Continuity of integrated density of states – independent randomness

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Dedicated to Michael Demuth on his 60th birthday

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Abstract

In this paper we discuss the continuity properties of the integrated density of states for random models based on that of the single site distribution. Our results are valid for models with independent randomness with arbitrary free parts. In particular in the case of the Anderson type models (with stationary, growing, decaying randomness) on the ν dimensional lattice, with or without periodic and almost periodic backgrounds, we show that if the single site distribution is uniformly α-Hölder continuous, $0 < \alpha \leq 1$, then the density of states is also uniformly α-Hölder continuous.

1 Introduction

In the spectral theory of random potentials one of the quantities of interest is the density of states which is an averaged total spectral measure. Often this measure is approximated using a sequence of operators (finite volume operators) and the continuity properties of the limit are proved using the total spectral measures of the approximants. One of the main questions in such an approximation procedure is the existence of the limiting density of states measure.

One of the first results on the α-Hölder continuity of the integrated density of states in the case of singular single site distributions (done for the approximating operators)) is by Carmona-Klein-Martinelli [1], (Lemma 6.1 and Theorem 6.2). They have essentially done what we present below, but because they restrict themselves to the approximants of the random operators restricted to boxes, they do not obtain the generality presented here.

The literature on the existence of the density of states and the Wegner estimate(Wegner [13]) is vast and we refer to the books Cycone-Froese-Kirsch-Simon [3], Carmona-Lacroix [2], Figotin-Pastur [6] for a historical development of the study of density of states and some of the recent papers of Combes-Hislop-Klopp-Nakamura [4], Kirsch-Veselic [10] and Hundertmark-Killip-Nakamura-Stollmann-Veselic [7] for complete references for more recent advances. The latest review paper of Werner Kirsch and Bernd Metzger [9] is a good starting point.

In this paper we consider a direct proof without going through the approximation process and this requires us to declare some average total spectral measure as the density of states and we will choose a definition as in Krishna [11] that agrees with the standard one in the case of the Anderson model.

To this end let $\mathcal{H}$ be a separable Hilbert space and let $(\Omega, \mathcal{B}_\Omega, \mathbb{P})$ be a probability space. We consider a self-adjoint operator valued random variable A and a real valued random variable q on Ω. Thus for each ω, A^ω is a self-adjoint operator and the resolvents $(A^\omega + i)^{-1}$ are weakly measurable in ω (and hence so are $(A^\omega - z)^{-1}$, $Im(z) \neq 0$). q^ω is a measurable real valued function.

Hypothesis 1.1. *Let $(\Omega, \mathcal{B}_\Omega, \mathbb{P})$ and A, q be as above. We shall assume that q and A are independent, which means that for any vectors $f, g \in \mathcal{H}$, the random variables q and $\langle f, Ag\rangle$ are independent and so are $\langle f, \psi(A)g\rangle$ for any bounded measurable function ψ on $\mathbb{R}$.*

Definition 1.2. *Let μ be a probability measure on $\mathbb{R}$. Then μ is said to be uniformly α-Hölder continuous, $0 < \alpha \leq 1$, if*

$$\sup_{x\in\mathbb{R}} \quad \sup_{0<\epsilon\leq 1} \frac{\mu((x-\epsilon, x+\epsilon))}{(2\epsilon)^\alpha} < \infty.$$

Note that the above condition is equivalent to

$$d_\mu^{\alpha,\infty} = \sup_{x\in\mathbb{R}} \sup_{\epsilon\in(0,\infty)} \frac{\mu((x-\epsilon, x+\epsilon))}{(2\epsilon)^\alpha} < \infty.$$

In the definition above we formulated the α-Hölder continuity of a measure, however this implies the same for the distribution function of the measure and so often one can say "the density of states" is uniformly α-Hölder continuous" or "the integrated density of states is uniformly α-Hölder continuous" interchangeably.

Let us consider a pair of operators A, q as in hypothesis 1.1. Let $\mathbb{E}$ denote taking averages of complex valued functions of A with respect to the measure $\mathbb{P}$. Thus if $f : \mathcal{L}(\mathcal{H}) \to \mathbb{C}$, is a bounded measurable function, then

$$\mathbb{E}(f(A)) = \int f(A^\omega) d\mathbb{P}(\omega).$$

Let q be distributed according to the probability measure μ.

