A New Interpretation of Flux Quantization

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Abstract

We study the effect of Aharonov-Bohm flux on the superconducting state in metallic cylinders. Although Byers and Yang attributed flux quantization to the flux-dependent minimum of kinetic energies of the Cooper pairs, it is shown that kinetic energies do not produce any discernible oscillations in the free energy of the superconducting state (relative to that of normal state) as a function of the flux. This result is indeed anticipated by the observation of persistent current in normal metal rings at low temperature. Instead, we have found that pairing interaction depends on the flux, leading to flux quantization. When the flux (Φ) is given by $\Phi = n \times hc/2e$ (with integer n), the pairing interaction and the free energy become unchanged (even n) or almost unchanged (odd n), due to degenerate-state pairing resulting from the energy level crossing. As a result, flux quantization and Little-Parks oscillations follow.

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I. INTRODUCTION

Flux quantization is one of the most fascinating properties of superconducting states. This phenomenon was found by Deaver and Fairbank[1] and Doll and Näbauer[2] in 1961. They observed that the magnetic flux trapped in a superconducting cylinder is quantized in units of hc/2e. Recently, half-integer flux quantization has attracted much attention in connection with the pairing symmetry in high Tc cuprates.[3]

Byers and Yang[4] proposed that flux quantization follows because the free energy of the superconducting state is periodic in the unit of the flux, hc/2e, if electrons are paired. In fact, they assumed that i) the free energy, F_N, of the normal state is independent of the flux, i.e., $F_N(\Phi)$ = constant, and ii) the pairing energy is constant as a function of the flux.[5] Consequently, they ascribed flux quantization to the minimum of the kinetic energy of the superconducting state at $\Phi/(hc/2e)$ = integer. Based on the theory of Byers and Yang, Little and Parks[6] pointed out that the transition temperature T_c is also a periodic function of the enclosed flux Φ, which is called Little-Parks oscillations.

However, recent discovery of persistent current in normal metal rings at low temperature casts a doubt on the first assumption, that the free energy of the normal state is essentially independent of the flux.[7–9] Since the electronic states of normal metal rings are periodic in flux Φ, with period hc/e, the persistent current is given by $I(\Phi) = -c \times \partial F_N(\Phi)/\partial\Phi$. Therefore, the first assumption is not compatible with the experimental observation of the persistent current in normal metal rings.

In this paper we report that flux quantization and the Little-Parks oscillations are caused by the flux dependence of the pairing energy. In addition, we show that the flux dependence of the kinetic energy does not lead to any significant oscillations in the transition temperature T_c and the energy gap parameter Δ_0 for superconducting cylinders. As the flux increases, the pairing matrix element is reduced since we must pair single-particle states which are not degenerate and have different density distributions.[10,11] When the flux (Φ) is equal to $\Phi = n \times hc/2e$ (with integer n), the pairing interaction and the free energy of the superconductor remain unchanged (even n) or almost unchanged (odd n) from their values with zero flux, since we can pair degenerate partners, with the same density distribution, due to the energy level crossing. Accordingly, we find flux quantization and Little-Parks oscillations. It is interesting to note that Schwartz and Cooper[12] obtained a similar result for *generalized pairing* with different angular momentum in superconducting cylinders.

II. THEORY OF BYERS AND YANG

To study the effect of the Aharonov-Bohm flux Φ on the superconducting states in cylinders, Byers and Yang[4] considered the variation of the energy levels E of a single particle in the presence of the flux:

$$E = \frac{1}{2m}[p_r^2 + p_z^2 + \frac{\hbar^2}{r^2}(m + \frac{e}{hc}\Phi)^2]. \tag{1}$$

Here p_r and p_z denote momenta in the radial and z directions. Employing the BCS pairing between states m and -m, they noted that the average (kinetic) energy for the states m and -m, near $\Phi = +0$, increases with Φ like

$$\text{constant} + \frac{\hbar^2}{2\text{mr}^2}(\frac{\text{e}\Phi}{\text{hc}})^2. \tag{2}$$

