Renormalization of Spontaneaously Broken SU(2) Yang-Mills Theory with Flow Equations

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Abstract

Abstract: We present a renormalizability proof for spontaneously broken $SU(2)$ gauge theory based on Flow Equations. It is a conceptually and technically simplified version of the earlier paper [KM] including some extensions. The proof of [KM] also was incomplete since an important assumption made implicitly in the proof of Lemma 2 there is not verified. So the present paper is also a corrected version of [KM].

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1 Introduction

The differential flow equations [WH] of the renormalization group [W] offer a powerful tool for a unified approach to the analysis of systems with infinitely many degrees of freedom. Although first conceived for an analysis of such systems beyond perturbation theory, it was realized by Polchinski [P] that these equations also paved the way for a new elegant approach to perturbative renormalization theory[1]. Local gauge theories, however, present particular difficulties in this approach because the momentum space regulator violates gauge invariance. Thus dimensional renormalization is in practice the most popular scheme for renormalizing such theories in perturbation theory. But at the same time this scheme is restricted to Feynman graphs. It not only defies to be given rigorous meaning in path integral formulations, it does not even directly apply in a mathematical sense to perturbative Green functions as a whole without splitting them into graphs. Thus, in some sense it is farthest away from nonperturbative analysis, and it does not allow to address a number of interesting conceptual, mathematical and quantitative questions. The authors analysed spontaneously broken SU(2)-Yang-Mills theory with flow equations in [KM]. This analysis was simplified in [M]. In an endeavour to further simplify and clarify the analysis, which was also caused by lecturing on the subject several times, we came across an error in [KM], which reappeared in [M] by quotation. In fact Lemma 2 in [KM] cannot be proven without an assumption made implicitly in its proof, which did not take into account the presence of irrelevant boundary terms in the bare action. These terms have been "forgotten" because the context of the proof had changed in the progress of our work, after the Lemma had been written. Since we have found quite a number of further simplifcations in the mean time, since the subject is important in physics, and since a correction of [KM] required quite a lot of changes, even if the line of argument stays the same, we preferred to write a self-contained modern and (hopefully !) mathematically correct version of our previous paper.

The strategy of proof remains that of [KM]. The (ultraviolet) power counting part of the flow equation renormalization proof is universal and simple for all renormalizable theories. For gauge theories we have to show that gauge invariance can be restored when the cutoffs are taken away. On the level of the Green functions (which are not gauge invariant) this means that we have to verify the Slavnov-Taylor identities (STI) of the theory. They then allow to argue that physical quantities such as the S-matrix are gauge-invariant [Z]. On analysing the flow equations (FE) for a gauge theory one realizes that the restoration of the STI depends on the choice of the renormalization conditions chosen and cannot be true in general. More

[1] Wilson himself remarked already in the late sixties that this should be possible, as we learned from E. Brézin.

precisely, since gauge invariance is violated in the regularized theory, the renormalization group flow will generally produce nonvanishing contributions to all those relevant parameters of the theory, which are forbidden by gauge invariance, e.g. a noninvariant gauge field self-coupling of the form $(\vec{A}^2)^2$. The question is then: Can we use the freedom in adjusting the renormalization conditions such that the STI are nevertheless restored in the end? To answer this question a first observation is crucial: The violation of the STI in the regularized theory can be expressed through Green functions carrying an operator insertion, which depends on the regulators. FE theory for such insertions tells us that these Green functions will vanish once the cutoffs are removed, if we achieve renormalization conditions on the noninserted Green functions such that the inserted ones, which are calculated from those, have vanishing renormalization conditions for all relevant terms, i.e. up to the dimension of the insertion (which is 5 in our case). Comparing the number of relevant terms for the SU(2) theory - 37 (see App.A)- and for the insertion - 53 (see App.C) -, we realize that it is not possible to make vanish 53 terms on adjusting 37 free parameters, unless there are linear interdependences. These interdependences are revealed in the analysis of the present paper. As compared to [KM] we also include the proof of the validity of the equation of the antighost in the renormalized theory for suitable renormalization conditions.

This paper is organized as follows. In Section 2 we introduce the classical action of the model and the BRST-transformations, [BRS], [T]. In Section 3 we introduce the concepts from FE theory and recall the statements on renormalizability we need. In particular we introduce the above mentioned operator insertions. When using FE it is natural to analyse the generating functional of free propagator amputated Schwinger functions. The analysis of the STI is however technically simpler for one-particle irreducible vertex functions so that we introduce the generating functionals of both, together with the corresponding renormalizability statements. In Section 4 we derive the violated Slavnov-Taylor identities (VSTI) for the regularized theory in various forms for the bare and the renormalized functionals. The Sections 1 to 4 follow closely the line of [KM]. In Section 5 we present the new tool required in view of the fact that Lemma 2 of [KM] has become obsolete. Namely the generating functional of the vertex functions is not only expanded w.r.t. to fields and momenta, but also w.r.t. the mass parameters, as far as their presence indicates improvement of UV power counting. The corresponding redefinition of relevant renormalization constants permits a *complete* analysis of the relevant part of the STI in terms of the renormalization conditions. We do not need any more to jump from bare to renormalized functionals and vice versa. It is then possible to show that for suitable renormalization conditions the inserted functional decribing the violation of the STI has no relevant part. This result together with an obvious bound on its irrelevant part at the regularization scale Λ_0, following directly from the properties of regulator, permits to prove that the violation disappears for $\Lambda_0 \to \infty$ so that the STI hold in this limit. This proof finally elucidates the fact the validity of the STI can directly and fully be settled by analysing the (large) system of equations describing its relevant part at the renormalization point. This aim was not achieved in [KM].

We reproduce the appendices of [KM] with slight notational changes. In Appendix A we list all 37 relevant terms allowed by the global symmetries of $SU(2)$-Yang-Mills theory. In Appendix B the 7 relevant terms appearing in the inserted functionals describing the BRST-transformations are listed. In Appendix C we list the 53 equations corresponding to the relevant contributions to the inserted functional describing the violation of the STI. By analysis of this system of equations we show restoration of gauge symmetry in the (properly) renormalized theory.

A reader familiar with the power counting results following from the flow equations can skip the major part of Section 3. He might use it for finding some notations also used in later Sections and to get acquainted with the mass expansion of the Schwinger functions which is used for the first time in this paper. It is described in the last part of Section 3.1 (from (55) onwards) and in the last page of Section 3.2 (from (88) onwards).

2 The classical action

Following closely the monograph of Faddeev and Slavnov [FS], we collect some basic properties of the classical Euclidean SU(2) Yang-Mills-Higgs model on four-dimensional Euclidean space-time. The fields of the model are a triplet $\{A^a_\mu\}_{a=1,2,3}$ of real vector fields and the complex scalar doublet $\{\phi_\alpha\}_{\alpha=1,2}$. The classical action has the form

$$S_{inv} = \int dx \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} (\nabla_\mu \phi)^* \nabla_\mu \phi + \lambda (\phi^* \phi - \rho^2)^2 \right\}, \tag{1}$$

with the field strength tensor

$$F^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g \epsilon^{abc} A^b_\mu(x) A^c_\nu(x) \tag{2}$$

and the covariant derivative

$$\nabla_\mu = \partial_\mu + g \frac{1}{2i} \sigma^a A^a_\mu(x) \tag{3}$$

acting on the SU(2)-spinor ϕ. The parameters g, λ, ρ are real positive, ϵ^{abc} is totally skew symmetric, $\epsilon^{123} = +1$, and $\{\sigma^a\}_{a=1,2,3}$ are the standard Pauli matrices. The action (1) is invariant under local gauge transformations of the fields

$$\begin{aligned} \frac{1}{2i} \sigma^a A^a_\mu(x) &\longrightarrow u(x) \frac{1}{2i} \sigma^a A^a_\mu(x) u^*(x) + g^{-1} u(x) \partial_\mu u^*(x), \\ \phi(x) &\longrightarrow u(x) \phi(x), \end{aligned} \tag{4}$$

with $u : \mathbf{R}^4 \to \mathrm{SU}(2)$, smooth. The choice of a stable equilibrium point of the action (1) leads to spontaneous symmetry breaking, dealt with by reparametrizing the complex scalar doublet as

$$\phi(x) = \begin{pmatrix} B^2(x) + iB^1(x) \\ \rho + h(x) - iB^3(x) \end{pmatrix} , \tag{5}$$

where $\{B^a(x)\}_{a=1,2,3}$ is a real triplet and $h(x)$ the real Higgs field. Moreover, in place of the parameters ρ, λ the masses

$$m = \frac{1}{2}\, g\rho, \quad M = (8\lambda\rho^2)^{\frac{1}{2}} \tag{6}$$

are used. Aiming at a quantized theory, pure gauge degrees of freedom have to be eliminated. We choose the 't Hooft gauge fixing, with $\alpha \in \mathbf{R}_+$,

$$S_{g.f.} = \frac{1}{2\alpha} \int dx (\partial_\mu A^a_\mu - \alpha m B^a)^2 \ . \tag{7}$$

With regard to functional integration this condition is implemented by introducing anticommuting Faddeev-Popov ghost and antighost fields $\{c^a\}_{a=1,2,3}$ and $\{\bar{c}^a\}_{a=1,2,3}$, respectively, and forming with these six independent scalar fields the additional term in the action

$$S_{gh} = - \int dx \bar{c}^a \{(-\partial_\mu \partial_\mu + \alpha m^2)\delta^{ab} + \frac{1}{2}\,\alpha g m h \delta^{ab} + \frac{1}{2}\,\alpha g m \epsilon^{acb} B^c - g \partial_\mu \epsilon^{acb} A^c_\mu\} c^b. \tag{8}$$

Hence, we have the total "classical action"

$$S_{\mathrm{BRS}} = S_{\mathrm{inv}} + S_{\mathrm{g.f.}} + S_{\mathrm{gh}}, \tag{9}$$

which is decomposed as

$$S_{\mathrm{BRS}} = \int dx \, \{\mathcal{L}_{\mathrm{quad}}(x) + \mathcal{L}_{\mathrm{int}}(x)\} \tag{10}$$

into its quadratic part, where $\Delta \equiv \partial_\mu \partial_\mu$,

$$\begin{aligned} \mathcal{L}_{\mathrm{quad}} \quad = \quad & \frac{1}{4}\,(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 + \frac{1}{2\alpha}\,(\partial_\mu A^a_\mu)^2 + \frac{1}{2}\,m^2 A^a_\mu A^a_\mu + \frac{1}{2}\,h(-\Delta + M^2)h \\ & + \frac{1}{2}\,B^a(-\Delta + \alpha m^2)B^a - \bar{c}^a(-\Delta + \alpha m^2)c^a \,, \end{aligned} \tag{11}$$

and into its interaction part

$$\begin{aligned} \mathcal{L}_{\mathrm{int}} \quad = \quad & g\epsilon^{abc}(\partial_\mu A^a_\nu) A^b_\mu A^c_\nu + \frac{1}{4}\,g^2(\epsilon^{abc} A^b_\mu A^c_\nu)^2 \\ & + \frac{1}{2}\,g\left\{(\partial_\mu h) A^a_\mu B^a - h A^a_\mu \partial_\mu B^a - \epsilon^{abc} A^a_\mu (\partial_\mu B^b) B^c\right\} \\ & + \frac{1}{8}\,g A^a_\mu A^a_\mu \left\{4mh + g(h^2 + B^a B^a)\right\} \\ & + \frac{1}{4}\,g \frac{M^2}{m} h(h^2 + B^a B^a) + \frac{1}{32}\,g^2 \left(\frac{M}{m}\right)^2 (h^2 + B^a B^a)^2 \\ & - \frac{1}{2}\,\alpha g m \bar{c}^a \left\{h\delta^{ab} + \epsilon^{acb} B^c\right\} c^b - g\,\epsilon^{acb}(\partial_\mu \bar{c}^a) A^c_\mu c^b \ . \end{aligned} \tag{12}$$

Inspecting the quadratic part (11) we recognize two favourable consequences of the particular gauge fixing (7) : this part is diagonal in the fields (no coupling $A^a_\mu \partial_\mu B^a$ appears) and all fields are massive.
As a prerequisite to state the symmetries of S_{BRS} (10), composite classical fields are introduced as follows:

$$
\begin{aligned}
\psi^a_\mu(x) &= \left\{\partial_\mu \, \delta^{ab} + g\epsilon^{arb} A^r_\mu(x)\right\} c^b(x), \\
\psi(x) &= -\frac{1}{2}\, g B^a(x) c^a(x), \\
\psi^a(x) &= \left\{(m + \frac{1}{2}\, g\, h(x))\delta^{ab} + \frac{1}{2}\, g\epsilon^{arb} B^r(x)\right\} c^b(x), \\
\Omega^a(x) &= \frac{1}{2}\, g\epsilon^{apq} c^p(x) c^q(x) \ .
\end{aligned}
\tag{13}
$$

We can then write (8) in the form

$$
S_{gh} = -\int dx\, \bar{c}^a \{-\partial_\mu \psi^a_\mu + \alpha m \psi^a\} \ . \tag{14}
$$

