{\varphi_g(T)}{2\pi J} \tag{13}$$

including relativistic correction γ. The dynamical phase evolution corresponds to the cyclic frequency $\omega_{M\pm}$ and characterizes the general expression for spin-rotation coupling observed in the laboratory frame which can be assumed to be on S^2

$$M_\pm \frac{\omega_{M\pm}}{\omega} = \pm 1 \mp \frac{\omega_p}{\gamma\omega} = \cos\theta_\pm \tag{14}$$

in accordance with eq.(7).
Omitting the $\pm$ polarity the relative dynamical coupling constant $\alpha(M)$ can be defined by the ratio dynamical phase evolution frequency ω_M divided by the carrier (Compton) frequency ω driven by particle spin J

$$\alpha(M) = \frac{J\triangle\varphi_d(T)}{\varphi(T)} = \frac{J\omega_M}{\omega}, \tag{15}$$

where the coupling is proportional to the evolution of the dynamical part $\triangle\varphi_d(T)/\varphi(T)$. With eq.(14) in eq.(15)

$$\frac{\varphi_g(T)}{2\pi J} = 1 - \frac{M\alpha}{J}, \tag{16}$$

the dynamical part of eq.(16) provides for

$$M\alpha = J\cos(\theta). \tag{17}$$

Comparing eq.(14) - eq.(17) the precession cone vertex angle 2θ is linearly related to the dynamic spin-orbit interaction and given by the feedback coupling relation of the most trivial kind

$$\theta = \pi\alpha. \tag{18}$$

eq.(18) has a simple geometric interpretation: spin-orbit coupling modelled by a 'rolling cone' representing a vector state or signal. Rotated once, the cone will change its orbital orientation by a special angle $2\pi/M$, rotated M-times in the quantum case, the cone will return to the initial position with integral M (providing for single-valuedness). If the base of the cone has radius θ/π, the side length is $M\theta/\pi = \cos(\theta)$. As shown in [10], for $M = 137$ and vitual photon vector coupling with $J = 1$ the coupling constant $\alpha \approx 1/137.03600941164$ fits within error range to a neutral and theory independent determination of the Sommerfeld fine structure constant. This suggests to assign intrinsic computational entities and capabilities to the electromagnetic field.

Conclusion

In the holonomic approach to information processing geometric phases serve as the carrier of physical information. In this case geometric phases are the primordial memory of orientation given by a path integral measure of curvature on $S^2 = SU(2)/U(1)$, where the coupling of intrinsic spin with rotation reveals the quantum of rotational inertia $\equiv$ memory $\equiv$ angular momentum quantum $\hbar$. The system carries pair-creation energy E and coupling energy $\triangle E$ stabilized by a phase-locked feedback loop, a periodical refreshment including precession as a form of phase-frequency modulation. The non-linear iteration converges quickly and provides for a fast and flip-flop-type situation: the prototype space-time imprint of a polar binary system. A virtual coupling bit-stream can be assigned to the navigational iteration on the closed path, since one non-linear iteration step requires to exchange one bit of information. Regarding the polar coupling constant and the magnetic monopole topology [1, 9], it can be proposed that natural memories optimized by iterative phase relationships can be found everywhere. The non-linear phase-locked feedback mechanism provides for an hidden and very fast bit-stream running at a nice bandwidth $\approx \omega_M$. In such a natural high-performance computer, fine structure as a pure number would divide hardware from software. Additional details can be found in [10], [11], and [12].

References

[1] M. V. Berry, Proc. Roy. Soc. Lond. A **392**, 45 (1984).

[2] P. Zanardi, M. Rasetti, Phys. Lett. **A264**, 94 (1999). quant-ph/9904011

[3] M. Nakahara, *Geometry, Topology and Physics*, IOP Publishing Ltd., 1990

[4] F. Wilczek and A. Zee., Phys. Rev. Lett. **52**, 2111 (1984).

[5] J.A. Jones, V. Vedral, A. Ekert, G. Castagnoli, Nature **403**, 869 (2000).

[6] R. Batterman, 'Falling Cats, Parallel Parking, and Polarized Light' (2002); PITT-PHIL-SCI00000583.

[7] Y. Aharonov, J. Anandan, 'Phase Change During a Cyclic Quantum Evolution', Phys. Rev. Lett. **58**, 1593 (1987).

[8] F.V. Gubarev, V.I. Zakharov, Int. J. Mod. Phys. **A17**, 157 (2002); hep-th/0004012.

[9] P. A. M. Dirac, Proc. Roy. Soc. London A **133**, 60 (1931).

[10] B. Binder, 'Berry's Phase and Fine Structure' (2002); PITT-PHIL-SCI00000682.

[11] B. Binder, 'Geometric Phase Locked in Fine Structure' (2002); PITT-PHIL-SCI00000774.

[12] B. Binder, alpha simulation; http://www.quanics.com/spinorbit.html.