Furthermore, at $2e\Phi/hc = 1$, since pairing between

$$m + \frac{e}{hc}\Phi = m + \frac{1}{2} = \frac{1}{2} \text{ and } -\frac{1}{2}, \frac{3}{2} \text{ and } -\frac{3}{2}, \text{ etc.,} \tag{3}$$

is preferred, the kinetic energy per particle remains the same as for the case $\Phi = 0$. Then the additional energy for each of these pairs, in the neighborhood of $2e\Phi/hc$, increases with Φ in a parabolic manner:

$$2 \times (\frac{\hbar^2}{2mr^2})[(\frac{e\Phi}{hc}) - \frac{1}{2}]^2. \tag{4}$$

Consequently, the kinetic energy of the Cooper pairs has the minimum at $\Phi = n \times hc/2e$ (with integer n). Based upon this observation, Byers and Yang[4] asserted that flux quantization is due to the flux-dependent minimum of the kinetic energy of the Cooper pairs. On the other hand, the corresponding BCS wavefunctions at $\Phi = n \times hc/2e$ are given by[13]

$$\prod(u_\nu + v_\nu a^+_{\nu+m\uparrow} a^+_{-\nu+m\downarrow})|0>; \;(n \text{ even}) \tag{5}$$

$$\prod(u_{\nu+1/2} + v_{\nu+1/2} a^+_{\nu+m+1\uparrow} a^+_{-\nu+m\downarrow})|0>; \;(n \text{ odd}) \tag{6}$$

with $n = 2m$ ($n = 2m + 1$) for even (odd) values of n. The operator $a^+_{\nu+m\uparrow}$ creates an electron in the state $(\nu + m \uparrow)$.

However, the kinetic energy of the normal electrons will show the same flux dependence, as confirmed by the observation of the persistent current in normal metal rings.[7−9] Therefore, we expect that this flux dependence will be canceled in the superconducting state. More precisely, using the BCS theory, we show that the variation of the kinetic energy with the flux does not lead to any oscillations of the condensation energy (i.e., Δ_0) and the free energy (i.e., T_c) of superconducting cylinders.

Assuming a constant pairing matrix element V,[4,5] one finds the BCS gap equation at $T = 0K$ and the T_c equation:[14−16]

$$\Delta_m = -V\sum_{m'} \frac{\Delta_{m'}}{2\sqrt{(\frac{\epsilon_{m'-\mu}+\epsilon_{-m'-\mu}}{2})^2 + \Delta^2_{m'}}} \tag{7}$$

$$1 = -V\sum_m \frac{tanh\frac{\epsilon_{m-\mu}}{2T_c} + tanh\frac{\epsilon_{-m-\mu}}{2T_c}}{2(\epsilon_{m-\mu} + \epsilon_{-m-\mu})} \tag{8}$$

where $\epsilon_{m-\mu}$ is the one-particle energy (measured from the Fermi energy E_F) with $\mu = e\Phi/hc$. Contrary to Byers and Yang, it is clear that the kinetic energy does not give rise to any significant oscillations in the gap parameter, $\Delta_m = \Delta_0$, since its effect is only to shift the BCS cutoff region from $-\omega_D < \epsilon < \omega_D$ to $-\omega_D + \epsilon_\mu < \epsilon < \omega_D + \epsilon_\mu$ (where $\epsilon_\mu \equiv \frac{\hbar^2\mu^2}{2mr^2}$):

$$\Delta_0 = -V\sum_m \frac{\Delta_0}{2\sqrt{(\epsilon_m + \epsilon_\mu)^2 + \Delta_0^2}}. \tag{9}$$