The classical action S_{BRS} (10), shows the following symmetries:

i) Euclidean invariance: S_{BRS} is an O(4)-scalar.

ii) Rigid SO(3)-isosymmetry: The fields $\{A^a_\mu\}, \{B^a\}, \{c^a\}, \{\bar{c}^a\}$ are isovectors and h an isoscalar; S_{BRS} is invariant under spacetime independent SO(3)-transformations.

iii) BRS-invariance:
The BRS-transformations of the basic fields [BRS] are defined as

$$
\begin{array}{rcl}
A^a_\mu(x) & \longrightarrow & A^a_\mu(x) - \psi^a_\mu(x)\, \varepsilon, \\
h(x) & \longrightarrow & h(x) - \psi(x)\, \varepsilon, \\
B^a(x) & \longrightarrow & B^a(x) - \psi^a(x)\, \varepsilon, \\
c^a(x) & \longrightarrow & c^a(x) - \Omega^a(x)\, \varepsilon, \\
\bar{c}^a(x) & \longrightarrow & \bar{c}^a(x) - \dfrac{1}{\alpha} \left(\partial_\nu A^a_\nu(x) - \alpha m B^a(x)\right) \varepsilon
\end{array}
\tag{15}
$$

with the composite fields (13), and ε is a Grassmann element not depending on space-time, that commutes with the fields $\{A^a_\mu, h, B^a\}$ but anticommutes with the (anti-) ghosts $\{c^a, \bar{c}^a\}$. To show the BRS-invariance of the total classical action (9) one first observes that the composite classical fields (13) are themselves invariant under the BRS-transformations (15). Herewith, and using (14), it follows easily that the sum $S_{\text{g.f.}} + S_{\text{gh}}$ is invariant under the transformation (15). Finally, on S_{inv} act only the BRS-transformations of the fields A^a_μ, B^a, h,

which amounts to local gauge transformations.
We observe that upon scaling the composite fields (13) entering the BRS-transformations as well as S_{gh} (14), by a factor of λ, the corresponding $S_{\rm BRS}$ remains invariant under such BRS-transformations.

3 Renormalization without Slavnov-Taylor identities

3.1 The Flow Equations for the Schwinger Functions

Quantization of the theory by means of functional integration in the realm of (formal) power series is based on a Gaussian measure related to the quadratic part (11) of $S_{\rm BRS}$ (10). Denoting the differential operators appearing there by

$$D_{\mu\nu} := (-\Delta + m^2)\,\delta_{\mu\nu} - \frac{1-\alpha}{\alpha}\,\partial_\mu\partial_\nu\,, \quad \tilde{D} := -\Delta + M^2, \quad D := -\Delta + \alpha m^2\,, \tag{16}$$

we write

$$\int dx\ \mathcal{L}_{\rm quad}(x) = \frac{1}{2}\,\langle A^a_\mu, D_{\mu\nu}\,A^a_\nu\rangle + \frac{1}{2}\,\langle h, \tilde{D}h\rangle + \frac{1}{2}\,\langle B^a, DB^a\rangle - \langle \bar{c}^a, Dc^a\rangle\,. \tag{17}$$

To these differential operators (16) are associated the (free) propagators

$$C_{\mu\nu}(x,y) = \frac{1}{(2\pi)^4}\int dk\ e^{ik(x-y)}\ C_{\mu\nu}(k)\ , \tag{18}$$

and similarly in the other cases, with

$$C_{\mu\nu}(k) = \frac{1}{k^2+m^2}\Big(\delta_{\mu\nu} - (1-\alpha)\frac{k_\mu k_\nu}{k^2+\alpha m^2}\Big), \quad C(k) = \frac{1}{k^2+M^2}, \quad S(k) = \frac{1}{k^2+\alpha m^2}\,. \tag{19}$$

A Gaussian product measure, the covariances of which are a regularized version of the propagators (18), (19), forms the point of departure. We choose the cutoff function, improving slightly the former one of [M],

$$\sigma_\Lambda(k^2)\ =\ \exp\Big(-\frac{(k^2+m^2)(k^2+\alpha m^2)(k^2+M^2)(k^2)^2}{\Lambda^{10}}\Big)\,. \tag{20}$$

It is positive, invertible and analytic, and has the property

$$\frac{d}{dk^2}\,\sigma_\Lambda(k^2)|_{k^2=0}\ =\ 0 \tag{21}$$

which will be helpful in the analysis of the relevant part of the STI later on. Employing this cutoff function we define the regularized propagators, with UV-cutoff $\Lambda_0 < \infty$ and a flow parameter Λ satisfying $0 \le \Lambda \le \Lambda_0$,

$$C^{\Lambda,\Lambda_0}_{\mu\nu}(k) \equiv C_{\mu\nu}(k)\ \sigma_{\Lambda,\,\Lambda_0}(k^2) := C_{\mu\nu}(k)\,\big(\sigma_{\Lambda_0}(k^2) - \sigma_\Lambda(k^2)\big) \tag{22}$$

and similarly for $C(k)$, $S(k)$. The particular choice (20) implies

$$\partial_\Lambda C^{\Lambda,\Lambda_0}_{\mu\nu}(k) = -\frac{10}{\Lambda^3}\cdot\frac{(k^2+\alpha m^2)\delta_{\mu\nu}-(1-\alpha)k_\mu k_\nu}{\Lambda^2}\cdot\frac{(k^2+M^2)(k^2)^2}{\Lambda^6}\,\sigma_\Lambda(k^2)\,,$$

and similarly in the other cases. Herefrom follow the bounds, using $C^{\Lambda,\Lambda_0}(k)$ as a collective symbol for the propagators considered,

$$\left|\partial^w\partial_\Lambda\, C^{\Lambda,\Lambda_0}(k)\right| \le \left\{ \begin{array}{ll} c_{|w|}\,\sigma_{2\Lambda}(k^2) & for \quad 0\le\Lambda\le m\ , \\ \Lambda^{-3-|w|}P_{|w|}(\frac{|k|}{\Lambda})\,\sigma_\Lambda(k^2) & for \quad \Lambda>m\ . \end{array}\right\} \tag{23}$$

On the l.h.s. ∂^w denotes a $|w|-$fold partial momentum derivative (see below (39)). Moreover, the polynomials $P_{|w|}$ have nonnegative coefficients, which, as well as the constants $c_{|w|}$, depend on $\alpha, m, M, |w|$ only. Considering $\sigma_\Lambda(k^2)$, (20), as a function of (Λ,k^2), it cannot be extended continuously to $(0,0)$. We set $\sigma_0(0):=\lim_{k^2\to 0}\sigma_0(k^2)=0$, and hence $\sigma_{0,\Lambda_0}(0)=\sigma_{\Lambda_0}(0)=1$.

It is convenient to introduce a short collective notation for the various fields and their sources:
i) We denote the bosonic fields and the corresponding sources, respectively, by

$$\varphi_\tau = (A^a_\mu\,,\, h,\, B^a)\ ,\quad J_\tau=(j^a_\mu\,,\, s,\, b^a)\ , \tag{24}$$

ii) and all fields and their respective sources by

$$\Phi = (\varphi_\tau\,,\, c^a,\, \bar c^a)\ ,\quad K=(J_\tau\,,\, \bar\eta^a,\, \eta^a)\ . \tag{25}$$

The sources η^a and $\bar\eta^a$ are Grassmann elements and have ghost number $+1$ and -1, respectively. In the sequel, we exclusively use left derivatives with respect to these quantities.
The characteristic functional of the Gaussian product measure with the covariances $\hbar C^{\Lambda,\Lambda_0}$ from (22), (19) is then given by

$$\int d\mu_{\Lambda,\Lambda_0}(\Phi)\, e^{\frac{1}{\hbar}\langle\Phi,K\rangle} \;=\; e^{\frac{1}{\hbar}P^{\Lambda,\Lambda_0}(K)}\ , \tag{26}$$

where

$$\langle\Phi,K\rangle := \int dx\Big(\sum_\tau \varphi_\tau(x)J_\tau(x)+\bar c^a(x)\eta^a(x)+\bar\eta^a(x)c^a(x)\Big)\ , \tag{27}$$

$$P^{\Lambda,\Lambda_0}(K) \;=\; \frac{1}{2}\,\langle j^a_\mu, C^{\Lambda,\Lambda_0}_{\mu\nu}\, j^a_\nu\rangle + \frac{1}{2}\,\langle s, C^{\Lambda,\Lambda_0}\, s\rangle + \frac{1}{2}\,\langle b^a, S^{\Lambda,\Lambda_0}\, b^a\rangle - \langle\bar\eta^a, S^{\Lambda,\Lambda_0}\,\eta^a\rangle\ . \tag{28}$$

Aiming at a quantized descendant of the classical theory, we consider the generating functional $L^{\Lambda,\Lambda_0}(\Phi)$ of the connected amputated Schwinger functions (CAS)

$$e^{-\frac{1}{\hbar}\left(L^{\Lambda,\Lambda_0}(\Phi)+I^{\Lambda,\Lambda_0}\right)} \;=\; \int d\mu_{\Lambda,\Lambda_0}(\Phi')e^{-\frac{1}{\hbar}L^{\Lambda_0,\Lambda_0}(\Phi'+\Phi)}\ , \tag{29}$$

$$L^{\Lambda,\Lambda_0}(0) \;=\; 0\ . \tag{30}$$

The constant I^{Λ,Λ_0} is the vacuum part of the theory which is proportional the volume because of translation invariance. It therefore requires to consider the theory at first in a finite volume $\Omega \subset \mathbf{R}^4$. For details see [KMR].

Since the regularization necessarily violates the local gauge symmetry, the bare functional

$$L^{\Lambda_0,\Lambda_0}(\Phi) = \int dx\ \mathcal{L}_{\rm int}(x)\ +\ L^{\Lambda_0,\Lambda_0}_{c.t.}(\Phi) \tag{31}$$

in a first stage has to be chosen sufficiently general in order to allow for the restoration of the Slavnov-Taylor identities at the end. Therefore, we add to the interaction part (12) of classical origin counter terms $L^{\Lambda_0,\Lambda_0}_{c.t.}$, which a priori include all local terms of mass dimension ≤ 4 permitted by the unbroken global symmetries, i.e. Euclidean $O(4)$-invariance and $SO(3)$-isosymmetry. There are 37 such terms, by definition all at least of order $\mathcal{O}(\hbar)$. The general bare functional is presented in Appendix A.
From (29) the corresponding flow equation follows upon differentiation with respect to the flow parameter Λ,

$$\partial_\Lambda\, e^{-\frac{1}{\hbar}\left(L^{\Lambda,\Lambda_0}(\Phi)+I^{\Lambda,\Lambda_0}\right)} = \hbar\,\dot{\Delta}_{\Lambda,\Lambda_0}\ e^{-\frac{1}{\hbar}\left(L^{\Lambda,\Lambda_0}(\Phi)+I^{\Lambda,\Lambda_0}\right)}\ , \tag{32}$$

where the r.h.s. is obtained on derivation of the Gaussian measure $d\mu_{\Lambda,\Lambda_0}(\Phi')$ and observing that the integrand is a function of $\Phi'+\Phi$. The "dot" appearing on the functional Laplace operator

$$\Delta_{\Lambda,\Lambda_0} = \frac{1}{2}\left\langle\frac{\delta}{\delta A^a_\mu}, C^{\Lambda,\Lambda_0}_{\mu\nu}\frac{\delta}{\delta A^a_\nu}\right\rangle + \frac{1}{2}\left\langle\frac{\delta}{\delta h}, C^{\Lambda,\Lambda_0}\frac{\delta}{\delta h}\right\rangle + \frac{1}{2}\left\langle\frac{\delta}{\delta B^a}, S^{\Lambda,\Lambda_0}\frac{\delta}{\delta B^a}\right\rangle + \left\langle\frac{\delta}{\delta c^a}, S^{\Lambda,\Lambda_0}\frac{\delta}{\delta \bar{c}^a}\right\rangle \tag{33}$$

denotes differentiation with respect to Λ. Hence, we arrive at the flow equation

$$\begin{aligned}\partial_\Lambda\left(L^{\Lambda,\Lambda_0}(\Phi)+I^{\Lambda,\Lambda_0}\right) = \frac{\hbar}{2}\Big(\sum_\tau\Big\langle\frac{\delta}{\delta\varphi_\tau}, \dot{C}^{\Lambda,\Lambda_0}_\tau\frac{\delta}{\delta\varphi_\tau}\Big\rangle + 2\Big\langle\frac{\delta}{\delta c^a}, \dot{S}^{\Lambda,\Lambda_0}\frac{\delta}{\delta\bar{c}^a}\Big\rangle\Big)L^{\Lambda,\Lambda_0}(\Phi)\\ -\frac{1}{2}\sum_\tau\Big\langle\frac{\delta L^{\Lambda,\Lambda_0}}{\delta\varphi_\tau}, \dot{C}^{\Lambda,\Lambda_0}_\tau\frac{\delta L^{\Lambda,\Lambda_0}}{\delta\varphi_\tau}\Big\rangle - \Big\langle\frac{\delta L^{\Lambda,\Lambda_0}}{\delta c^a}, \dot{S}^{\Lambda,\Lambda_0}\frac{\delta L^{\Lambda,\Lambda_0}}{\delta\bar{c}^a}\Big\rangle .\end{aligned} \tag{34}$$