Then we have the following theorem, where we denote by $E_B(\cdot)$, the (projection valued) spectral measure of the self-adjoint operator B. We also denote by P_ϕ the orthogonal projection onto the one dimensional subspace generated by the vector ϕ. In what follows the constant $d_\mu^{\alpha,\infty}$ associated with a measure μ is given in Definition 1.2.

Theorem 1.3. *Let A, q be as in hypothesis 1.1 and let ϕ be a unit vector in $\mathcal{H}$. Consider the operators*

$$H^\omega = A^\omega + q^\omega P_\phi$$

and consider the measures

$$\nu_{_{A^\omega}} = \int \langle \phi, E_{H^\omega}(\cdot)\phi \rangle \; d\mu(q^\omega).$$

Suppose q is distributed according to a probability measure μ which is uniformly α-Hölder continuous, $0 < \alpha \leq 1$.

1. *Then $\nu_{_{A^\omega}}$ is uniformly α-Hölder continuous with the same exponent α for each fixed A^ω.*
2. *We have the following uniform bound for each ω.*
$$d^{\alpha,\infty}_{\nu_{_{A^\omega}}} \leq 2^{2-\alpha}\pi \; d_\mu^{\alpha,\infty}.$$
3. *$\mathbb{E}\left(\nu_{_{A^\omega}}\right)$ is also uniformly α-Hölder continuous.*

Strictly speaking one should state the theorem with $H = A + qP_\phi$, and use the notation ν_A etc., but we follow the spectral theory community's convention of writing H^ω etc., to distinguish the "random" operators from the "deterministic" operators.

Remark 1.4. *In the above theorem instead of uniform α-Hölder continuity we could also take some modulus of continuity*

$$s(\mu, \epsilon) = sup_{\{I:|I|<\epsilon\}}\mu(I)$$

the sup taken over intervals I, then the theorem is true for such a modulus of continuity. This remark is private communication by Peter Stollmann [12].

As an application of the above theorem we have the following. Consider $\Omega = \mathbb{R}^{\mathbb{Z}^d}, \mathbb{P} = \prod \mu$, $\mathcal{H} = \ell^2(\mathbb{Z}^\nu)$ and consider the models

$$H^\omega = \Delta + BV^\omega \tag{1}$$

with $(\Delta u)(n) = \sum_{|i|=1} u(n+i)$, $(V^\omega u)(n) = \omega(n)u(n)$, so $\{V(n)\}$ are real valued i.i.d random variables. Here B is a real valued diagonal operator $(Bu)(n) = a_n u(n)$, with the sequence $\{a_n\}$ being non-zero. In the case when $B = I$ the identity operator, one has the Anderson model which is stationary.

Remark 1.5. *We note that if μ is uniformly α-Hölder continuous and if c is a non-zero real number, then the measures μ_c, μ^c defined by*

$$\mu_c(B) = \mu(cB), \text{ for all } B \in \mathcal{B}_\mathbb{R} \text{ or } \mu^c(B) = \mu(B+c), \text{ for all } B \in \mathcal{B}_\mathbb{R},$$

are also uniformly α-Hölder continuous. We note also that by an easy calculation one has for $c \neq 0$,

$$d_{\mu_c}^{\alpha,\infty} = |c|^\alpha d_\mu^{\alpha,\infty} \text{ and } d_{\mu^c}^{\alpha,\infty} = d_\mu^{\alpha,\infty}. \tag{2}$$

The following theorem is then a corollary of theorem 1.3, which is seen by setting, $\Omega = \mathbb{R}^{\mathbb{Z}^\nu}$, $\mathbb{P} = \prod \mu$, $\phi = \delta_n$, V^ω is multiplication by $\omega(n)$ (ω coming from the support of $\mathbb{P}$) and $A^\omega = \Delta + BV^\omega - a_n\omega(n)P_{\delta_n}$ with $q^\omega = a_n\omega(n)$ for each $n \in \mathbb{Z}^\nu$. Therefore $q^\omega(n)$ are distributed according to $\mu_{a_n^{-1}}$ and we have the following theorem whose proof mimics the proof of theorem 1.3 using the above facts in the last few steps.