Use of $\epsilon_{m-\mu} + \epsilon_{-m-\mu} = 2(\epsilon_m + \epsilon_\mu)$ has been made. For the T_c, Eq. (8) shows indeed a minor decrease in T_c (for nanoscale systems) due to the flux, which is *inconsistent* with the ($T = 0K$) gap equation, Eq. (7), and has been elusive.[5,6] However, this decrease is completely negligible for bulk systems and it originates from the exclusion of the Cooper pairs with negative excitation energy.[17,18] Since analytic solution is not available, we rely on the numerical calculations and merely emphasize that the correction is in the denominator:

$$\begin{aligned} 1 &= -V \sum_m \frac{tanh\frac{\epsilon_{m-\mu}}{2T_c} + tanh\frac{\epsilon_{-m-\mu}}{2T_c}}{2(\epsilon_m + \epsilon_\mu)} \\ &= -V \sum_m \frac{1}{2(\epsilon_m + \epsilon_\mu)} \frac{sinh\frac{\epsilon_m + \epsilon_\mu}{T_c}}{cosh\frac{\epsilon_{m-\mu}}{2T_c} cosh\frac{\epsilon_{-m-\mu}}{2T_c}}. \end{aligned} \quad (10)$$

Note that for bulk systems m is of the order of 1,000 near E_F, while μ is of the order of 1.

For example, we solve Eqs. (7) and (8) numerically for a superconducting cylinder with the inner radius $r_a = 7,200\mathring{A}$, the outer radius $r_b = 8,000\mathring{A}$, and the height $L = 500\mathring{A}$. For a thin cylinder, i.e. $r_b - r_a << r_b$, the electron energy is approximately written as $\epsilon_{m-\mu} \cong \frac{\hbar^2}{2m}[\frac{(m-\mu)^2}{r_a r_b} + (\frac{\ell\pi}{r_b - r_a})^2 + (\frac{n_z \pi}{L})^2]$.[19,20] (More calculational details will be explained in the next section.) We assumed $E_F = 1.0eV$ and $\omega_D = 0.02eV$. There are 1,286,200 states in the BCS cutoff range. Figure 1 shows Δ_0 and T_c versus $\mu = e\Phi/hc$. As can be seen, there is no change in Δ_0 (i.e., the condensation energy) and T_c (i.e., the free energy of superconducting state) due to the variation of the kinetic energy with the flux Φ. For a certain V (= 17.0/volume [eV]), the gap parameters Δ_0 are the same 2.144 886 meV for $\mu = 0$ and $\mu = 0.25$, whereas the $T_c's$ are 1.212 888 meV ($\mu = 0$) and 1.211 921 meV ($\mu = 0.25$), respectively. It is noteworthy that the decrease in T_c scales approximately $\sim 1/\sqrt{N}$, with N= number of states in the BCS cutoff range. Consequently, the T_c decrease due to the exclusion of the Cooper pairs with negative excitation energy[17,18] can be completely ignored in the bulk limit. Note also that $2\Delta_0/T_c \cong 3.537$, in agreement with the bulk result.

III. FLUX-DEPENDENT PAIRING INTERACTION

Now we consider flux-dependence of the pairing matrix element and the pairing interaction in a superconducting cylinder whose axis coincides with the z-axis. The inner and outer radii are r_a and r_b, and the height is L. First, we determine one-particle eigenfunctions and eigenvalues in the presence of the Aharonov-Bohm flux Φ. The Hamiltonian is given by

$$H_0 = -\frac{\hbar^2}{2m_e}[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}(-i\frac{\partial}{\partial \phi} - \mu)^2 + \frac{\partial^2}{\partial z^2}] \tag{11}$$

where $\mu = e\Phi/hc$. Assuming a solution of the form

$$\phi(r, \phi, z) = R(r)sin(\frac{n_z \pi z}{L})e^{im\phi}. \tag{12}$$

we obtain the Bessel equation for $R(r)$:

$$R'' + \frac{R'}{r} + [k^2 - \frac{(m-\mu)^2}{r^2}]R = 0. \tag{13}$$

From the boundary condition $R(r = r_a) = R(r = r_b) = 0$, we find the radial solutions and eigenvalues:[19,20]

$$R_{m-\mu}(r) = \mathcal{N}[N_{m-\mu}(kr_a)J_{m-\mu}(kr) - J_{m-\mu}(kr_a)N_{m-\mu}(kr)] \tag{14}$$

$$E_{lmn_z} = \frac{\hbar^2}{2m_e}[k_{ml}^2 + (\frac{n_z \pi}{L})^2] \ (n_z = 1, 2, 3, \cdots, \ m = 0, \pm 1, \pm 2, \cdots) \tag{15}$$

where $\mathcal{N}$ is the normalization constant and k_{ml} is the lth root of the transcendental equation

$$N_{m-\mu}(kr_a)J_{m-\mu}(kr_b) - J_{m-\mu}(kr_a)N_{m-\mu}(kr_b) = 0. \tag{16}$$

From Eq. (16) we calculate the single-particle energy as a function of the flux Φ, shown in Figure 2, near $E_F = 1.0$ eV for $r_a = 75\mathring{A}$ and $r_b = 150\mathring{A}$. Ignoring the z coordinate, we find 15 basis pair states within the BCS cutoff range for $\omega_D = 0.02eV$. We consider a small size ring for simplicity and because of the difficulty in calculating the high order Bessel functions. As μ increases, some states near 1.02 eV get out of the BCS cutoff range, while some states below 0.98 eV are coming in the range. We disregard this complication, since this effect will be negligible in the bulk limit. Moreover, it is caused by the simple approximation, which can be avoided in the strong-coupling theory.[21] Near $\mu = 0.5$ we chose $E_F \cong 0.99$ eV, which is obtained from the E_F for $\mu = 0$ by applying the flux. Although the spectra near $\mu = 0$ and $\mu = 0.5$ are similar, they are not exactly the same due to the finite size effect. Notice the crossings of energy levels at $\Phi = hc/2e \times n$ (with integer n).

Second, we derive the pairing matrix element for an Einstein phonon model with the phonon Green's function $D(x - x')$. Using the equivalent electron-electron potential in the electron-phonon problem,[11,22,23]

$$V(x - x') \rightarrow V_0 D(x - x'), \tag{17}$$

with $x = (\mathbf{r}, t)$, we get the pairing matrix element:

$$V_{m,m'} = V_0 \int_{r_a}^{r_b} R^*_{m'-\mu}(r) R^*_{-m'-\mu}(r) R_{-m-\mu}(r) R_{m-\mu}(r) r dr;\ -\frac{1}{2} + \delta < \mu < \frac{1}{2} - \delta \tag{18}$$

$$V_{m,m'} = V_0 \int_{r_a}^{r_b} R^*_{m'-\mu}(r) R^*_{-m'+1-\mu}(r) R_{-m+1-\mu}(r) R_{m-\mu}(r) r dr;\ \frac{1}{2} - \delta < \mu < 1 - \delta \tag{19}$$

$$V_{m,m'} = V_0 \int_{r_a}^{r_b} R^*_{m'+1-\mu}(r) R^*_{-m'+1-\mu}(r) R_{-m+1-\mu}(r) R_{m+1-\mu}(r) r dr;\ 1 - \delta < \mu < \frac{3}{2} - \delta \tag{20}$$

where $\delta \sim 1/4$. We have made use of the BCS wavefunction with the flux:

$$\tilde{\phi}_{BCS} = \prod_m (u_m + v_m a^+_{m-\mu\uparrow} a^+_{-m-\mu\downarrow}|0>;\ -\frac{1}{2} + \delta < \mu < \frac{1}{2} - \delta \tag{21}$$

$$\tilde{\phi}_{BCS} = \prod_m (u_m + v_m a^+_{m-\mu\uparrow} a^+_{-m+1-\mu\downarrow}|0>;\ \frac{1}{2} - \delta < \mu < 1 - \delta \tag{22}$$

$$\tilde{\phi}_{BCS} = \prod_m (u_m + v_m a^+_{m-\mu\uparrow} a^+_{-m+2-\mu\downarrow}|0>;\ 1 - \delta < \mu < \frac{3}{2} - \delta. \tag{23}$$