Since we restrict to perturbation theory, the generating functional will be considered within a formal loop expansion

$$L^{\Lambda,\Lambda_0}(\Phi) = \sum_{l=0}^\infty \hbar^l L^{\Lambda,\Lambda_0}_l(\Phi)\ . \tag{35}$$

Furthermore, decomposing into particular n-point Schwinger functions we use a multiindex n, the components of which denote the number of each source field species appearing:

$$n = (n_A,\, n_h,\, n_B,\, n_{\bar{c}},\, n_c)\ ,\quad |n| = n_A+n_h+n_B+n_{\bar{c}}+n_c\ . \tag{36}$$

Because of (12) there will not appear 1- and 2-point functions at the tree level ($l = 0$). If we do not regard the vacuum part, we can study the flow of the n-point functions in the infinite volume limit $\Omega \to \mathbf{R}^4$. Due to translation invariance, it is convenient to consider also the Fourier transformed source field $\hat{\Phi}$, the conventions used are

$$\int_p := \int_{\mathbf{R}^4} \frac{d^4p}{(2\pi)^4} \;, \quad \Phi(x) = \int_p e^{ipx} \hat{\Phi}(p) \longrightarrow \delta_{\Phi(x)} := \frac{\delta}{\delta\Phi(x)} = (2\pi)^4 \int_p e^{-ipx}\, \delta_{\hat{\Phi}(p)} \,. \tag{37}$$

Given these conventions, the momentum representation of the n-point function with multiindex n, (36), at loop order l is obtained as an $|n|$-fold functional derivative

$$(2\pi)^{4(|n|-1)}\, \delta^{\,n}_{\hat{\Phi}(p)} L_l^{\Lambda,\Lambda_0}(\Phi)|_{\Phi=0} \;=\; \delta(p_1 + \cdots + p_{|n|})\, \mathcal{L}^{\Lambda,\Lambda_0}_{l,n}(p_1, \cdots, p_{|n|}) \;. \tag{38}$$

For the sake of a slim appearance, the notation does not reveal how the momenta are assigned to the multiindex n, and in addition, the $O(4)$- and $SO(3)$-tensor structure remains hidden. By definition the n-point function is completely symmetric (antisymmetric) if the variables that belong to each of the bosonic (fermionic) species occurring are permuted. As momentum derivatives of n-point functions have to be considered, too, we also introduce the shorthand notation

$$w = (w_{1,1}, \cdots, w_{n-1,4}) \;, \quad w_{i,\mu} \in \mathbf{N}_0 \;, \quad \partial^w := \prod_{i=1}^{n-1} \prod_{\mu=1}^{4} \Big(\frac{\partial}{\partial p_{i,\mu}}\Big)^{w_{i,\mu}} \;, \quad |w| = \sum_{i,\mu} w_{i,\mu} \;. \tag{39}$$

The system of flow equations (FE) for the connected amputated Schwinger functions (CAS) then follows from (34), using (35),(38), and finally performing the momentum derivatives (39)

$$\begin{aligned} \partial_\Lambda \partial^w \mathcal{L}^{\Lambda,\Lambda_0}_{l,n}(p_1, \cdots, p_{|n|}) \;=\; & \sum_{n',|n'|=|n|+2} c_{n-n'} \int_k (\partial_\Lambda C^{\Lambda,\Lambda_0}(k))\, \partial^w \mathcal{L}^{\Lambda,\Lambda_0}_{l-1,n'}(k, -k, p_1, \cdots, p_{|n|}) \\ & - \sum_{\substack{l_1+l_2=l,\, w_1+w_2+w_3=w \\ n_1,n_2,|n_1|+|n_2|=|n|+2}} c_{\{w_i\}} \Big[c_{n_1,n_2}\, \partial^{w_1} \mathcal{L}^{\Lambda,\Lambda_0}_{l_1,n_1}(p_1, \ldots, p_{|n_1|-1}, p') \\ & \qquad \cdot (\partial^{w_3} \partial_\Lambda C^{\Lambda,\Lambda_0}(p'))\;\; \partial^{w_2} \mathcal{L}^{\Lambda,\Lambda_0}_{l_2,n_2}(-p', \ldots, p_{|n|}) \Big]_{s,a} \;. \end{aligned} \tag{40}$$

The field assignment of the propagators C^{Λ,Λ_0} on the r.h.s. is not written, it is implicit in the multiindices n', n_1, n_2 related to n. In the linear term the integrated momentum k refers to that of the fields from $n'-n$ and the factor $c_{n-n'}$ has the value $1/2$ and 1 in the case of bosons and fermions, respectively. In the bilinear term we have $-p' = p_1 + \ldots + p_{|n_1|-1}$. Furthermore the subscripts s, a indicate full (anti)symmetrization according to the statistics of the various fields, requiring the combinatorial constants c_{n_1,n_2} to rule out those permutations, which act solely within a given CAS.[2] The combinatoric coefficients $c_{\{w_i\}}$ stem form the Leibniz rule and have the values $c_{\{w_i\}} = \frac{w!}{w_1!w_2!w_3!}$, where $w! = \prod_{i,\mu} w_{i,\mu}!$.

To end up with Schwinger functions fulfilling the Slavnov-Taylor identities (STI), we have to consider Schwinger functions with a composite field inserted, too. Two kinds of such insertions have to be dealt with: local insertions implementing the BRS-variations, and a space-time integrated insertion representing the intermediate violation of the STI.
The classical composite BRS-fields (13) all have mass dimension 2 and transform as vector-isovector, scalar-isoscalar, scalar-isovector and scalar-isovector, respectively. Moreover, the first three have ghost number 1, whereas the last one has ghost number 2. Hence, adding counterterms, we introduce the bare composite fields

$$(\psi_\mu^a)^{0,\Lambda_0}(x) = R_1^0\, \partial_\mu c^a(x) + R_2^0\, g\, \epsilon^{arb} A_\mu^r(x)\, c^b(x) \ , \tag{41a}$$

$$(\psi)^{0,\Lambda_0}(x) = -R_3^0\, \frac{1}{2} g\, B^a(x) c^a(x) \ , \tag{41b}$$

$$(\psi^a)^{0,\Lambda_0}(x) = R_4^0\, m\, c^a(x) + R_5^0\, \frac{1}{2} g\, h(x)\, c^a(x) + R_6^0\, \frac{1}{2} g\, \epsilon^{arb} B^r(x)\, c^b(x) \ , \tag{41c}$$

$$(\Omega^a)^{0,\Lambda_0}(x) = R_7^0\, \frac{1}{2} g\, \epsilon^{apq} c^p(x) c^q(x) \ , \tag{41d}$$

keeping the notation from (13) but using it henceforth exclusively according to (41a)-(41d). We set

$$R_i^0 \ = \ 1 + \mathcal{O}(\hbar) \ , \tag{42}$$

thus viewing the counterterms again as formal power series in $\hbar$; the tree order $\hbar^0$ provides the classical terms (13). Observe that for $l > 0$ the field products appearing in the classical composite fields ψ_μ^a and ψ^a of (13) do require R_1^0 and R_4^0 , respectively, as counterterms. Moreover, it is important to note that the modified composite fields (41a)-(41d) remain *invariant* under the BRS-transformations (15) upon assuming the conditions

$$R_6^0 = R_7^0 = R_2^0\,, \qquad R_3^0\, R_5^0 = (R_2^0\,)^2 \tag{43}$$

and employing the generalized composite fields (41a)-(41d) in place of the original ones, (13). To deal with Schwinger functions showing *one* insertion, the bare interaction (31) is modified adding the composite fields (41a)-(41d) coupled to corresponding sources

$$\tilde{L}^{\Lambda_0,\Lambda_0}(\xi;\Phi) := L^{\Lambda_0,\Lambda_0}(\Phi) + L^{\Lambda_0,\Lambda_0}(\xi) \ , \tag{44}$$

[2]For details see [M], eq.(2.28).

$$L^{\Lambda_0,\Lambda_0}(\xi) = \int dx\ \{\gamma^a_\mu(x)\psi^a_\mu(x) + \gamma(x)\psi(x) + \gamma^a(x)\psi^a(x) + \omega^a(x)\Omega^a(x)\}\ . \tag{45}$$

According to the properties of these composite fields, the sources $\gamma^a_\mu, \gamma, \gamma^a$ are Grassmann elements, they all have canonical dimension 2 and ghost number -1 , whereas ω^a has canonical dimension 2 and ghost number -2 . For the insertions and their respective sources we also introduce a short collective notation

$$\psi_\tau = (\psi^a_\mu\,,\,\psi,\,\psi^a)\ ,\quad \gamma_\tau = (\gamma^a_\mu\,,\gamma,\,\gamma^a)\ ,\quad \xi = (\gamma_\tau\,,\,\omega^a)\ . \tag{46}$$

Using now (44) in place of L^{Λ_0,Λ_0} as the bare action in the representation (29) provides the functional $\tilde L^{\Lambda,\Lambda_0}(\xi\,;\Phi)$, from which the generating functional of the regularized CAS with *one* insertion $\psi(x)$ follows as

$$L^{\Lambda,\Lambda_0}_\gamma(x\,;\Phi) := \frac{\delta}{\delta\gamma(x)}\tilde L^{\Lambda,\Lambda_0}(\xi\,;\Phi)|_{\,\xi=0}\ , \tag{47}$$

and similarly for the other insertions from (45). In the infinite volume limit, and performing a Fourier transform of the insertion position we obtain

$$\hat L^{\Lambda,\Lambda_0}_\gamma(q\,;\Phi) = \int dx\ e^{iqx}\ L^{\Lambda,\Lambda_0}_\gamma(x\,;\Phi)\ . \tag{48}$$

After loop expansion the n-point function with one insertion ψ is obtained as

$$\delta(q+p_1+\cdots+p_{|n|})\ \mathcal{L}^{\Lambda,\Lambda_0}_{\gamma;\,l,n}(q\,;p_1,\cdots,p_{\,|n|}) := (2\pi)^{4(|n|-1)}\delta^n_{\hat\Phi(p)}\hat L^{\Lambda,\Lambda_0}_{\gamma;\,l}(q\,;\Phi)|_{\Phi=0}\ , \tag{49}$$

and similarly as regards the other insertions.
Starting from the analog of (34) for the modified generating functional $\tilde L^{\Lambda,\Lambda_0}(\xi;\Phi)$, which emerges from the bare action (44), and restricting to one insertion by the operation (47), leads to a *linear* flow equation for $L^{\Lambda,\Lambda_0}_\gamma(x\,;\Phi)$. Proceeding then as before in the derivation of (40), yields the system of differential FE for the CAS with *one insertion* ψ

$$\begin{aligned}\partial_\Lambda\partial^w\,\mathcal{L}^{\Lambda,\Lambda_0}_{\gamma;\,l,n}(q\,;p_1,\cdots,p_{|n|})\ =& \sum_{n',|n'|=|n|+2} c_{n-n'}\int_k(\partial_\Lambda C^{\Lambda,\Lambda_0}(k))\,\partial^w\mathcal{L}^{\Lambda,\Lambda_0}_{\gamma;\,l-1,\,n'}(q;k,-k,p_1,\cdots,p_{|n|})\\
&-\sum_{\substack{l_1+l_2=l,\,w_1+w_2+w_3=w\\ n_1,n_2,|n_1|+|n_2|=|n|+2}} c_{\{w_i\}}\Bigg[c^{(1)}_{n_1,n_2}\,\partial^{w_1}\mathcal{L}^{\Lambda,\Lambda_0}_{\gamma;\,l_1,n_1}(q;p_1,\cdots,p_{|n_1|-1},p')\\
&\qquad\cdot(\partial^{w_3}\partial_\Lambda C^{\Lambda,\Lambda_0}(p'))\,\partial^{w_2}\mathcal{L}^{\Lambda,\Lambda_0}_{l_2,n_2}(-p',\cdots,p_{|n|})\Bigg]_{s,a}\ .\end{aligned} \tag{50}$$

The notation is that of (40), with $-p' = q+p_1+\cdots+p_{|n_1|-1}$, however. Since ghost and antighost in (34) do not appear symmetrically, the $\bar c$ (c)-derivative appears once in n_1 (n_2)

and once in n_2 (n_1). It is obvious that each of the other insertions (45) leads to a similar system of flow equations.

