# A circularly polarized beam carries the double angular momentum

Radi I. Khrapko*
*Moscow Aviation Institute, 125993, Moscow, Russia*

**Abstract**

It is shown that, in reality, a circularly polarized light beam without an azimuthal phase structure carries the double angular momentum in comparison with the prediction of the standard Maxwell electrodynamics, and an experimental check of the statement is suggested. The Maxwell electrodynamics does not explain a result of the Beth experiment and solves the problem of the two-element absorber incorrectly. A conception of classical electrodynamics spin is presented.

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## 1 The standard point of view

All physicists [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] consider

$$J = \mathcal{E}/\omega \tag{1}$$

where $\mathcal{E}$ is energy of a circularly polarized light beam without an azimuthal phase structure as total angular momentum of the beam, and the angular momentum is given by the formula

$$\mathbf{J} = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{H}) dV. \tag{2}$$

Heitler wrote [3, p.401],

> A plane wave traveling in the $z$-direction and with infinite extension in the $xy$-directions can have no angular momentum about the $z$-axis, because $\mathbf{E} \times \mathbf{H}$ is in the $z$-direction and $(\mathbf{r} \times (\mathbf{E} \times \mathbf{H}))_z = 0$. However, this is no longer the case for a wave with finite extension in the $xy$- plane. Consider a cylindrical wave with its axis in the $z$-direction and traveling in this direction. At the wall of the cylinder $r = R$, say, we let the amplitude drop to zero. It can be shown that the wall of such a wave packet gives a finite contribution to $J_z$.

Ohanian wrote [6, p.502]

*Email addresses: khrapko_ri@mai.ru, khrapko_ri@hotmail.com

In a wave of finite transverse extent, the **E** and **B** fields have a component parallel to the wave vector (the field lines are closed loops) and the energy flow has components perpendicular to the wave vector. ... The circulating energy flow in the wave implies the existence of angular momentum, whose direction is along the direction of propagation. This angular momentum is the spin of the wave.

Simmonds and Guttman [5, p.227] wrote

The electric and magnetic field can have a nonzero $z$-component only within the skin region of the wave. Having $z$-component within this region implies the possibility of a nonzero $z$-component of angular momentum within this region. Since the wave is identically zero outside the skin and constant inside the skin region, the skin region is the only one in which the $z$-component of angular momentum does not vanish.

All physicists think that the angular momentum has been measured by Beth [11] by letting the beam pass through a doubly refracting plate which alters the polarization. The plate was allowed to rotate about the axis of the beam. It was found that the plate, indeed, begins to rotate as soon as the beam passes through, and the angular momentum acquired by the plate was in agreement with (1) [3, p.404].

# 2 The Beth experiment is a puzzle

The Beth classical experiment [11] was made almost 70 years ago.

A beam of circularly polarized light should exert a torque on a doubly refracting plate which changes the state of polarization of the light beam.

The apparatus used involves a torsional pendulum with about a ten- minute period consisting of a round quartz half-wave plate one inch in diameter suspended with its plane horizontal from a quartz fiber about 25 centimeters long. About 4 millimeters above this is mounted a fixed quartz quarter-wave plate. The *top* side of the upper plate was coated by evaporation with a reflecting layer of aluminum. The rotation of the pendulum is observed by a telescope.

A circularly polarized light beam (power, $\mathcal{P}$ = 80 mW; $\lambda$ = 1.2 mu; $\omega = 1.6 \cdot 10^{15}$ 1/s) passes through the half-wave plate, then it passes twice through the quarter-wave plate, and then returns through the half-wave plate. The torque, $\tau$, exerting on the half-wave plate is 20 dyne-cm. This result seems to be agreed with (1)

$$\tau = 4\mathcal{P}/\omega. \tag{3}$$

At the same time, it is evident that the Poynting vector $\mathbf{E} \times \mathbf{H}$ equals to zero in the Beth experiment because the passed beam interferes with the reflected one. So, $\mathcal{P} = 0$. Indeed, let us start from the Jackson's expression for a circularly polarized beam [4]. ($\omega = k = 1$).

$$\mathbf{E} = [\mathbf{x}\cos(z-t) - \mathbf{y}\sin(z-t) + \mathbf{z}(-\sin(z-t)\partial_x - \cos(z-t)\partial_y)]E_0(x,y),$$

$$\mathbf{H} = -\int \mathrm{rot}\mathbf{E}dt = [\mathbf{x}\sin(z-t) + \mathbf{y}\cos(z-t) + \mathbf{z}(\cos(z-t)\partial_x - \sin(z-t)\partial_y)]E_0(x,y),$$

Here $E_0(x,y) = \text{Const}$ inside the beam, and $E_0(x,y) = 0$ outside the beam.