Finally, we obtain the BCS gap equation[14–16]

$$\Delta_m = -\sum_{m'} V_{mm'} \frac{\Delta_{m'}}{4\sqrt{(\frac{\epsilon_{m'-\mu}+\epsilon_{-m'-\mu}}{2})^2 + \Delta^2_{m'}}} (tanh\frac{E_{m'-\mu}}{2T} + tanh\frac{E_{-m'-\mu}}{2T}) \tag{24}$$

where the excitation energy $E_{m-\mu}$ is

$$E_{m-\mu} = \frac{1}{2}(\epsilon_{m-\mu} - \epsilon_{-m-\mu}) + \sqrt{(\frac{\epsilon_{m-\mu} + \epsilon_{-m-\mu}}{2})^2 + \Delta^2_m}. \tag{25}$$

Since the analytic expression of the matrix element is not available, we rely on the numerical method to calculate T_c from Eq. (24). Note that Eq. (24) leads to the negative excitation energy, when $\epsilon_{m-\mu}$ and $\epsilon_{-m-\mu}$ have different signs near E_F, as in the cases of a superconductor with a uniform exchange field[17] and the current-carrying superconducting state.[18] Nevertheless, in this case the possibility of the negative excitation energy is not significant as shown in Fig. 1, because $m \sim 1,000$ near E_F while $\mu \sim 1$ in the bulk limit. In other words, the exclusion of the contribution of the negative excitation does not lead to any noticeable effect in T_c, as shown in Fig. 1.

IV. EXPLANATION OF FLUX QUANTIZATION

Flux quantization may be explained from the flux-dependence of the pairing matrix element $V_{mm'}$. We stress the periodicity of the matrix element with the flux:

$$V_{mm'}(\Phi = \frac{hc}{2e} \times \text{even n}) = \mathrm{V_{mm'}}(\Phi = 0). \tag{26}$$

$$V_{mm'}(\Phi = \frac{hc}{2e} \times \text{odd n}) \cong \mathrm{V_{mm'}}(\Phi = 0). \tag{27}$$

This behavior is caused by the energy level crossings at $\Phi = hc/2e \times n$ (or $\mu =$ integer or half $-$ integer). It was shown that the level crossings (at integer μ) are due to the cylindrical symmetry of the system.[4,20] Since for odd n the energy level crossing occurs at lower energy than for even n, the matrix elements are slightly different for even and odd n's.

In the neighborhood of $\Phi = 0+$ we pair states $(m - \mu \uparrow)$ and $(-m - \mu \downarrow)$ which are not degenerate and have a different density distribution.[10,11] Accordingly, the pairing matrix element $V_{mm'}$ is decreasing with the flux, compared to $V_{mm'}$ at zero flux. For a large cylinder, we may employ the asymptotic radial wavefunction:[24]

$$R_{m-\mu}(r) = \mathcal{N}\{[1 + a_{m-\mu}\frac{1}{r_a r} - b_{m-\mu}(\frac{1}{2r^2} + \frac{1}{2r_a^2})]\frac{sink_l(r - r_a)}{\sqrt{r}}\} \tag{28}$$

where

$$a_{m-\mu} = [\frac{4(m-\mu)^2 - 1}{8k_l}]^2,\ b_{m-\mu} = \frac{[4(m-\mu)^2 - 1][4(m-\mu)^2 - 9]}{(8k_l)^2}, \tag{29}$$

and

$$k_l = \frac{l\pi}{r_b - r_a}. \tag{30}$$

It is obvious that the algebraic terms lead to the different density distributions of states $(m-\mu\uparrow)$ and $(-m-\mu\downarrow)$ and the decrease of the matrix element:

$$V_{mm'}(\Phi) \approx [1 - C'(\frac{2m^3}{k_l^2})^2\mu^2][1 - C'(\frac{2m'^3}{k_{l'}^2})^2\mu^2] \times V_{mm'}(\Phi = 0), \tag{31}$$

where $C' \cong (1/r_a r_b - 1/2r_b^2 - 1/2r_a^2)^2$.