Theorem 1.6. *Consider the self-adjoint operators H^ω given in equation (1). Suppose $V(n)$ are distributed according to a probability measure μ. Let ν_n denote the measure*

$$\nu_n(\cdot) = \mathbb{E}\left(\langle \delta_n, E_{H^\omega}(\cdot)\delta_n\rangle\right).$$

Suppose μ is uniformly α-Hölder continuous for some $0 < \alpha \leq 1$, then ν_n is also uniformly α-Hölder continuous with exponent α. Any total spectral measure $\nu = \sum_n \beta_n \nu_n$, $\beta_n > 0$, $\sum \beta_n = 1$ such that $\sum \beta_n |a_n|^{-\alpha} < \infty$ is also uniformly α-Hölder continuous.

In the case when $B = I$, the above model in equation (1) reduces to the Anderson model and all the measures ν_n are the same and agree with the density of states of the Anderson model. Therefore we have the following corollary.

Corollary 1.7. *Consider the Anderson model $H^\omega = \Delta + V^\omega$, on $\ell^2(\mathbb{Z}^\nu)$, with $V(n)$ i.i.d distributed according to μ. Suppose the stationary distribution μ is uniformly α-Hölder continuous with exponent α, $0 < \alpha \leq 1$, then the density of states is also uniformly α-Höder continuous with the same exponent α.*

Remark 1.8.

- *In the equation (1), we could have replaced Δ with any self-adjoint operator. Thus Δ perturbed by a periodic perturbation is covered. In fact we can take any orbit $\mathcal{O}_S$ of a subset S of Ω under the $\mathbb{Z}^d$ action (by translation) and take a nice probability measure on this orbit. Then if we take any real valued random variable W supported by $\mathcal{O}$ and take the operators $\Delta + W + V$, the theorem is still valid when we average over all the randomness W and V. Thus the above theorems cover periodic and almost periodic backgrounds.*
- *Since theorem 1.3 is quite abstract it can be phrased in terms of ergodic and non-ergodic dynamical systems and gives numerous corollaries for average spectral measures of the associated self-adjoint operators.*

Finally we mention that in a forthcoming paper with Werner Kirsch [8] we will consider models of the form

$$-\Delta + W + \sum_{i \in \mathbb{Z}^d} q_i \chi_{\Lambda_i},$$

on $L^2(\mathbb{R}^d)$ where Λ_i are cubes centred at $i \in \mathbb{Z}^d$. We show that a class of averaged total spectral measures have the same continuity properties as the single site distribution q_i provided q_i are independent. Here again the cases cover periodic backgrounds and other free parts.

2 Proofs

We begin with a Lemma on Borel transforms, where given a probability measure σ we denote $F_\sigma(z) = \int \frac{1}{x-z}\, d\sigma(x)$.

Lemma 2.1. *Let σ be a probability measure on $\mathbb{R}$. Then for any $y \in \mathbb{R}$ and any $a \in \mathbb{R} \setminus \{0\}$ we have the uniform bound*

$$|\frac{a}{Im(F_\sigma^{-1}(y+ia))}| \leq 2.$$

Proof: We have

$$\begin{aligned}\frac{a}{Im(F_\sigma^{-1}(y+ia))} &= \frac{1}{Im((aF_\sigma)^{-1}(y+ia))} \\ &= -\frac{[Re(aF_\sigma(y+ia))]^2 + [Im(aF_\sigma(y+ia))]^2}{Im(aF_\sigma(y+ia))},\end{aligned} \tag{3}$$

using the fact that

$$Im(z^{-1}) = \frac{-Im(z)}{(Re(z))^2 + (Im(z))^2)}.$$

Now we have

$$aF_\sigma(y+ia) = \int \frac{a}{x-y-ia}\, d\sigma(x) = \int \frac{1}{\frac{x-y}{a} - i}\, d\sigma(x).$$

If we set $\frac{x-y}{a} = \beta(x,y,a)$, then $\beta(x,y,a)$ is real valued and we have

$$aF_\sigma(y+ia) = \int \frac{1}{\beta(x,y,a) - i}\, d\sigma(x).$$

Using this relation and computing the real and imaginary parts of $aF_\sigma(y+ia)$ we have

$$\begin{aligned} Re(aF_\sigma)(y+ia)) &= \int \frac{\beta(x,y,a)}{\beta(x,y,a)^2+1}\, d\sigma(x) \\ |Re(aF_\sigma)(y+ia))| &\leq \int \frac{1}{\sqrt{(\beta(x,y,a)^2+1)}}\, d\sigma(x) \\ Im(aF_\sigma)(y+ia)) &= \int \frac{1}{\beta(x,y,a)^2+1}\, d\sigma(x) \end{aligned} \tag{4}$$