As will turn out in Section 4, the initial regularization, necessarily violating the STI, leads to a bare space-time integrated insertion of the form

$$L_1^{\Lambda_0,\Lambda_0}(\Phi) = \int dx\, N(x)\,, \qquad N(x) = Q(x) + Q'(x\,;\Lambda_0^{-1})\ . \tag{51}$$

The individual terms of $N(x)$ involve at most five fields and have ghost number 1. Furthermore, $Q(x)$ is a local polynomial in the fields and their derivatives, having canonical mass dimension $D = 5$, whereas $Q'(x\,;\Lambda_0^{-1})$ is nonpolynomial in the field momenta but suppressed by powers of ${\Lambda_0}^{-1}$. To obtain the generating functional $L_1^{\Lambda,\Lambda_0}(\Phi)$ with one (bare) insertion (51) we can resort to the local case, considering the bare local insertion

$$L^{\Lambda_0,\Lambda_0}(\varrho) = \int dx\, \varrho(x) N(x) \tag{52}$$

and proceed as before. Observing (47), (48) we obtain

$$L_1^{\Lambda,\Lambda_0}(\Phi) \;=\; \int dx\, \frac{\delta}{\delta\varrho(x)}\tilde{L}^{\Lambda,\Lambda_0}(\varrho\,;\Phi)|_{\varrho=0} \;=\; \int dx\; L_\varrho^{\Lambda,\Lambda_0}(x\,;\Phi) \;=\; \hat{L}_\varrho^{\Lambda,\Lambda_0}(0;\Phi)\ . \tag{53}$$

Performing again a loop expansion, the CAS n-point function with one insertion (51) is obtained as

$$\delta(p_1 + \cdots + p_{|n|})\; \mathcal{L}_{1;\,l,n}^{\Lambda,\Lambda_0}(p_1,\cdots,p_{|n|}) := (2\pi)^{4(|n|-1)}\delta^{\,n}_{\hat{\Phi}(p)} L_{1;\,l}^{\Lambda,\Lambda_0}(\Phi)|_{\,\Phi=0}\ . \tag{54}$$

For these CAS holds again a system of linear FE. According to the preceding treatment of the integrated insertion we only have to take (50) at the fixed momentum value $q = 0$ of the insertion, and then replace each symbol $\mathcal{L}_{\gamma;\,l,n}^{\Lambda,\Lambda_0}(0\,;\cdots)$ by the new symbol $\mathcal{L}_{1;\,l,n}^{\Lambda,\Lambda_0}(\cdots)$.

Polchinski realized the flow equations (40) to open the way for a simple inductive proof of renormalizability. The mathematical proof was carried through in [KKS] on simplifying still Polchinski's argument. The FE for composite operators (50) were introduced and analysed in [KK]. For a recent presentation see [M].

The analysis of the STI, however, as will be shown in Section 4, requires to trace in the perturbative expansion the effect of the super-renormalizable three-point couplings present in the interaction. To this end we scale in the tree-level part (12) of (31) the mass parameters appearing in the three-point couplings, as well as in the BRS-insertions the part proportional to m, see (41c), by a common factor of $\lambda > 0$:

$$m \to \lambda m\ , \quad M \to \lambda M\ . \tag{55}$$

Note however that we do not scale the mass parameters which are present in the regularized propagators appearing in the flow equations. All CAS will then depend smoothly on λ, and we expand them as

$$\mathcal{L}^{\Lambda,\Lambda_0}_{l,n}(\lambda;\vec{p}) \;=\; \sum_{\nu=0}^{\infty}(m\,\lambda)^{\nu}\;\mathcal{L}^{(\nu),\Lambda,\Lambda_0}_{l,n}(\vec{p})\;, \quad \vec{p}=(p_1,\cdots,p_{|n|})\,, \tag{56}$$

$$\mathcal{L}^{\Lambda,\Lambda_0}_{\gamma;\,l,n}(\lambda;q;\vec{p}) \;=\; \sum_{\nu=0}^{\infty}(m\,\lambda)^{\nu}\;\mathcal{L}^{(\nu),\Lambda,\Lambda_0}_{\gamma;\,l,n}(q;\vec{p})\;, \tag{57}$$

where for suitable (physically natural !) renormalization schemes the sum is finite, its size depending on l and n, as will be shown below. We adopt the following
Renormalization scheme : Relevant terms are those which satisfy
$|n|+|w|+\nu\le 4$ in case of the functional L^{Λ,Λ_0}, $|n|+|w|+\nu\le 2$ in case of $L^{\Lambda,\Lambda_0}_{\gamma}$,
in agreement with the bounds to be derived below.
At tree level we then have [3]

$$(\partial^w\mathcal{L}^{(\nu),\Lambda,\Lambda_0}_{0,n})(\vec{0}) \;=\; 0\,, \quad \text{if} \quad |n|+|w|+\nu\;<\;4\;. \tag{58}$$

For $l\ge 1$, we use renormalization conditions on the relevant terms as follows: we impose

$$(\partial^w\mathcal{L}^{(\nu),0,\Lambda_0}_{l,n})(\vec{0}) \stackrel{!}{=} 0\,, \quad \text{if} \quad |n|+|w|+\nu\;<\;4\;, \tag{59}$$

whereas if $|n|+|w|+\nu\;=\;4$, on the r.h.s. a free constant $r_{(\nu),\,l,\,n}$ can be chosen.
Correspondingly, in the case of an insertion, we have at the tree level

$$(\partial^w\mathcal{L}^{(\nu),\Lambda,\Lambda_0}_{\gamma;\,0,n})(0;\vec{0}\,)=0\,, \quad \text{if} \quad |n|+|w|+\nu<2\,, \tag{60}$$

and employ renormalization conditions

$$(\partial^w\mathcal{L}^{(\nu),0,\Lambda_0}_{\gamma;\,l,n})(0;\vec{0}\,)\stackrel{!}{=}0\,, \quad \text{if} \quad |n|+|w|+\nu<2\,, \tag{61}$$

but if $|n|+|w|+\nu\;=\;2$, on the r.h.s. again a free constant can be chosen.
Because of the expansions (56) and (57) the FE (40) and (50) have to be adjusted attributing a superscript (ν) to the CAS and to sum $\nu_1+\nu_2=\nu$, in complete analogy to the loop index l. Using these extended FE the following bounds can be deduced,

Proposition 1

Let $l\in\mathbf{N}_0$ and $0\le\Lambda\le\Lambda_0$, then

$$|\,\partial^w\mathcal{L}^{(\nu),\Lambda,\Lambda_0}_{l,n}(\vec{p})|\;\le\;(\Lambda+m)^{4-|n|-|w|-\nu}\,\mathcal{P}_1(\log\frac{\Lambda+m}{m})\,\mathcal{P}_2(\frac{|\vec{p}|}{\Lambda+m})\;, \tag{62}$$

[3] Notice, that for $l=0$ there are no CAS with $|n|\le 2$.

$$|\,\partial^w \mathcal{L}^{(\nu),\Lambda,\Lambda_0}_{\gamma;\,l,n}(q;\vec{p})| \;\le\; (\Lambda+m)^{2-|n|-|w|-\nu}\,\mathcal{P}_1(\log\frac{\Lambda+m}{m})\,\mathcal{P}_2(\frac{|q,\vec{p}|}{\Lambda+m})\;. \tag{63}$$

In these bounds $\mathcal{P}_i$, $i = 1,2$, denote (each time they appear possibly new) polynomials with nonnegative coefficients independent of Λ, Λ_0, $\vec{p}$, q, m . The coefficients may depend on n, l, w, and the other free parameters of the theory α, M/m , g.

These bounds are uniform in Λ_0. The proof is solely based on power counting for renormalizable theories, it does not involve the symmetry structure of the Yang-Mills theory.

Proof: To prove (62) one proceeds *by induction* as follows: ascending in $N := 2l+|n|$, for given N ascending in l, for given N, l ascending in ν, and for given N, l, ν descending in $|w|$. Given n , the irrelevant cases $|n|+|w|+\nu > 4$ are treated first, integrating from the initial point $\Lambda = \Lambda_0$ "downwards" with initial conditions equal to zero. In contrast, the relevant ones, i.e. $|n|+|w|+\nu \le 4$, choosing the particular momentum value $\vec{p} = 0$, are integrated from the initial point $\Lambda = 0$ "upwards" with initial conditions (59) and the remaining ones chosen freely, hereafter this result has to be extended to general $\vec{p}$ via the Taylor formula

$$f(\vec{p}) = f(0) + \vec{p}\cdot \int_0^1 (\vec{\partial} f)(t\vec{p})\,dt\,.$$

Descending in $|w|$, the integrand in the respective remainder of the Taylor extension has already been bounded previously. A derivative by induction provides another factor of $(\Lambda+m)^{-1}$, which can be combined with the momentum factor of the remainder to increase the degree of the bounding polynomial. A key to this induction is the property that in the tree order there are no CAS with $|n| \le 2$. Bounding the linear term [4] of the FE

$$\Big|\sum_{n',|n'|=|n|+2} c_{n-n'} \int_k (\partial_\Lambda C^{\Lambda,\Lambda_0}(k))\,\partial^w \mathcal{L}^{(\nu),\Lambda,\Lambda_0}_{l-1,\,n'}(k,-k,\vec{p})\Big|$$
$$\le \sum_{n',|n'|=|n|+2} \Lambda \int_{k'} |\,\Lambda^3 \partial_\Lambda C^{\Lambda,\Lambda_0}(\Lambda k')|\,|\,\partial^w \mathcal{L}^{(\nu),\Lambda,\Lambda_0}_{l-1,\,n'}(\Lambda k',-\Lambda k',\vec{p})|$$
$$\le \Lambda \sum_{n',|n'|=|n|+2} (\Lambda+m)^{4-|n'|-|w|-\nu}\,\mathcal{P}_1(\log\frac{\Lambda+m}{m})\,\mathcal{P}_2(\frac{|\vec{p}|}{\Lambda+m})$$
$$\le (\Lambda+m)^{4-|n|-|w|-\nu-1}\,\mathcal{P}_3(\log\frac{\Lambda+m}{m})\,\mathcal{P}_4(\frac{|\vec{p}|}{\Lambda+m})\,,$$

after a change of the integration variable $k = \Lambda k'$ one uses the bounds (23) and (62) and then performs the k'-integration.

The proof of (63) is analogous to the proof of (62): One has to observe the inherent demarcation between relevant and irrelevant, and to employ the bound (62) required to treat the

[4] This term generates a new loop.

bilinear term on the r.h.s. of the FE (50) . ■

Our renormalization scheme implies

$$\mathcal{L}^{(\nu),\Lambda,\Lambda_0}_{l,n}(\vec{p}) \equiv 0 \ , \quad \text{if} \ \ \nu > 2l + |n| - 2 \ , \qquad \mathcal{L}^{(\nu),\Lambda,\Lambda_0}_{\gamma;\, l,n}(q,\vec{p}) \equiv 0 \ , \quad \text{if} \ \ \nu > 2l + |n| - 1 \ . \tag{64}$$

These statements follow inductively from the FE, once they hold for the terms fixed by the boundary conditions. Note that the first of these relations can be understood in terms of Feynman graphs as following from the upper bound on the number of trivalent vertices at a given loop-order. The second one takes into account additionally that the BRS-insertions (41c) also include one factor of m .

To also prove convergence for $\Lambda_0 \to \infty$ (which a physicist would grant as a consequence of uniformity) one has to analyse the FE, derived w.r.t. Λ_0 , using the same inductive technique. It is then possible to prove [M] that

$$|\partial_{\Lambda_0} \partial^w \mathcal{L}^{(\nu),\,\Lambda,\Lambda_0}_{l,n}(\vec{p})| \ \leq \ \Lambda_0^{-2} \ (\Lambda + m)^{5-|n|-|w|-\nu}\, \mathcal{P}_1(\log \frac{\Lambda_0}{m})\, \mathcal{P}_2(\frac{|\vec{p}|}{\Lambda + m}) \ , \tag{65}$$

$$|\partial_{\Lambda_0} \partial^w \mathcal{L}^{(\nu),\,\Lambda,\Lambda_0}_{\gamma;\, l,n}(q;\vec{p})| \ \leq \ \Lambda_0^{-2} \ (\Lambda + m)^{3-|n|-|w|-\nu}\, \mathcal{P}_1(\log \frac{\Lambda_0}{m})\, \mathcal{P}_2(\frac{|q,\vec{p}|}{\Lambda + m}) \ , \tag{66}$$

for Λ_0 large enough. Herefrom we can infer the existence of the limits $\Lambda_0 \to \infty$ at fixed value of Λ .