Figure 3 shows the radial density distributions $|R(r)|^2 r$ of the $m = \pm 54$ states when $\mu = 0.25$, (i.e., $m - \mu = 53.75, -m - \mu = -54.25$) for the same ring as Fig. 2. The solid line corresponds to $m - \mu = 54.75$, whereas the dotted line corresponds to $-m - \mu = -54.25$, in which the electron is rotating faster, due to bigger angular momentum projection along the z-axis, leading to the shift of the radial distribution towards the outer radius. The difference in the radial density distributions then induces the decrease of the pairing matrix element with increasing μ. Figure 4 shows the matrix element $V_{mm'} \equiv V_{ij}$ for $\mu = 0$. Notice that it is 15×15 symmetric matrix, since there are 15 pair states in the BCS cutoff range. The decrease of the matrix element with μ is not big enough to be noticeable in Fig. 4, though. We just mention the decrease of biggest matrix element $V_{m=53,m'=51} \equiv V_{i=2,j=3}$, between pair states with the same l value, from 0.0002450 to 0.0002448.

This observation and the periodicity of $V_{mm'} \equiv V_{i,j}$ prove that flux quantization is due to the minimum of the pairing energy of the superconducting cylinders as a function of the flux. Accordingly, the free energy of the superconducting state also has minimum at $\Phi = hc/2e \times n$ since the contribution of the kinetic energy is constant. In other words, pairing between the degenerate states caused by the energy level crossing at $\Phi = hc/2e \times n$ is the physical origin of flux quantization. Thus the Little-Parks oscillations follow accordingly. From Eqs. (24) and (31) we may estimate roughly the decreases of the energy gap at $T = 0K$ and T_c:

$$\Delta_m(\Phi) \sim \Delta_m(\Phi=0)(1-8C' < \frac{m^6}{k_l^4} > \mu^2/N_0V_0), \tag{32}$$

$$T_c \sim T_{c0}(1-8C' < \frac{m^6}{k_l^4} > \mu^2/N_0V_0), \tag{33}$$

where N_0 is the density of states at E_F and $<>$ denotes the average over the states in the BCS cutoff range. T_{c0} is the transition temperature in the absence of the magnetic flux.

Figure 5 shows the $T = 0K$ *anisotropic* gap parameter Δ_i for $\mu = 0$, calculated from Eq. (24) by employing the matrix element $V_{i,j} \equiv V_{mm'}$ as in Fig. 4. Even though the coupling between the second and third states is strongest, we find the biggest gap at the tenth state, i.e., $\Delta_{i=10}$, due to its strong couplings to several neighbor states.

Figure 6 displays the decrease of the (average) energy gap with μ. The solid line denotes the ($T = 0K$) average gap parameter, $\bar{\Delta} = \sum_i \Delta_i/15$. It should be noticed that this decrease, so much enhanced as a result of the finite size effect, persists even in the bulk limit, as Eq. (32) shows. The different variations near $\mu = 0$ and $\mu = 0.5$ are also due to the finite size effect. The dashed line shows the isotropic gap parameter, according to the assumptions of Byers and Yang.[4] It exhibits a small decrease with the flux too, which quickly disappears with the increase of the system size. For the isotropic solution, we included $V_{ii} = V$ besides $V_{ij} = V =$ constant. Therefore, we get the bigger gap. The semicircle is the T_c when $\mu = 0$. Since, as the flux is increased, the finite temperature gap equation, Eq. (24) eliminates the contributions from the Cooper pairs with negative excitation energy,[17,18] T_c drops to zero immediately. If we consider the Cooper problem,[25] it is easy to understand why we need to eliminate the Cooper pair basis states with different signs in energy near E_F. However, this behavior is greatly amplified because of the finite size effect and it will not cause any problem in realistic systems, as shown in Fig. 1. Furthermore, the strong coupling theory[21] will reduce the effect even more. For nanoscale systems it may be desirable to use the canonical ensemble approach.[26] We expect that for a reasonably large system, we find a continuous (small) variation of the T_c, consistent with the variation of the energy gap.