Using these two inequalities and the fact that σ is a probability measure we see that

$$|Im(aF_\sigma)(y+ia))| = \int \frac{1}{\beta(x,y,a)^2+1}\, d\sigma(x) \leq 1. \tag{5}$$

By using the inequality (4) and the Schwarz inequality and the inequality (5), we also have

$$
\begin{aligned}
|Re(aF_\sigma)(y+ia))| &\leq \int \frac{1}{\sqrt{(\beta(x,y,a)^2+1)}}\, d\sigma(x) \\
&\leq \left(\int \frac{1}{\beta(x,y,a)^2+1}\, d\sigma(x) \right)^{\frac{1}{2}} \\
&= |Im(aF_\sigma(y+ia))|^{\frac{1}{2}}.
\end{aligned} \tag{6}
$$

Therefore we immediately get the bounds

$$
\frac{(Re(aF_\sigma(y+ia)))^2}{|Im(aF_\sigma(y+ia))|} \leq 1 \quad \text{and} \quad \frac{(Im(aF_\sigma(y+ia)))^2}{|Im(aF_\sigma(y+ia))|} \leq 1. \tag{7}
$$

This estimate together with equation (3) gives the lemma. □

In the following proposition we give an equivalent condition, in terms of a wavelet transform of the probability measure μ, for it to be a uniformly α-Hölder continuous measure.

Proposition 2.2. *Suppose ψ is a continuously differentiable positive even function on $\mathbb{R}$ satisfying $|\psi(x)| + (1+|x|)|\psi'(x)|$ is integrable and $\psi(0) = 1$. Suppose μ is a probability measure on $\mathbb{R}$. Then, for each $0 < \alpha \leq 1$,*

$$
\sup_{x\in\mathbb{R}} \sup_{a>0} \frac{1}{a^\alpha}(\psi_a * \mu)(x) < \infty \iff d_\mu^{\alpha,\infty} < \infty.
$$

Proof: The lemma is proved if we show that $\frac{1}{a^\alpha}(\psi_a * \mu)(x)$ to be uniformly bounded in x, a if and only if μ is α-Hölder continuous.

To see the if part of this statement, we note the relation

$$
\frac{1}{a^\alpha}(\psi_a * \mu)(x) = -\int_0^\infty \psi'(y)(2y)^\alpha \frac{\mu((x-ay, x+ay))}{(2ay)^\alpha}\, dy, \tag{8}
$$

as in equation (1.3.4) of Demuth-Krishna [5]. We note that for any $x \in \mathbb{R}$ and any $a, y \in (0, \infty)$,

$$
\begin{aligned}
\frac{\mu((x-ay, x+ay))}{(2ay)^\alpha} &\leq \sup_{ay>0} \frac{\mu((x-ay, x+ay))}{(2ay)^\alpha} \\
&\leq \sup_{x\in\mathbb{R}} \sup_{ay>0} \frac{\mu((x-ay, x+ay))}{(2ay)^\alpha} \\
&= d(\mu, \alpha) < \infty,
\end{aligned} \tag{9}
$$

for some constant $d(\mu, \alpha)$, by the uniform α-Hölder continuity of μ (see definition 1.2)). Therefore the right hand side is uniformly bounded in x, a since $|-\psi'(y)(2y)^\alpha| \leq 2|(1+|y|)\psi'(y)|$ is an integrable function, $0 < \alpha \leq 1$ showing that $\frac{1}{a^\alpha}(\psi_a * \mu)(x)$ is uniformly bounded in x, a.

To see the only if part of the statement, note that since ψ is positive and continuous with $\psi(0) = 1$, there is a $\beta > 0$ depending only on ψ such that $\psi(x) \geq \frac{1}{2},\ x \in (-\beta, \beta)$. So we have

$$\frac{1}{a^\alpha}\psi_a * \mu(x) \geq \frac{1}{a^\alpha}\int_{x-\beta a}^{x+\beta a} \psi_a(y-x)d\mu(y) \geq \frac{1}{2}\frac{1}{a^\alpha}\mu((x-\beta a, x+\beta a)). \tag{10}$$

Since β is a fixed positive number, it is easy to see that the μ is uniformly α-Hölder continuous whenever the left hand side of the above inequality is uniformly bounded in (x, a). $\square$

We have a corollary of the above for Borel transforms. Recall that $F_\sigma(z) = \int \frac{1}{x-z}\, d\sigma(x)$.