3.2 The Flow Equations for the Proper Vertex Functions

Our analysis of the Slavnov-Taylor identities (STI) and the proof of their restoration will be based on a presentation in terms of proper vertex functions (1PI), since the extraction of relevant parts from the STI is simpler and more transparent in terms of those than in terms of the CAS. To present their relation with the CAS considered so far, we introduce the shorthand notation

$$\tilde{L}(\xi;\Phi) := \tilde{L}^{\Lambda,\Lambda_0}(\xi;\varphi_\tau,c,\bar{c}) \ , \qquad C_\tau := C_\tau^{\Lambda,\Lambda_0} \ , \qquad S := S^{\Lambda,\Lambda_0} \ , \tag{67}$$

for the generating functional of the CAS with insertion (45) and for the regularized propagators. From $\tilde{L}(\xi;\Phi)$ we define the "classical fields" $\underline{\Phi} \equiv (\underline{\varphi}_\tau, \underline{c}, \underline{\bar{c}})$ by

$$\begin{gathered}
\underline{\varphi}_\tau(x) = \varphi_\tau(x) - \int dy\, C_\tau(x-y)\, \frac{\delta \tilde{L}(\xi;\Phi)}{\delta \varphi_\tau(y)} \ , \\
\underline{c}^a(x) = c^a(x) + \int dy\, S(x-y)\, \frac{\delta \tilde{L}(\xi;\Phi)}{\delta \bar{c}^a(y)} \ , \quad \underline{\bar{c}}^a(x) = \bar{c}^a(x) - \int dy\, S(x-y)\, \frac{\delta \tilde{L}(\xi;\Phi)}{\delta c^a(y)} \ .
\end{gathered} \tag{68}$$

The generating functional of the proper vertex functions $\tilde{\Gamma}(\xi;\underline{\Phi}) \equiv \tilde{\Gamma}^{\Lambda,\Lambda_0}(\xi;\underline{\varphi}_\tau,\underline{c},\underline{\bar{c}})$ is then given by the transform [5]

$$\tilde{\Gamma}(\xi;\underline{\Phi}) \;=\; \tilde{L}(\xi;\Phi) - \frac{1}{2}\sum_\tau \langle\varphi_\tau, C_\tau^{-1}\varphi_\tau\rangle + \langle\bar{c}, S^{-1}c\rangle + \sum_\tau \langle\underline{\varphi}_\tau C_\tau^{-1}\varphi_\tau\rangle - \langle\bar{c}, S^{-1}\underline{c}\rangle - \langle\underline{\bar{c}}, S^{-1}c\rangle\,, \tag{69}$$

with $\Phi = \Phi(\underline{\Phi})$ on the r.h.s., according to (68). Since we are only interested in the kernels to be derived from the generating functional Γ we may always assume the field variables to be sufficiently regular so that the application of the inverted regularized propagators makes sense. By functional derivation we deduce the relations

$$\begin{aligned}
&\frac{\delta\tilde{\Gamma}(\xi;\underline{\Phi})}{\delta\underline{\varphi}_\tau(x)} = \int dy\, C_\tau^{-1}(x-y)\,\varphi_\tau(y)\ ,\\
&\frac{\delta\tilde{\Gamma}(\xi;\underline{\Phi})}{\delta\underline{c}^a(x)} = \int dy\, S^{-1}(x-y)\,\bar{c}^a(y)\ ,\quad \frac{\delta\tilde{\Gamma}(\xi;\underline{\Phi})}{\delta\underline{\bar{c}}^a(x)} = -\int dy\, S^{-1}(x-y)\,c^a(y)\ ,
\end{aligned} \tag{70}$$

forming the inverse of the relations (68). Moreover, acting on the "classical fields" (68) with the respective inverse propagators C_τ^{-1} and S^{-1}, and then using (70), provides the crucial relations between the generating functionals $\tilde{L}(\xi;\Phi)$ and $\tilde{\Gamma}(\xi;\underline{\Phi})$

$$\begin{aligned}
(2\pi)^{-4}\, C_\tau^{-1}(p)\,\underline{\varphi}_\tau(-p) &= \frac{\delta\tilde{\Gamma}(\xi;\underline{\Phi})}{\delta\underline{\varphi}_\tau(p)} - \frac{\delta\tilde{L}(\xi;\Phi)}{\delta\varphi_\tau(p)}\,,\\
(2\pi)^{-4} S^{-1}(p)\,\underline{c}^a(-p) &= -\frac{\delta\tilde{\Gamma}(\xi;\underline{\Phi})}{\delta\underline{\bar{c}}^a(p)} + \frac{\delta\tilde{L}(\xi;\Phi)}{\delta\bar{c}^a(p)}\,,\\
(2\pi)^{-4} S^{-1}(p)\,\underline{\bar{c}}^a(-p) &= \frac{\delta\tilde{\Gamma}(\xi;\underline{\Phi})}{\delta\underline{c}^a(p)} - \frac{\delta\tilde{L}(\xi;\Phi)}{\delta c^a(p)}\,,
\end{aligned} \tag{71}$$

written in terms of Fourier transformed fields. Functional derivation of (69) with respect to the source $\gamma(x)$ at fixed $\underline{\Phi}$ leads to

$$\frac{\delta\tilde{\Gamma}(\xi;\underline{\Phi})}{\delta\gamma(x)}\Big|_{\xi=0} = \frac{\delta\tilde{L}(\xi;\Phi)}{\delta\gamma(x)}\Big|_{\xi=0}\,, \tag{72}$$

and to analogous equations as regards the other sources γ_μ^a, γ^a, ω^a.
Restricting again to perturbation theory we consider the proper vertex functions which correspond to the various types of CAS dealt with up to now. Hence, we define proper vertex functions without insertion, with one local insertion as in (47), (48), and with a global one as in (53), keeping the same notations. Since by definition $\tilde{\Gamma}(\xi;\underline{\Phi})$ has no vacuum

[5] This transform corresponds to the familiar Legendre transform of the connected (non-amputated) Schwinger functions.

part, we can extend to infinite volume and use Fourier transformed "classical fields" (68), with the conventions (37) (but omitting the "hat" by abuse of notation). Hence, from the generating functionals $\Gamma_l^{\Lambda,\Lambda_0}$, $\Gamma_{\gamma;\,l}^{\Lambda,\Lambda_0}$, $\Gamma_{1;\,l}^{\Lambda,\Lambda_0}$ we obtain the corresponding n-point proper vertex functions of loop order l in analogy with (38), (49), (54),

$$(2\pi)^{4(|n|-1)}\delta^n_{\underline{\Phi}(p)}\Gamma_l^{\Lambda,\Lambda_0}(\underline{\Phi})|_{\underline{\Phi}\equiv 0} \;=\; \delta(p_1+\cdots+p_{|n|})\,\Gamma_{l,n}^{\Lambda,\Lambda_0}(p_1,\cdots,p_{|n|})\;, \tag{73}$$

$$(2\pi)^{4(|n|-1)}\delta^n_{\underline{\Phi}(p)}\Gamma_{\gamma;\,l}^{\Lambda,\Lambda_0}(q;\,\underline{\Phi})|_{\underline{\Phi}\equiv 0} \;=\; \delta(q+p_1+\cdots+p_{|n|})\,\Gamma_{\gamma;\,l,n}^{\Lambda,\Lambda_0}(q;p_1,\cdots,p_{|n|})\;, \tag{74}$$

$$(2\pi)^{4(|n|-1)}\delta^n_{\underline{\Phi}(p)}\Gamma_{1;\,l}^{\Lambda,\Lambda_0}(\underline{\Phi})|_{\underline{\Phi}\equiv 0} \;=\; \delta(p_1+\cdots+p_{|n|})\,\Gamma_{1;\,l,n}^{\Lambda,\Lambda_0}(p_1,\cdots,p_{|n|})\;. \tag{75}$$

The FE for the $\tilde{L}$-functional implies a corresponding flow equation for the proper vertex functional $\tilde{\Gamma}$. Performing the Λ-derivative of the transform (69) [6] and observing that the classical fields $\underline{\Phi}$, (68), themselves depend on Λ due to (70), eventually yields

$$\begin{aligned}(\partial_\Lambda\tilde{\Gamma})(\xi;\underline{\Phi}) = \partial_\Lambda\tilde{L}(\xi;\Phi) \;&-\; \frac{1}{2}\sum_\tau\langle\varphi_\tau,\partial_\Lambda C_\tau^{-1}\varphi_\tau\rangle + \langle\bar{c}\,,\partial_\Lambda S^{-1}c\rangle \\ &+\; \sum_\tau\langle\underline{\varphi}_\tau,\partial_\Lambda C_\tau^{-1}\varphi_\tau\rangle - \langle\bar{c}\,,\partial_\Lambda S^{-1}\underline{c}\rangle - \langle\underline{\bar{c}}\,,\partial_\Lambda S^{-1}c\rangle\;,\end{aligned} \tag{76}$$

where $(\partial_\Lambda\tilde{\Gamma})$ denotes the derivative of the functional $\tilde{\Gamma}$ itself. Inserting now the flow equation for $\tilde{L}(\xi;\Phi)$ which has the same form as (34), and eliminating in its bilinear terms the functionals $\frac{\delta}{\delta\Phi}\tilde{L}$ using the equations (68), provides the flow equation of the vertex functional

$$(\partial_\Lambda\tilde{\Gamma})(\xi;\underline{\Phi}) + (\partial_\Lambda\tilde{I})(\xi) - \frac{1}{2}\sum_\tau\langle\underline{\varphi}_\tau,\partial_\Lambda C_\tau^{-1}\underline{\varphi}_\tau\rangle + \langle\underline{\bar{c}}\,,\partial_\Lambda S^{-1}\underline{c}\rangle = \hbar\,\dot{\Delta}_{\Lambda,\Lambda_0}\,\tilde{L}(\xi;\Phi)\;, \tag{77}$$

where one should remember the dependence on the parameters Λ,Λ_0 from (67) and the definition (33). At this stage the fields $\underline{\Phi}$ can be considered as autonomous (test) functions of the functional $\tilde{\Gamma}$, not depending on Λ. On the l.h.s. the second term is the vacuum part, since $\tilde{\Gamma}(\xi;0)=0$, and the subsequent terms subtract the (regularized) two-point tree order from $(\partial_\Lambda\tilde{\Gamma})(\xi;\underline{\Phi})$. The resulting functional still has to be expressed in terms of proper vertex functions. Performing a loop expansion and functional derivatives w.r.t. the fields we obtain from (77) for $|n|\geq 1$

$$\delta^n_{\underline{\Phi}}|_{\underline{\Phi}\equiv 0}:\;(\partial_\Lambda\tilde{\Gamma}_l)(\xi;\underline{\Phi}) \;=\; \dot{\Delta}_{\Lambda,\Lambda_0}\,\tilde{L}_{l-1}(\xi;\Phi)\,,\quad l\geq 1\;. \tag{78}$$

Since the vacuum part has disappeared we can now pass to the infinite volume limit. On the right hand side the functional $\tilde{L}_{l-1}(\xi;\Phi)$ is first acted upon by two particular Φ-derivatives from the functional Laplace operator, then followed by an n-fold functional derivative with

[6] again to be viewed on finite volume before passing to correlation functions

respect to (the classical field) $\underline{\Phi}$. The resulting object has to be expressed in terms of proper vertex functions. There is no closed formula for the r.h.s. in terms of proper vertex functions, and the presence of various types of fields increases the combinatorial complexity. To indicate the procedure we employ a collective notation. We perform a $\underline{\Phi}$-derivative of the crucial relation (71), as regards $\tilde{L}(\xi;\Phi)$ via the chain rule together with (70), and hereafter consider the outcome within a loop expansion, [7]

$$\frac{\delta(p+q)\delta_{l,0}}{(2\pi)^4\, C_{\Phi,\Phi'}(p)} = \frac{\delta^2\tilde{\Gamma}_l(\xi;\underline{\Phi})}{\delta\underline{\Phi}(p)\,\delta\underline{\Phi}'(q)} - (2\pi)^8 \sum_{\substack{\Phi'' \\ l_1+l_2=l}} \int_k \frac{\delta^2\tilde{L}_{l_1}(\xi;\Phi)}{\delta\Phi(p)\,\delta\Phi''(k)} C_{\Phi''\Phi'''}(k) \frac{\delta^2\tilde{\Gamma}_{l_2}(\xi;\underline{\Phi})}{\delta\underline{\Phi}'''(-k)\,\delta\underline{\Phi}'(q)}\,. \tag{79}$$

This identity forms the point of departure to relate successively n-point functions of the L- and the Γ- functional. We have to deal with it in the case without insertion, setting $\xi\equiv 0$, as well as in the case of one local insertion. In the latter one, (79) has to be derived with respect to the source at zero source, cf. (47),(48). By this operation, both the L-functional with and without insertion appear,

$$\begin{aligned}\frac{\delta^2\Gamma^{\Lambda,\Lambda_0}_{\gamma;l}(q;\underline{\Phi})}{\delta\underline{\Phi}(p)\,\delta\underline{\Phi}'(p')} \;&=\; (2\pi)^8 \sum_{\substack{\Phi'' \\ l_1+l_2=l}} \Big(\int_k \frac{\delta^2 L^{\Lambda,\Lambda_0}_{\gamma;l_1}(q;\Phi)}{\delta\Phi(p)\,\delta\Phi''(k)}\, C^{\Lambda,\Lambda_0}_{\Phi''\Phi'''}(k) \frac{\delta^2\Gamma^{\Lambda,\Lambda_0}_{l_2}(\underline{\Phi})}{\delta\underline{\Phi}'''(-k)\,\delta\underline{\Phi}'(p')} \\ &\quad + \int_k \frac{\delta^2 L^{\Lambda,\Lambda_0}_{l_1}(\Phi)}{\delta\Phi(p)\,\delta\Phi''(k)}\, C^{\Lambda,\Lambda_0}_{\Phi''\Phi'''}(k) \frac{\delta^2\Gamma^{\Lambda,\Lambda_0}_{\gamma;l_2}(q;\underline{\Phi})}{\delta\underline{\Phi}'''(-k)\,\delta\underline{\Phi}'(p')} \Big)\,. \end{aligned} \tag{80}$$