The reflected beam may be got by changing the sign of $z$ and $y$. Adding up the passed and reflected beams, we get interesting expressions,

$$\mathbf{E}_{\text{tot}} = 2[(\mathbf{x}\cos z - \mathbf{y}\sin z)E_0 - \mathbf{z}(\sin z\,\partial_x E_0 + \cos z\,\partial_y E_0)]\cos t, \tag{4}$$

$$\mathbf{H}_{\text{tot}} = -2[(\mathbf{x}\cos z - \mathbf{y}\sin z)E_0 - \mathbf{z}(\sin z\,\partial_x E_0 + \cos z\,\partial_y E_0)]\sin t. \tag{5}$$

The $\mathbf{E}_{\text{tot}}$ and $\mathbf{H}_{\text{tot}}$ fields are parallel to each other everywhere. So, the Poynting vector is zero, and, according to (2), angular momentum of the light in the Beth's experiment is zero! So, the light should not exert a torque on a plate. Why did the torque exert the plate then?

As a matter of fact, the all-recognized equation (2) seems to be incorrect. As a matter of fact, the Maxwell electrodynamics is not complete [12, 13, 14, 15].

# 3 The problem of the two-element absorber

An important question was raised at the V. L. Ginsburg Moscow Physical Seminar in the spring of 1999. The question was about absorption of circularly polarized light by a round flat target, which is divided concentrically into an inner disc and a closely fitting outer annulus [16].

If the target absorbs a circularly polarized beam, the annulus absorbs the surface of the beam, which carries the angular momentum, and the disc absorbs the body of the beam, which has no angular momentum. According to the Maxwell theory, the expression $d\mathcal{F}^j = T^{ji}da_i$ gives an infinitesimal force acting on a surface element $da_i$ (here $T^{ji}$ is the Maxwell stress tensor, i.e. a space part of the Maxwell-Minkowski energy-momentum tensor). So, the disc does not perceive a torque when the target absorbs a circularly polarized beam, since the Poynting vector is perpendicular to the disc. There are no ponderomotive forces, which are capable to twist the disc. Tangential forces act only on the annulus.

But it is clear that in reality the disc does perceive a torque from the wave, since the disc gets spin which is not described by the Maxwell theory. Beth [11], Feynman [17] clearly showed how circularly polarized wave transferred torque to medium. The disc will be twisted in contradiction with the Maxwell paradigm.

Allen and Padgett [10] have attempted to explain the torque acting on the disc within the scope of the standard electrodynamics. They have written, "Any form of aperture introduces an intensity gradient ... and a field component is induced in the propagation direction and so the dilemma is potentially resolved." They decompose a plane wave into three beams: the inner beam, the annulus beam, and the remainder.

Alas! A small clearance between the inner disc and outer annulus does not aperture a beam and does not induce longitudinal field components. The imaginary decomposition of the plane wave is not capable to create longitudinal field components and, correspondingly, transverse momentum and torque acting on the disc. Maxwell stress tensor cannot supply the disc with a torque. According to the Maxwell theory, the disc absorbs energy and feels pressure only.

# 4 Classical electrodynamics' spin

To resolve the problem, in our opinion, we must use the conception of classical electrodynamics' spin which is described by a four-spin tensor $\Upsilon^{\mu\nu\alpha}$ [18, 19, 20, 14] (more precisely, tensor density). Electrodynamics' spin tensor is not zero, and ponderomotive forces acting on a surface element $da_i$ consist of both, the force itself and a torque.

$$d\mathcal{F}^j = T^{ji} da_i, \qquad d\tau^{jk} = \Upsilon^{jki} da_i \qquad (6).$$

So, we must recognize that the standard classical electrodynamics is not complete. Therefore, the angular momentum (2) is not spin. It is orbital angular momentum $L$. This is the main point of the paper.