Lemma 2.3. *Suppose μ is a probability measure on $\mathbb{R}$. Let, $0 < \alpha \leq 1$, then*

$$\sup_{z:Im(z)\neq 0} ||Im(z)|^{1-\alpha} Im(F_\mu(z))| < \infty \iff d_\mu^{\alpha,\infty} < \infty.$$

In addition we have the bound,

$$\sup_{z:Im(z)\neq 0} ||Im(z)|^{1-\alpha} Im(F_\mu(z))| \leq 2^\alpha \pi d_\mu^{\alpha,\infty}. \tag{11}$$

Proof: We set $\psi(x) = \frac{1}{1+x^2}$ in which case the first part of the lemma is valid by setting $z = E + ia$ so that

$$Im(F_\mu(z)) = \frac{1}{a}(\psi_a * \mu)(x), \text{ and } |Im(z)|^{1-\alpha}F_\mu(z) = \frac{1}{a^\alpha}(\psi_a * \mu)(x),$$

where we have taken $\phi * \mu(x) = \int \phi(y-x)d\mu(y)$. Hence the result follows for the case of $a > 0$ from Proposition 2.2. From the equation 8 and the inequality (9) we see that,

$$|\frac{1}{a^\alpha}(\psi_a * \mu)(x)| \leq d_\mu^{\alpha,\infty}\int_0^\infty |\psi'(y)(2y)^\alpha|\ dy,$$

which gives the bound

$$\sup_{x\in\mathbb{R}}\sup_{a>0} |\frac{1}{a^\alpha}(\psi_a * \mu)(x)| \leq d_\mu^{\alpha,\infty}2^\alpha\pi,$$

by making use of the fact that $\psi(x) = \frac{1}{1+x^2}$ in the present case. Since ψ is even, $\psi_{-a}(x) = \psi_a(x)$, so that the lemma for $Im(z) < 0$ follows from that for $Im(z) > 0$. $\square$

Lemma 2.4. *Let σ be a probability measure and let μ be a probability measure which is uniformly α-Hölder continuous, $0 < \alpha \leq 1$, and let $d_\mu^{\alpha,\infty}$ be the constant given in the Definition 1.2. Then we have*

$$\sup_{y\in\mathbb{R}} \sup_{a>0} |a^{1-\alpha} Im\left(\int \frac{1}{x + F_\sigma(y+ia)^{-1}}\ d\mu(x)\right)| < 2\pi\ d_\mu^{\alpha,\infty}. \quad (12)$$

Proof: We first note that since σ is a probability measure on $\mathbb{R}$, the function $F_\sigma(z)$ has non-zero imaginary part whenever z has non-zero imaginary part. Therefore we have

$$\begin{aligned} &\left|a^{1-\alpha} Im\left(\int \frac{1}{x + F_\sigma(y+ia)^{-1}}\ d\mu(x)\right)\right| \\ &= \left|\left(\frac{a}{Im(F_\sigma^{-1}(y+ia))}\right)^{1-\alpha}\right| \\ &\quad \times \left|(Im(F_\sigma^{-1}(y+ia)))^{1-\alpha} Im\left(\int \frac{1}{x + F_\sigma(y+ia)^{-1}}\ d\mu(x)\right)\right| \\ &\leq 2^{1-\alpha} 2^\alpha \pi\ d_\mu^{\alpha,\infty}, \end{aligned} \quad (13)$$

where Lemma 2.1 gives the bound $2^{1-\alpha}$ for the first factor while the second factor is bounded by $\sup_{w:Im(w)\neq 0} |Im(w)|^{1-\alpha} |Im(F_\mu(-w)|$, by taking $w = F_\sigma(y+ia)^{-1}$, which is bound using Lemma 2.3, in view of the uniform α-Hölder continuity of μ. $\square$

Proof of Theorem 1.3 : The parts (1) and (3) are obvious if we prove (2), so we restrict ourselves to proving (2). Let us define

$$\begin{aligned} F^\omega(E+ia) &= \langle \phi, (A^\omega + q^\omega P_\phi - E - ia)^{-1}\phi\rangle, \text{ and} \\ F_0^\omega(E+ia) &= \langle \phi, (A^\omega - E - ia)^{-1}\phi\rangle. \end{aligned} \quad (14)$$