Taking (80) at momentum $q=0$ and replacing the subscript γ by the subscript 1 provides the relation in the case of the integrated insertion.
From (79) without insertion, considered at loop order $l=0$ and at $\Phi=\underline{\Phi}\equiv 0$, follows in the first step, because of the key property $\mathcal{L}^{\Lambda,\Lambda_0}_{0,n}(k,-k)\equiv 0$, if $|n|=2$,

$$1 = C^{\Lambda,\Lambda_0}_{\Phi,\Phi'}(p)\,\Gamma^{\Lambda,\Lambda_0}_{0,n}(p,-p), \quad n\,\hat{=}\,(\Phi,\Phi')\ . \tag{81}$$

Before returning to the flow equation we note, that in order to obtain from (79) with $\xi\equiv 0$ or from (80) the relation between the various n-point functions of the L- and the Γ- functional, we have to act upon these equations repeatedly by $\underline{\Phi}$ - derivation, to be performed on the L-functional via the chain rule. The chain rule derivatives $\delta\Phi/\delta\underline{\Phi}$ can be read from (70). In particular, on account of the propagators $C^{\Lambda,\Lambda_0}(k)$ vanishing at $\Lambda=\Lambda_0$ and observing (81), one realizes, ascending with $|n|$,

$$\Gamma^{\Lambda_0,\Lambda_0}_{l,n}(p_1,\cdots,p_{|n|}) = \mathcal{L}^{\Lambda_0,\Lambda_0}_{l,n}(p_1,\cdots,p_{|n|}), \quad (l,|n|)\neq(0,2)\,, \tag{82}$$

[7] Here Φ''' is determined by Φ'', cf. (17).

$$\Gamma^{\Lambda_0,\Lambda_0}_{1;\,l,n}(p_1,\cdots,p_{|n|}) = \mathcal{L}^{\Lambda_0,\Lambda_0}_{1;\,l,n}(p_1,\cdots,p_{|n|})\,, \tag{83}$$

and similarly in the case of the local insertions.
We now return to the FE (78) and first treat the case without insertion, thus we set there $\xi\equiv 0$. Performing in addition the momentum derivatives (39) we obtain the system, for $|n|\geq 1$, $(l,|n|)\neq(0,2)$,

$$\partial_\Lambda\partial^w\,\Gamma^{\Lambda,\Lambda_0}_{l,n}(p_1,\cdots,p_{|n|}) \;=\; \frac{1}{2}\sideset{}{'}\sum_{|n'|=|n|+2}\int_k(\partial_\Lambda C^{\Lambda,\Lambda_0}(k))\,\partial^w L^{\Lambda,\Lambda_0}_{l-1,\,n'}(k,-k;p_1,\cdots,p_{|n|})\,. \tag{84}$$

The summation extends on the various propagators as stated in (79), not distinguished here notationally, the corresponding pair of fields together with n determine n'. Moreover, the momentum derivative ∂^w concerns the momenta $p_1,\cdots,p_{|n|}$ of the configuration n. To generate the functions on the r.h.s. of (84) we have to act on (78), after setting $\xi\equiv 0$, with $\delta^n_{\underline{\Phi}}|_{\underline{\Phi}\equiv 0}$, and these derivatives are *directly* applied on the L- functional. Hence the functions $L^{\Lambda,\Lambda_0}_{l,n}$ in (84), differing from the CAS $\mathcal{L}^{\Lambda,\Lambda_0}_{l,n}$. The vanishing 2-point CAS in the tree order, together with its correspondence (81) then allow to express *inductively* the functions $L^{\Lambda,\Lambda_0}_{l,n}$ on the r.h.s. of (84) in terms of proper vertex functions, ascending in l, and for fixed l ascending in $|n|$. The r.h.s of (84) then emerges in the form

$$L^{\Lambda,\Lambda_0}_{l-1,\,n'}(k,-k;p_1,\cdots,p_{|n|}) \;=\Gamma^{\Lambda,\Lambda_0}_{l-1,\,n'}(k,-k,p_1,\cdots,p_{|n|})\;+\cdots\,, \tag{85}$$

where the dots represent chains $\Gamma\,C\,\Gamma$ and higher iterations, formed of proper vertex functions $\Gamma^{\Lambda,\Lambda_0}_{l',\,n''}$ with (l',n'') prior to $(l-1,n')$, joined via (free) propagators.
In the case of one local insertion the equation (78) has to be derived with respect to the source at zero source, cf. (47),(48). Performing again the momentum derivation leads to the the system of flow equations for proper vertex functions with one local insertion, $|n|\geq 1$,

$$\partial_\Lambda\partial^w\,\Gamma^{\Lambda,\Lambda_0}_{\gamma;\,l,n}(q;p_1,\cdots,p_{|n|}) \;=\; \frac{1}{2}\sideset{}{'}\sum_{|n'|=|n|+2}\int_k(\partial_\Lambda C^{\Lambda,\Lambda_0}(k))\,\partial^w L^{\Lambda,\Lambda_0}_{\gamma;\,l-1,n'}(q;k,-k;p_1,\cdots,p_{|n|})\;, \tag{86}$$

The r.h.s. of (86) is now obtained in complete analogy to the case without insertion, the r.h.s. is now extracted inductively from (80) in place of (79). By this operation, both the L-functions with and without insertion appear. Proceeding inductively as before, and using the already determined L-functions without insertion, provides the function on the r.h.s. of the system (86), as

$$L^{\Lambda,\Lambda_0}_{\gamma;\,l-1,\,n'}(q;k,-k;p_1,\cdots,p_{|n|}) \;=\Gamma^{\Lambda,\Lambda_0}_{\gamma;\,l-1,\,n'}(q;k,-k,p_1,\cdots,p_{|n|})\;+\cdots\,, \tag{87}$$

where the dots again represent a sum of chains, each of which contains exactly *one* inserted factor $\Gamma^{\Lambda,\Lambda_0}_{\gamma;\,l'',n''}$, which has already been determined previously in the inductive procedure. Finally, in the case of an integrated insertion, we obtain the system (86) at the particular momentum value $q \equiv 0$.

Once (85) and (87) have been inductively fixed, we can again perform the mass scaling (55) in the tree-level interaction and insertions. It then leads to expansions corresponding to (56), (57) for the vertex functions

$$\Gamma^{\Lambda,\Lambda_0}_{l,n}(\lambda;\vec{p}\,) \;=\; \sum_{\nu=0}^{\infty}(m\lambda)^{\nu}\;\Gamma^{(\nu),\Lambda,\Lambda_0}_{l,n}(\vec{p}\,)\;, \quad \vec{p}=(p_1,\cdots,p_{|n|})\,, \tag{88}$$

$$\Gamma^{\Lambda,\Lambda_0}_{\gamma;\,l,n}(\lambda;q;\vec{p}\,) \;=\; \sum_{\nu=0}^{\infty}(m\lambda)^{\nu}\;\Gamma^{(\nu),\Lambda,\Lambda_0}_{\gamma;\,l,n}(q;\vec{p}\,)\;. \tag{89}$$

We first consider the tree level $l=0$. In the case of (88) the scaling (55) of the interaction results in

$$(\partial^{w}\,\Gamma^{(\nu),0,\Lambda_0}_{0,n})(\vec{0}\,)=0\,, \quad |n|=3, \quad |w|+\nu\neq 1\;. \tag{90}$$

Whereas there is no $|n|=1$ content, the 2-point functions are fixed by the regularized propagators (81) (the masses of which are not scaled). The vertex functions with insertion (89) satisfy

$$(\partial^{w}\,\Gamma^{(\nu),0,\Lambda_0}_{\gamma;\,0,n})(0;\vec{0}\,)=0\,, \quad |n|+|w|+\nu<2\,. \tag{91}$$

Owing to the expansions (88) and (89), in both FE (84) and (86) a superscript (ν) has to be attached to the respective n-point function on the l.h.s. and on the n'-point functions present on the r.h.s. We then use the same inductive scheme which leads to the bounds (62),(63) on the CAS and may deduce renormalizability of the proper vertex functions. For the relevant terms the choice of the renormalization conditions is as follows , $l\geq 1$,

$$(\partial^{\,w}\,\Gamma^{(\nu),0,\Lambda_0}_{l,\,n})(\vec{0}\,)\overset{!}{=}0\,, \quad \text{if} \quad |n|+|w|+\nu<4\,, \tag{92}$$

but if $|n|+|w|+\nu=4$, a nonvanishing constant can be chosen on the r.h.s.,
whereas in the case of an insertion

$$(\partial^{\,w}\,\Gamma^{(\nu),0,\Lambda_0}_{\gamma;\,l,\,n})(0;\,\vec{0}\,)\overset{!}{=}0\,, \quad \text{if} \quad |n|+|w|+\nu<2\,, \tag{93}$$

but if $|n|+|w|+\nu=2$, again a nonvanishing constant on the r.h.s. may be imposed.
Proceeding inductively as indicated we obtain the bounds:

Proposition 2

$$|\,\partial^{w}\,\Gamma^{(\nu),\Lambda,\Lambda_0}_{l,n}(\vec{p})|\;\leq\;(\Lambda+m)^{4-|n|-|w|-\nu}\,\mathcal{P}_1(\log\frac{\Lambda+m}{m})\,\mathcal{P}_2(\frac{|\vec{p}|}{\Lambda+m})\;, \quad (l,|n|)\neq(0,2)\,, \tag{94}$$

$$| \partial^w \Gamma^{(\nu),\Lambda,\Lambda_0}_{\gamma;\,l,n}(q;\vec{p})| \;\leq\; (\Lambda+m)^{2-|n|-|w|-\nu}\,\mathcal{P}_1(\log\frac{\Lambda+m}{m})\,\mathcal{P}_2(\frac{|q,\vec{p}|}{\Lambda+m})\;, \tag{95}$$

The notations are those from (62),(63).

Moreover, we can also obtain the bounds (65) - (66) in the case of proper vertex functions derived w.r.t. Λ_0 .

4 Violated Slavnov-Taylor identities

To examine the violation of the STI produced by the UV cutoff Λ_0 we depart from the generating functional of the regularized Schwinger functions at the physical value $\Lambda = 0$ of the flow parameter, [8]

$$Z^{\,0,\Lambda_0}(K) \;=\; \int d\mu_{\,0,\Lambda_0}(\Phi)\, e^{-\frac{1}{\hbar}L^{\Lambda_0,\Lambda_0}(\Phi)+\frac{1}{\hbar}\langle\Phi,K\rangle}\;. \tag{96}$$

The Gaussian measure $d\mu_{\,0,\Lambda_0}(\Phi)$ corresponds to the quadratic form $\frac{1}{\hbar}\,Q^{\,0,\Lambda_0}(\Phi)$, cf. (26),

$$Q^{\,0,\Lambda_0}(\Phi)=\frac{1}{2}\langle A^a_\mu,\big(C^{\,0,\Lambda_0}\big)^{-1}_{\mu\nu}A^a_\nu\rangle+\frac{1}{2}\langle h,(C^{\,0,\Lambda_0})^{-1}h\rangle+\frac{1}{2}\langle B^a,(S^{\,0,\Lambda_0})^{-1}B^a\rangle-\langle\bar{c}^a,(S^{\,0,\Lambda_0})^{-1}c^a\rangle. \tag{97}$$

We define *regularized* BRS-variations (15),(41a)-(41d) of the fields by

$$\begin{aligned}
\delta_{BRS}\,\varphi_\tau(x) &= -\,(\sigma_{0,\Lambda_0}\,\psi_\tau)(x)\,\varepsilon,\\
\delta_{BRS}\,c^a(x) &= -\,(\sigma_{0,\Lambda_0}\,\Omega^a)(x)\,\varepsilon,\\
\delta_{BRS}\,\bar{c}^a(x) &= -\,\big(\sigma_{0,\Lambda_0}\,(\,\frac{1}{\alpha}\,\partial_\nu A^a_\nu-mB^a)\big)(x)\,\varepsilon\;.
\end{aligned} \tag{98}$$

The BRS-variation of the Gaussian measure has the form

$$d\mu_{\,0,\Lambda_0}(\Phi)\mapsto d\mu_{\,0,\Lambda_0}(\Phi)\Big(1-\frac{1}{\hbar}\,\delta_{BRS}\,Q^{\,0,\Lambda_0}(\Phi)\Big)\;, \tag{99}$$

and inspecting (97) we observe that the factor σ_{0,Λ_0} of the variations (98) just cancels its inverse entering the inverted propagators. Hence, the BRS-variation of the Gaussian measure has mass dimension $D=5$. Requiring the regularized generating functional $Z^{\,0,\Lambda_0}(K)$, (96), to be invariant under the BRS-variations (98) of the integration variables, provides the *violated Slavnov-Taylor identities* (VSTI)

$$0\stackrel{!}{=}\int d\mu_{\,0,\Lambda_0}(\Phi)\;e^{-\frac{1}{\hbar}L^{\Lambda_0,\Lambda_0}(\Phi)+\frac{1}{\hbar}\langle\Phi,K\rangle}\Big(\delta_{BRS}\,\langle\Phi,K\rangle-\delta_{BRS}\,(Q^{\,0,\Lambda_0}+L^{\Lambda_0,\Lambda_0})\Big)\;. \tag{100}$$

[8] Again one should stay in finite volume as long as the vacuum part is involved.