So, if a target absorbs a circularly polarized electromagnetic beam, tangential forces act on the target at the area of absorption of the surface of the beam. Besides, a torque acts on the target at the all territory of absorption of the beam. As it was shown, the both parts of acting are equal to each other, $L = S$. Therefore the resultunt angular momentum received by a target will be equal to

$$J = L + S = 2\mathcal{E}/\omega,$$

instead of $J = L$, as it is predicted by the Maxwell theory. So, we double the angular momentum of a beam.

This result was submitted to "JETP" on February 25, 1999.

# 5 An experimental check of the electrodynamics

An experimental check of this result in a terrestrial laboratory is complicated by thermal streams which is related to absorption of energy. However, these difficulties disappear if a target is in free space. So, consider [14] an aluminum disk with mass $m$ = 27 g and area $a$ = 1 m$^2$ (the thickness is 10 microns, the rotational inertia is $I$ = 4.3 g$\cdot$m$^2$). I offer to illuminate it by circularly polarization electromagnetic radiation, the wavelength is $\lambda$ = 10 cm, and power is $\mathcal{P}$ = 100 W. According to the Maxwell theory, the torque is

$$\tau = \mathcal{P}/\omega = \mathcal{P}\lambda/2\pi c = 5.3 \cdot 10^{-9} \mathrm{N} \cdot \mathrm{m}.$$

Therefore, the disk will rotate through 1 radian in a time of 27 minutes, and, according to my prediction, in a time of 19 minutes ($t = \sqrt{2I/\tau}$).

# 6 A theory of the Beth experiment

Let us apply the conception of the classical electrodynamics' spin to the Beth experiment. Consider the expression [18, 19, 20, 14]

$$\Upsilon^{\mu\nu\alpha} = \underset{e}{\Upsilon^{\mu\nu\alpha}} + \underset{m}{\Upsilon^{\mu\nu\alpha}} = A^{[\mu}\partial^{|\alpha|}A^{\nu]} + \Pi^{[\mu}\partial^{|\alpha|}\Pi^{\nu]}. \qquad (7)$$

as the electrodynamics' spin tensor (see Sect. 7). Here $A^\mu$ is the magnetic vector potential, and $\Pi^\mu$ is the electric vector potential,

$$2\partial_{[\nu}A_{\mu]} = F_{\nu\mu}, \quad \Pi_\mu = \epsilon_{\mu\alpha\beta\sigma}\Pi^{\alpha\beta\sigma}, \quad \partial_\sigma\Pi^{\alpha\beta\sigma} = F^{\alpha\beta}.$$

Using vector notations, the potentials can be introduced as follows.

If div$\mathbf{B} = 0$, then $\mathbf{B} = $ curl$\mathbf{A}$. If also $\partial\mathbf{B}/\partial t = -$curl$\mathbf{E}$ then $\partial\mathbf{A}/\partial t = -\mathbf{E}$.
If div$\mathbf{D} = 0$, then $\mathbf{D} = $ curl$\Pi$. If also $\partial\mathbf{D}/\partial t = $ curl$\mathbf{H}$, then $\partial\Pi/\partial t = \mathbf{H}$.

We consider the Beth's light beams as plane waves because the Poynting vector is zero, and the wall effects are of no importance. In front of the half-wave plate we have (3) and (4) with $E_0 = 1$,

$$\mathbf{E} = 2(\mathbf{x}\cos z - \mathbf{y}\sin z)\cos t,$$

$$\mathbf{A} = -\int \mathbf{E}dt = 2(\mathbf{x}\cos z - \mathbf{y}\sin z)(-\sin t).$$

So, the electrical part of spin current density is uniform, but pulses,

$$\underset{e}{\Upsilon}^{xyz} = 2\sin^2 t.$$

The magnetic part is

$$\mathbf{H} = 2(-\mathbf{x}\cos z + \mathbf{y}\sin z)\sin t,$$

$$\Pi = \int \mathbf{H}dt = 2(\mathbf{x}\cos z - \mathbf{y}\sin z)\cos t, \quad \underset{m}{\Upsilon}^{xyz} = 2\cos^2 t.$$

So, the total spin current density is constant, $\Upsilon^{xyz} = \underset{e}{\Upsilon}^{xyz} + \underset{m}{\Upsilon}^{xyz} = 2$.