V. DISCUSSION

It is clear that more study is needed to understand the details of flux quantization. In particular, it is essential to consider a big hollow cylinder for comparison with the experiment, which requires high order Bessel functions. This study indicates that the parabolic background[5,6] in the T_c oscillations may be due to the magnetic field penetration into the superconductor, which is not addressed here. It is also interesting that disordered $Au_{0.7}In_{0.3}$ cylinders[27] show half-integer flux quantization as in the case of high T_c cuprates.[3]

This study also implies that the exact calculation of the eigenenergies using high order Bessel functions and the magnetic field penetration into the metals are important in calculating the persistent current in normal metals.

Finally, flux quantization plays an important role in the vortex state[28] which may be understood by generalizing this approach.

VI. CONCLUSION

We have shown that flux quantization and the Little-Parks oscillations in superconducting cylinders are due to the flux dependence of the pairing interaction, while the flux dependence of the kinetic energy is canceled as evidenced by the observation of the persistent current in normal metals. When the flux Φ is given by $\Phi = hc/2e \times n$ (with integer n), the pairing interaction and the free energy are unchanged (even n) or almost unchanged (odd n), due to the degenerate-state pairing, compared to those in the absence of the flux. Accordingly, flux quantization and the Little-Parks oscillations are obtained.

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FIGURES

FIG. 1. Energy gap and T_c as a function of the flux Φ for a cylinder with $r_a = 7,200\mathring{A}$, $r_b = 8,000\mathring{A}$, and $L = 500\mathring{A}$, following Byers and Yang. Energy gap does not change at all, while T_c shows negligible oscillations.

FIG. 2. One-particle energy levels vs $\mu = e\Phi/hc$ near the Fermi energy ($E_F = 1.0eV$ for $\mu = 0$ and $E_F \simeq 0.99eV$ for $\mu = 0.5$) for a ring with $r_a = 75\mathring{A}$ and $r_b = 150\mathring{A}$. Note the level crossings at $\mu = 0$ and 0.5.

FIG. 3. Radial probability density $|R(r)|^2 r$ of states $m - \mu = 53.75$ (solid line) and $-m - \mu = -54.25$ (dotted line) for the same ring as Fig. 2. Notice the shift of the radial distribution of $-m - \mu = -54.25$ state to the outer radius.

FIG. 4. Pairing matrix element $V_{mm'} \equiv V_{ij}$ for the same ring as Figs. 2 and 3. The biggest element $V_{i=2,j=3} \equiv V_{m=53,m'=51}$ corresponds to the pair states with the same l value. The diagonal term $V_{mm} \equiv V_{ii}$ does not contribute to superconductivity.

FIG. 5. Anisotropic gap parameter Δ_i for $\mu = 0$ at $T = 0K$, corresponding to the matrix element, V_{ij}, as in Fig. 4.

FIG. 6. Average energy gap parameter (solid line), $\bar{\Delta} = \sum_i \Delta_i/15$, (at $T = 0K$) vs μ for the same ring as Figs. 2-5. The dashed line shows the isotropic gap parameter vs μ following Byers and Yang. The semicircle denotes the T_c for $\mu = 0$, corresponding to the anisotropic gap parameter, Δ_i.