The BRS-variations appearing in (100) can be dealt with, considering corresponding modified generating functionals:
i) With the modified bare interaction (44) we define

$$\tilde{Z}^{\,0,\Lambda_0}(K,\xi) := \int d\mu_{\,0,\Lambda_0}(\Phi)\, e^{-\frac{1}{\hbar}\tilde{L}^{\Lambda_0,\Lambda_0}(\xi;\Phi)+\frac{1}{\hbar}\langle\Phi,K\rangle} \;, \tag{101}$$

and introduce a *regularized* BRS-operator

$$\mathcal{D}_{\Lambda_0} = \sum_\tau \big\langle J_\tau\,, \sigma_{0,\Lambda_0}\frac{\delta}{\delta\gamma_\tau}\big\rangle + \big\langle \bar{\eta}^a, \sigma_{0,\Lambda_0}\frac{\delta}{\delta\omega^a}\big\rangle + \big\langle\,\frac{1}{\alpha}\,\partial_\nu\frac{\delta}{\delta j^a_\nu} - m\frac{\delta}{\delta b^a}\,, \sigma_{0,\Lambda_0}\eta^a\big\rangle \;. \tag{102}$$

ii) The BRS-variations of the bare action and of the Gaussian measure

$$L_1^{\Lambda_0,\Lambda_0}\varepsilon := \,-\delta_{BRS}\Big(Q^{\,0,\Lambda_0} + L^{\Lambda_0,\Lambda_0}\Big) = \int dx\, N(x)\,\varepsilon \tag{103}$$

form a space-time integrated insertion with ghost number 1. The variation of L^{Λ_0,Λ_0}, however, keeps the regularizing factor σ_{0,Λ_0} of (98), thus the integrand $N(x)$ is no longer a polynomial in the fields and their derivatives. We can initially treat the integrand $N(x)$ as a local insertion with a source $\rho(x)$, cf. (52). Introducing the corresponding bare action $\tilde{L}^{\Lambda_0,\Lambda_0}(\rho;\Phi)$ similarly to (44), we define the functional [9] $\tilde{Z}^{\,0,\Lambda_0}(K,\rho)$ in analogy to (101).
In terms of these modified Z-functionals the VSTI (100) can now be written

$$\mathcal{D}_{\Lambda_0}\,\tilde{Z}^{\,0,\Lambda_0}(K,\xi)|_{\,\xi=0} \;=\; \int dx \frac{\delta}{\delta\varrho(x)}\tilde{Z}^{\,0,\Lambda_0}(K,\rho)|_{\,\rho=0} \;. \tag{104}$$

The modified Z-functional (101) is related to the corresponding generating functional of modified CAS by [10]

$$\tilde{Z}^{\,0,\Lambda_0}(K,\xi) = e^{\frac{1}{\hbar}P^{\,0,\Lambda_0}(K)}\, e^{-\frac{1}{\hbar}(\tilde{L}^{\,0,\Lambda_0}(\xi;\,\varphi_\tau,\,c,\,\bar{c})+I^{\,0,\Lambda_0})} \;, \tag{105}$$

and analogously in case of $\tilde{Z}^{\,0,\Lambda_0}(K,\rho)$. Furthermore, the variables of the Z- and the L-functional satisfy

$$\begin{aligned}
\varphi_\tau(x) &= \int dy\, C_\tau^{0,\Lambda_0}(x-y)\, J_\tau(y) \;, \\
c^a(x) &= -\int dy\, S^{\,0,\Lambda_0}(x-y)\,\eta^a(y) \;, \qquad \bar{c}^a(x) = -\int dy\, S^{\,0,\Lambda_0}(x-y)\,\bar{\eta}^a(y) \;.
\end{aligned} \tag{106}$$

[9] Abusing notation we let the variables ρ and ξ, respectively, denote different functions.
[10] The vacuum part $I^{\,0,\Lambda_0}$ is the same as in the case without insertion, since the latter has nonzero ghost number

From (104), via (105) and the analogous relation for $\tilde{Z}^{0,\Lambda_0}(K,\rho)$, we derive, using the definitions (47), (53) and denoting the differential operators (16) by D_τ in accord with φ_τ, the *violated Slavnov-Taylor identities of the CAS:*

$$\begin{aligned}\Big\langle c^a, D\,\big(\frac{1}{\alpha}\,\partial_\nu A^a_\nu - mB^a\big)\Big\rangle - \Big\langle c^a, \sigma_{0,\Lambda_0}\big(\,\partial_\nu\frac{\delta L^{0,\Lambda_0}}{\delta A^a_\nu} - m\frac{\delta L^{0,\Lambda_0}}{\delta B^a}\big)\Big\rangle \\ + \sum_\tau \big\langle \varphi_\tau\,, D_\tau L^{0,\Lambda_0}_{\gamma_\tau}\big\rangle - \big\langle \bar{c}^a, DL^{0,\Lambda_0}_{\omega^a}\big\rangle \;=\; L^{0,\Lambda_0}_1\;.\end{aligned} \tag{107}$$

Starting from the relations (72) between the generating functionals of the vertex- and Schwinger-functions we can convert (107) at the (physical) value $\Lambda = 0$ into the *violated Slavnov-Taylor identities for proper vertex functions*, on substituting there the fields Φ according due to (70), and employing (71), (72),

$$\begin{aligned}\sum_\tau \Big\langle \frac{\delta\Gamma^{0,\Lambda_0}}{\delta\underline{\varphi}_\tau}\,, \sigma_{0,\Lambda_0}\Gamma^{0,\Lambda_0}_{\gamma_\tau}\Big\rangle - \Big\langle \frac{\delta\Gamma^{0,\Lambda_0}}{\delta\underline{c}^a},\, \sigma_{0,\Lambda_0}\Gamma^{0,\Lambda_0}_{\omega^a}\Big\rangle - \Big\langle \frac{1}{\alpha}\,\partial_\nu\underline{A}^a_\nu - m\underline{B}^a,\, \sigma_{0,\Lambda_0}\frac{\delta\Gamma^{0,\Lambda_0}}{\delta\underline{\bar{c}}^a}\Big\rangle \\ = \Gamma^{0,\Lambda_0}_1(\underline{\varphi}_\tau\,, \underline{c}^a, \underline{\bar{c}}^a)\;,\end{aligned} \tag{108}$$

with

$$\Gamma^{0,\Lambda_0}_1(\underline{\varphi}_\tau, \underline{c}^a, \underline{\bar{c}}^a) \;=\; L^{0,\Lambda_0}_1(\varphi_\tau, c^a, \bar{c}^a)\;. \tag{109}$$

In the analysis of the STI it will turn out that we need the form of their explicit violation "on the bare side", $\Gamma^{\Lambda_0,\Lambda_0}_1(\underline{\Phi})$, too. From the definition (103) we directly determine the bare functional $L^{\Lambda_0,\Lambda_0}_1(\Phi)$, using (44) and (45),

$$\begin{aligned}L^{\Lambda_0,\Lambda_0}_1(\Phi) \;=\; \langle c^a, D\,(\frac{1}{\alpha}\,\partial_\nu A^a_\nu - mB^a)\rangle + \sum_\tau \langle\varphi_\tau\,, D_\tau L^{\Lambda_0,\Lambda_0}_{\gamma_\tau}\rangle - \langle\bar{c}^a, DL^{\Lambda_0,\Lambda_0}_{\omega}\rangle \\ -\Big\langle\frac{\delta L^{\Lambda_0,\Lambda_0}}{\delta\bar{c}^a}, \sigma_{0,\Lambda_0}(\frac{1}{\alpha}\,\partial_\nu A^a_\nu - mB^a)\Big\rangle + \sum_\tau\Big\langle\frac{\delta L^{\Lambda_0,\Lambda_0}}{\delta\varphi_\tau}, \sigma_{0,\Lambda_0}L^{\Lambda_0,\Lambda_0}_{\gamma_\tau}\Big\rangle - \Big\langle\frac{\delta L^{\Lambda_0,\Lambda_0}}{\delta c^a}, \sigma_{0,\Lambda_0}L^{\Lambda_0,\Lambda_0}_{\omega}\Big\rangle.\end{aligned} \tag{110}$$

The functional $L^{\Lambda_0,\Lambda_0}_1(\Phi)$ generates n-point functions with $2 \le |n| \le 5$. Moreover, we observe that only the terms emerging from the BRS-variation of the bare interaction L^{Λ_0,Λ_0} have mass dimension greater than $D = 5$, because of the cutoff function $\sigma_{0,\Lambda_0}(k^2)$ (cf. remark after (99)). Given the functional $L^{\Lambda_0,\Lambda_0}_1$, its n-point functions coincide with those of the functional $\Gamma^{\Lambda_0,\Lambda_0}_1$, due to the identity (83).

5 Restoration of the Slavnov-Taylor Identities

5.1 Mass expansions of Vertex Functionals

To restore the STI, it is in particular necessary to make vanish the relevant part of the violating functional Γ_1^{0,Λ_0}. It will then turn out that this is also sufficient in the limit $\Lambda_0 \to \infty$. Namely the irrelevant contributions to this functional at the bare scale $\Gamma_1^{\Lambda_0,\Lambda_0}$, which stem from the regulating function σ_{0,Λ_0}, are sufficiently bounded in terms of inverse powers of Λ_0 so that we may apply Proposition 3 providing the bound (119).

The freedom we dispose of to achieve this task is the freedom of choosing the renormalization conditions for the relevant terms appearing in the functionals $\Gamma_{l,n}^{0,\Lambda_0}$ and $\Gamma_{\gamma;l,n}^{0,\Lambda_0}$. On inspection of the VSTI (108) one realizes that there is an obstacle on this way of proceeding : Since the insertion defining the functional $\Gamma_1^{\Lambda,\Lambda_0}$ is of dimension 5, we have to apply up to 5 field- and momentum-derivatives on (108) in order to exhaust all relevant terms. We first notice that momentum derivatives of the cutoff function $\sigma_{0,\Lambda_0}(k^2) = \sigma_{\Lambda_0}(k^2)$ do not contribute to the relevant terms looked for, [11] cf. (21). Hence, in the terms generated from (108) by these field- or momentum-derivatives there apply d_1 (field or momentum)-derivatives to the factors of the form $\delta\Gamma/\delta\varphi$ in (108), and d_2 (field or momentum)-derivatives apply to the factors of the form Γ_γ, ∂A^a, or mB^a, where $d_1 + d_2 \le 5$. If $d_2 \ge 3$ derivatives apply to the functionals $\Gamma_{\gamma;l,n}^{0,\Lambda_0}$, they generate irrelevant contributions, since the insertions in $\Gamma_{\gamma;l,n}^{0,\Lambda_0}$ are of dimension 2. In our earlier paper [KM] such contributions to the VSTI were denoted by "irr" in its Appendix C. They hampered the analysis of the relevant part of the VSTI at the renormalization scale in our previous efforts since they cannot be controlled *explicitly* in terms of the renormalization conditions. The only way out can be that the relevant terms from $\Gamma_{l,n}^{0,\Lambda_0}$ multiplying these irrelevant terms can always be made to vanish so as to avoid the a priori unknown irrelevant terms to appear. One then realizes however that there are contributions in $\Gamma_{l,n}^{0,\Lambda_0}$, present already at the tree level $l = 0$, which do not satisfy this criterion, namely the nonvanishing super-renormalizable three-point couplings, as well as the mass term of the 2-point functions (see Appendix A).

We present the following solution to this problem : The functionals $\Gamma_{\gamma;l,n}^{0,\Lambda_0}$ and $\Gamma_{l,n}^{0,\Lambda_0}$ are expanded at zero momentum not only w.r.t. the fields and the momenta but also w.r.t. to the number of super-renormalizable vertices, or otherwise stated w.r.t. to the number of mass parameters appearing in these couplings, see Section 3.2, (88) and (89). The degree of divergence then diminishes with this number, in fact the corresponding bounds (94) and (95) show that the presence of an explicit mass term produces a gain in power counting by

[11] This property is at the origin of our particular choice of the cutoff function.

one unit. Disposing then of all relevant terms in this new sense, we will realize that there do not remain uncontrollable contributions to the VSTI of the form mentioned above. One should note that counting a power of a mass parameter as a power of a field, is intuitively in accord with the fact that these mass parameters stem from the vacuum expectation value of the scalar field.