Using the spectral theorem we have

$$\int \frac{1}{x - E - ia}\ d\langle \phi, E_{A^\omega + q^\omega P_\phi}(x)\phi\rangle = F^\omega(E+ia).$$

Then we have, taking average of q^ω with respect to μ, using the definition of ν_{A^ω} and using Fubini,

$$\int \frac{1}{x - E - ia}\ d\nu_{A^\omega}(x) = \int F^\omega(E+ia)\ d\mu(q^\omega) \quad (15)$$

and from Lemma 2.3 it is enough to show that

$$\sup_{E\in\mathbb{R}}\sup_{a>0}|a^{1-\alpha}\int Im(F^\omega(E+ia))\ d\mu(q^\omega)| < d_\mu^{\alpha,\infty}. \tag{16}$$

Let $E \in \mathbb{R}$ and $a > 0$, then we have using the well known rank one perturbation formula for the resolvents (see Lemma 3.1.1 Demuth-Krishna [5] for example) that

$$Im(F^\omega(E+ia)) = Im(\frac{1}{q^\omega + F_0^\omega(E+ia)^{-1}}) \tag{17}$$

and the assumption on A^ω and q^ω imply that the random variables $F_0(E+ia)$ and q are independent. Therefore we have

$$Im(\int F^\omega(E+ia)\ d\mu(q^\omega)) = Im(\int \frac{1}{q^\omega + F_0^\omega(E+ia)^{-1}})\ d\mu(q^\omega)) \tag{18}$$

Thus using equations (15) and (19)

$$Im(\int \frac{1}{x-E-ia}\ d\nu_{A^\omega}(x)) = Im(\int \frac{1}{q^\omega + F_0^\omega(E+ia)^{-1}}\ d\mu(q^\omega)). \tag{19}$$

Now $F_0^\omega(E+ia) = \int \frac{1}{x-E-ia} d\sigma^\omega(x)$ for some probability measure σ^ω (in fact $\sigma^\omega(\cdot) = \langle \phi, E_{A^\omega}(\cdot)\phi \rangle$), which is independent of q^ω by assumption, so fixing it we have

$$Im(\int \frac{1}{x-E-ia}\ d\nu_{A^\omega}(x)) = Im(\int \frac{1}{x + F_0^\omega(E+ia)^{-1}}\ d\mu(x)). \tag{20}$$

From this we see that

$$a^{1-\alpha} Im(\int \frac{1}{x-E-ia}\ d\nu_{A^\omega}(x)) = a^{1-\alpha} Im(\int \frac{1}{x + F_0^\omega(E+ia)^{-1}}\ d\mu(x)). \tag{21}$$

The integral in the expectation on the right hand side is uniformly bounded by $2\pi\ d_\mu^{\alpha,\infty}$ by Lemma 2.4). since μ is uniformly α-Hölder continuous. On the other hand using the inequality (10), noting that $\beta = 1$ there, in the case when $\psi(x) = \frac{1}{1+x^2}$, the left hand side has the lower bound,

$$\begin{aligned} a^{1-\alpha} Im(\int \frac{1}{x-E-ia}\ d\nu_{A^\omega}(x)) &= \frac{1}{a^\alpha}(\psi_a * \mu)(E) \\ &\geq 2^{\alpha-1}\frac{\nu_{A^\omega}((E-a, E+a))}{(2a)^\alpha}. \end{aligned}$$

Therefore we get

$$\frac{\nu_{A^\omega}((x-a,x+a))}{(2a)^\alpha} \leq 2^{1-\alpha} 2\pi \ d_\mu^{\alpha,\infty},$$

which gives the required bound by taking sup over a and x on the left hand side. $\square$

Proof of theorem 1.6: The proof of this theorem proceeds on the same lines of that of theorem 1.3 since $q^\omega(n)$ is distributed according to the probability measure $\mu_{a_n^{-1}}$, using the comments after Remark 1.5. Using this fact and the equation (1) we obtain the bound

$$d_{\nu_n}^{\alpha,\infty} \leq |a_n|^{-\alpha} d_\mu^{\alpha,\infty}.$$

This estimate gives the bound

$$d_\nu^{\alpha,\infty} \leq \sum \beta_n |a_n|^{-\alpha} d_\mu^{\alpha,\infty},$$

from which the stated uniform α-Hölder continuity of ν follows. $\square$

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