The same calculation for the domain behind the plate gives $\Upsilon^{xyz} = -2$. This means that the $S^{xy}$-component of the spin moves opposite the $z$-direction, i.e. towards the plate also. In result, the plate receives the spin current density of 4, as in (3), with the absence of an energy current!

# 7 Appendix. The Belinfante-Rosenfeld mistake

The Poynting vector, $\mathbf{E} \times \mathbf{H}$, is a component of the Maxwell-Minkowski energy-momentum tensor,

$$T^{\alpha}_{\mu} = -F_{\mu\nu}F^{\alpha\nu} + \delta^{\alpha}_{\mu}F_{\sigma\nu}F^{\sigma\nu}/4, \tag{8}$$

and the Eq. (2) looks as

$$J^{\mu\nu} = \int 2r^{[\mu}T^{\nu]\alpha}dV_{\alpha} \tag{9}$$

in the Minkowski space.

There are no simple theoretical method to obtain the Maxwell-Minkowski tensor. The canonical Lagrangian,

$$\mathcal{L} = -F_{\mu\nu}F^{\mu\nu}/4, \quad F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]},$$

according to Noether's theorem, gives the canonical energy- momentum and spin tensors [21, Sec. 7g]:

$$\underset{c}{T}^{\mu\alpha} = \partial^{\mu}A_{\sigma}\frac{\partial\mathcal{L}}{\partial(\partial_{\alpha}A_{\sigma})} - g^{\mu\alpha}\mathcal{L} = -\partial^{\mu}A_{\sigma}F^{\alpha\sigma} + g^{\mu\alpha}F_{\sigma\rho}F^{\sigma\rho}/4, \tag{10}$$

$$\underset{c}{\Upsilon}^{\mu\nu\alpha} = -2A^{[\mu}\delta^{\nu]}_{\sigma}\frac{\partial\mathcal{L}}{\partial(\partial_{\alpha}A_{\sigma})} = -2A^{[\mu}F^{\nu]\alpha}. \tag{11}$$

But the canonical energy-momentum tensor is asymmetric and has an incorrect divergence,

$$\partial_{\alpha}\underset{c}{T}^{\mu\alpha} = -\partial^{\mu}A_{\nu}j^{\nu}.$$

The divergence of a true energy-momentum tensor must be equal to $-F_{\mu\nu}j^{\nu}$.

To symmetrize the canonical energy-momentum tensor and to turn it to the Maxwell-Minkowski tensor, theorists has to add a term

$$\partial_\beta A_\mu F^{\alpha\beta} \tag{12}$$

to the canonical energy-momentum tensor (10). This term divides in two parts:

$$T^\alpha_\mu = \underset{c}{T}{}^{\alpha}_{\mu} + \partial_\beta A_\mu F^{\alpha\beta} = \underset{c}{T}{}^{\alpha}_{\mu} + \partial_\beta (A_\mu F^{\alpha\beta}) + A_\mu j^\alpha.$$

The second part,

$$A_\mu j^\alpha, \tag{13}$$

repairs the divergence of the canonical tensor, and the first part,

$$\partial_\beta (A_\mu F^{\alpha\beta}), \tag{14}$$

symmetrizes the contravariant form of the tensor.

The only reason for adding the term (12) to the canonical tensor (10) is to obtain the Maxwell-Minkowski tensor (8) which was known beforehand. The term (12) is not even a divergence.

Theorists ignore the second part (13) of the term. They do not see it. But Belinfante and Rosenfeld [22, 23] made a considerable study of the first part (14). They pointed out that an antisymmertization of the first part yields the divergence of the canonical spin tensor (11) with the minus sign,

$$2\partial_\beta (A^{[\mu} F^{\nu]\beta}) = -\partial_\beta \underset{c}{\Upsilon}{}^{\mu\nu\beta}. \tag{15}$$

We shall use the fact below, but now we have to emphasize that the Belinfante's part (14) itself turns the canonical energy-momentum tensor (10) not to the Maxwell-Minkowski tensor (8), but to a tensor that may be named the Belinfante's tensor,

$$\underset{B}{T}{}^{\mu\alpha} = \underset{c}{T}{}^{\mu\alpha} + \partial_\beta (A^\mu F^{\alpha\beta}) = T^{\mu\alpha} - A^\mu j^\alpha.$$

Nevertheless, theorists believe that they obtain the Maxwell-Minkowski tensor (8) by adding the Belinfante's divergence (14) to the canonical tensor (10). Moreover, on the grounds of (15), theorists add $(-\underset{c}{\Upsilon}{}^{\mu\nu\beta})$ to the canonical spin tensor (11) and arrive to a zero as the electrodynamics' spin tensor. So, the Belinfante's divergence (14) does not lead to the Maxwell-Minkowski tensor (8), but it eliminates spin tensor. That is why classical spin is absent in the Maxwell electrodynamics. The classical spin tensor is considered zero. That is why they consider a circularly polarized plane wave has no angular momentum [16, 4, 1, 2, 6, 7, 5, 8, 9].

Here a problem arises: what is an electrodynamics' true spin tensor. What must we add to the canonical spin tensor to get the true spin tensor?

Our answer is as follows: a spin addition, $\Delta \underset{c}{\Upsilon}{}^{\mu\nu\alpha}$, and the energy-momentum addition (12) must satisfy an equation of type (15) which uses (12) instead of (14),

$$2\partial_\beta A^{[\mu} F^{\nu]\beta} = \partial_\beta \Delta \underset{c}{\Upsilon}{}^{\mu\nu\beta}. \tag{16}$$

A simple expression satisfies the Eq. (16),

$$\Delta \underset{c}{\Upsilon}{}^{\mu\nu\alpha} = 2A^{[\mu}\partial^{\nu]} A^\alpha.$$

So, we obtain [18, 19, 20]

$$\Upsilon^{\mu\nu\alpha} = \underset{c}{\Upsilon}^{\mu\nu\alpha} + \Delta \underset{c}{\Upsilon}^{\mu\nu\alpha} = 2A^{[\mu}\partial^{|\alpha|}A^{\nu]}. \tag{17}$$

This result was submitted to "JETP Letters" on May 14, 1998.

The spin tensor (17) is a function of the vector potential $A_\mu$ and is not gauge invariant. We greet this fact [20]. As is shown, $A^\mu$ must satisfy the Lorentz condition, $\partial_\mu A^\mu = 0$.

The expression (17) is not final. As a matter of fact, the electrodynamics is asymmetric. Magnetic induction is closed, but magnetic field strength has electric current as a source:

$$\partial_{[\alpha}F_{\beta\gamma]} = 0, \quad \partial_\nu F^{\mu\nu} = j^\mu.$$

So, a magnetic vector potential exists, but, generally speaking, an electric vector potential does not exist. However, when currents are absent the symmetry is restored, and a possibility to introduce an electric multivector potential $\Pi^{\mu\nu\sigma}$ appears. The electric multivector potential satisfies the equation

$$\partial_\sigma \Pi^{\mu\nu\sigma} = F^{\mu\nu}.$$

A covariant vector, dual relative to the multivector potential,

$$\Pi_\alpha = \epsilon_{\alpha\mu\nu\sigma}\Pi^{\mu\nu\sigma},$$

is an analog of the magnetic vector potential $A_\alpha$. We name it the electric vector potential.

The symmetry of the electrodynamics forces us to offer a symmetric expression for the spin tensor consisting of two parts, electric and magnetic (7).

# Conclusion

The Maxwell electrodynamics is not complete. We have to put a spin tensor in the modern electrodynamics. Theorists did not catch sight of the classical spin because of ignoring a localization of energy-momentum and due to the concept of the ambiguity of an energy-momentum tensor. A significance of the Lagrange formalism was overestimated.

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# Notes

The material of this paper was rejected or ignored by the journals (dates of the submissions are in brackets):
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The arXiv blacklisted all computers of the Moscow Aviation Institute and rejected my submissions 21 Jan 2002, 18 Feb 2002, 02 June 2002, 13 June 2002, 12 Apr 2003, 15 May 2003.

Prof. Ginsparg ignored my objections.

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