We start introducing the expansion of the functionals L_1^{Λ,Λ_0} and $\Gamma_1^{\Lambda,\Lambda_0}$ inherited from the mass scaling (55),

$$\mathcal{L}^{\Lambda,\Lambda_0}_{1;\,l,n}(\lambda;\vec{p}) \;=\; \sum_{\nu=0}^{\infty}(m\lambda)^{\nu}\;\mathcal{L}^{(\nu),\Lambda,\Lambda_0}_{1;\,l,n}(\vec{p})\;, \quad \vec{p}=(p_1,\cdots,p_{|n|})\,, \tag{111}$$

$$\Gamma^{\Lambda,\Lambda_0}_{1;\,l,n}(\lambda;\vec{p}) \;=\; \sum_{\nu=0}^{\infty}(m\lambda)^{\nu}\;\Gamma^{(\nu),\Lambda,\Lambda_0}_{1;\,l,n}(\vec{p})\;. \tag{112}$$

Since we aim at a consistent mass expansion of the VSTI, (108), we first observe, that we also have to perform the mass scaling (55) of the BRS-variation $\frac{1}{\alpha}(\partial_\nu A^a_\nu(x)-\alpha m B^a(x))$ of the antighost appearing, cf.(15), in accord with our treatment of the BRS-insertions. We then want to determine via (108) the relevant part of the functional Γ_1^{0,Λ_0}, given by the values $(\partial^w\Gamma^{(\nu),\,0,\,\Lambda_0}_{1;\,l,n})(\vec{0})\,,\ |n|+|w|+\nu\le 5\,.$ It is important to note that irrelevant contributions only emerge from the functionals containing a BRS-insertion. Requiring the vertex functions in (108) to satisfy the boundary conditions, $l\in\mathbf{N}_0$,

$$(\partial^w\Gamma^{(\nu),\,0,\,\Lambda_0}_{l,\,n})(\vec{0}) \;\stackrel{!}{=}\; 0\,, \quad \text{if} \quad |n|+|w|+\nu<4\,, \tag{113}$$

irrelevant contributions from the functionals $\Gamma^{(\nu),\,0,\Lambda_0}_{\gamma_\tau},\Gamma^{(\nu),\,0,\Lambda_0}_{\omega}$ then are annihilated by multiplication and only contributions of these functionals with $|n_2|+|w_2|+\nu_2\le 2$ field-, momentum- and mass-derivatives, i.e. relevant terms, do appear. The condition (113) is satisfied for $l\ge 1$ by the renormalization conditions (92), and in the tree order, if $|n|=3\,,$ (90).

Here, we remind the reader that we do not apply the mass expansion to the free propagator, but only to the boundary terms appearing in the FE. Now the inverted free propagators $\Gamma^{0,\Lambda_0}_{0,\,n}$, $|n|=2,$ appear in (108) as boundary terms at $\Lambda=0$ for the functions $\Gamma^{\Lambda,\Lambda_0}_{1;l,n}$, and they are then mass expanded, (55), thus satisfying (113), too. Therefore it is important to remember that the FE and the VSTI are derived *before* mass expanding. Afterwards we consistently apply the mass expansion to all boundary terms and make the corresponding statement on the bounds for the vertex functions which is verified inductively.

The renormalization conditions (92) imposed on (a subset of) the relevant terms of the vertex functions imply zero renormalization conditions for the leading contributions to all the two-point functions :

$$\delta m^2_{(\nu)} = 0\ , \quad \Sigma^{\bar{c}c(\nu)}(0) = 0\ , \quad \Sigma^{BB(\nu)}(0) = 0\ , \quad \Sigma^{hh(\nu)}(0) = 0 \ \text{ for } \ \nu \le 1\ , \tag{114}$$

and also

$$\Sigma^{AB(\nu)}(0) = 0 \ \text{ for } \ \nu = 0\ ; \quad \kappa^{(\nu)} = 0 \ \text{ for } \ \nu \le 2\ . \tag{115}$$

Here we use the notations of App. A. The respective relevant parts of the inserted functionals Γ_γ are collected in App. B. The restricted set of renormalization conditions (93) is automatically satisfied, even in the nonvoid case with $|n| = 1$,

$$n \equiv c^a\ : \quad \Gamma^{0,\Lambda_0}_{\gamma^a;\,n}(0;\vec{0}\,) = m\, R_4\,, \tag{116}$$

due to the explicit factor of m to be scaled according to (55).

The functionals $L_1^{\Lambda,\Lambda_0}(\Phi)$, $\Gamma_1^{\Lambda,\Lambda_0}(\underline{\Phi})$ serve to control the violation of the STI. They contain irrelevant boundary terms at $\Lambda = \Lambda_0$, in contrast to the functionals without insertion or with a BRS-insertion. These boundary terms are due to the presence of the factors σ_{0,Λ_0}, cf. the remarks after (110). They are proportional to $\sigma_{0,\Lambda_0}(p) - 1 = O((p^2)^2/\Lambda_0^4)$, as follows from (20), since the terms proportional to $\sigma_{0,\Lambda_0}(0) = 1$ are relevant.
We first assert the bound on the bare functional $\Gamma_1^{\Lambda_0,\Lambda_0}$, valid for $l \in \mathbf{N}_0$,

$$|\,\partial^w \Gamma^{(\nu),\,\Lambda_0,\Lambda_0}_{1;\,l,n}(\vec{p})| \ \le\ (\Lambda_0 + m)^{\,5-|n|-|w|-\nu} \Big(\log \frac{\Lambda_0}{m}\Big)^r\, \mathcal{P}\big(\frac{|\vec{p}\,|}{\Lambda_0}\big)\ , \tag{117}$$

and trivially satisfied, unless $2 \le |n| \le 5$. Because of the identity (83) we can establish the corresponding bound on $L_1^{\Lambda_0,\Lambda_0}$ and making use of (110). We employ the previous bounds on $\partial^w \mathcal{L}^{(\nu),\,\Lambda,\Lambda_0}_{l,n}$, (62), and on $\partial^w \mathcal{L}^{(\nu),\,\Lambda,\Lambda_0}_{1;\,l,n}$, (63), at the value $\Lambda = \Lambda_0$. For $\sigma_{0,\Lambda_0}(k^2) = \sigma_{\Lambda_0}(k^2)$ we use the bounds

$$|\,\partial^w \sigma_{\Lambda_0}(k^2)| \ \le\ \Lambda_0^{-|w|}\, \mathcal{P}_{|w|}\big(\frac{|k|}{\Lambda_0}\big)\ ,$$

which are an easy consequence of (20), the polynomials $\mathcal{P}_{|w|}$ having nonnegative coefficients not depending on k. With these ingredients we prove (117).
The bound on the functional $\Gamma_1^{\Lambda,\Lambda_0}$ (119) does not follow from the choice of standard renormalization conditions for insertions. We rather assume its relevant part at the physical value $\Lambda = 0$ of the flow parameter to vanish, $l \in \mathbf{N}_0$,

$$(\partial^w \Gamma^{(\nu),\ 0,\Lambda_0}_{1;\,l,n})(\vec{0}\,) = 0\,, \qquad |n| + |w| + \nu \le 5\,. \tag{118}$$

In Section 5.3 we will be able to verify these conditions from the VSTI (108), choosing for the functionals entering the l.h.s. suitable renormalization conditions within the class (92), (93) considered. Assuming (118), we want to show that the corresponding irrelevant part

vanishes upon shifting the UV- cutoff to infinity:
Proposition 3
Given (118), then for $l \in \mathbf{N}_0, |n| \geq 2$ *and* $0 \leq \Lambda \leq \Lambda_0$,

$$|\,\partial^w \Gamma^{(\nu),\Lambda,\Lambda_0}_{1;\,l,n}(\vec{p})| \;\leq\; \frac{1}{\Lambda_0}\,(\Lambda+m)^{\,5+1-|n|-|w|-\nu}\Big(\,\log\frac{\Lambda_0}{m}\Big)^{r}\,\mathcal{P}\big(\frac{|\vec{p}\,|}{\Lambda+m}\big)\;. \tag{119}$$

with a positive integer r *depending on* n, l, w *, and a polynomial* $\mathcal{P}$ *as in (62),(63).*
Proof: We first notice, that the bound (119) at $\Lambda = \Lambda_0$ agrees with the bound (117) , and at $\Lambda < \Lambda_0$ majorizes this bound, if $|n| + |w| + \nu > 5$. The functions $\partial^w \Gamma^{(\nu),\Lambda,\Lambda_0}_{1;\,l,n}$ with flow parameter $0 \leq \Lambda \leq \Lambda_0$ are bounded integrating inductively the FE (86), adapted to an integrated insertion and to the λ-expansion, however, as stated. We proceed in the inductive order as in the proof of the Proposition 1, but observing that the relevant terms of the functional treated here satisfy $|n| + |w| + \nu \leq 5$.
Considering the tree order first we notice, that the r.h.s. of the FE does vanish. Hence, this order is already fixed by its boundary value at $\Lambda = \Lambda_0$. If $|n| = 2$, the boundary value even vanishes and thus the function itself, satisfying (119) trivially. Proceeding, for given n in the irrelevant cases $|n| + |w| + \nu > 5$ the bound (119) follows from the bound on their boundary values. Integrating the relevant cases $|n| + |w| + \nu \leq 5$ with initial values (118) yields $\partial^w\,\Gamma^{(\nu),\Lambda,\Lambda_0}_{1;\,0,n}(\vec{0}\,) = 0$. Descending in $|w|$, the integrand in the respective remainder of the Taylor extension has already been bounded before, providing the bound for general value $\vec{p}$. Hence, the assertion is established in the tree order.
Proceeding for $l > 0$ inductively as indicated, the L-functions appearing on the r.h.s. of the FE (86) have to be determined within this inductive process via (80), as expounded in presenting the FE and supplemented in the text after (87), leading to the Proposition 2. Therefore, to bound the r.h.s. one also needs the bound (94) on the vertex functions without insertions, to be dealt with independently before. As a result the bound deduced on $|\partial^w L^{(\nu),\,\Lambda,\Lambda_0}_{1;\,l-1,\,n'}|$ essentially coincides with the bound on $|\partial^w \Gamma^{(\nu),\,\Lambda,\Lambda_0}_{1;\,l-1,\,n'}|$, cf. (87), i.e. has the same form and power behaviour of $\Lambda + m$. This bound allows to estimate the r.h.s. of the FE and hereafter the integrations "downwards" with initial conditions (117), and "upwards" with initial conditions (118), of the irrelevant and relevant cases, respectively. Extending finally the relevant cases via the Taylor formula to general $\vec{p}$ completes the proof. ■
Thus, given the condition (118), the bound (119) implies that *the Slavnov-Taylor-Identities are restored in the limit* $\Lambda_0 \to \infty$ *.*

5.2 Equation of motion of the anti-ghost

Renormalization theory for nonabelian gauge theories in gauge invariant renormalization schemes is generally based on the STI, complemented by the equation of motion of the antighost [Z], [FS]. In our scheme we rather start from a derivation of this equation from the functional integral. In Section 5.3 we will then show that this equation is satisfied for renormalization conditions compatible with the STI if *in addition the renormalization condition for the longitudinal part of the gauge field propagator is fixed uniquely* to vanish at zero momentum.

The field equation follows from the representation (29). After functional derivation of (29) with respect to $\bar{c}^a(x)$ we reexpress the r.h.s. as

$$\frac{\delta L^{\Lambda,\Lambda_0}(\Phi)}{\delta \bar{c}^a(x)}\, e^{-\frac{1}{\hbar}\left(L^{\Lambda,\Lambda_0}(\Phi)+I^{\Lambda,\Lambda_0}\right)} = \frac{\delta}{\delta \zeta^a(x)} \int d\mu_{\Lambda,\Lambda_0}(\Phi')\; e^{-\frac{1}{\hbar}\left(L^{\Lambda_0,\Lambda_0}(\Phi'+\Phi)+L^{\Lambda_0,\Lambda_0}(\zeta;\,\Phi'+\Phi)\right)}\Big|_{\zeta=0}$$

on extending the original bare interaction $L^{\Lambda_0,\Lambda_0}(\Phi)$ by the insertion

$$L^{\Lambda_0,\Lambda_0}(\zeta;\Phi) \;=\; \int dx\; \zeta^a(x)\, \frac{\delta L^{\Lambda_0,\Lambda_0}(\Phi)}{\delta \bar{c}^a(x)}\;. \tag{120}$$

The source $\zeta^a(x)$ is a Grassmann element carrying ghost number -1. Treating now the r.h.s. analogously as in (44) - (47), we obtain the field equation of the antighost

$$\frac{\delta L^{\Lambda,\Lambda_0}(\Phi)}{\delta \bar{c}^a(x)} \;=\; L^{\Lambda,\Lambda_0}_{\zeta^a}(x;\Phi)\;, \tag{121}$$

employing the notation introduced there. On the r.h.s. appears the generating functional of the CAS with one local insertion corresponding to (120). The classical BRS-invariant action (9) satisfies the classical field equation $\delta/\delta\bar{c}^a(x) S_{BRS} = \partial_\mu \psi^a_\mu(x) - \alpha m \psi^a(x)$, observing (14). The aim is to show that the relation following from the classical action at the tree level for the physical value $\Lambda = 0$ of the flow parameter

$$\frac{\delta L^{0,\Lambda_0}(\Phi)}{\delta \bar{c}^a(x)} \;=\; \partial_\mu L^{0,\Lambda_0}_{\gamma^a_\mu}(x\,;\Phi)|_{mod} - \alpha m L^{0,\Lambda_0}_{\gamma^a}(x;\Phi)|_{mod}\,, \tag{122}$$

still holds in the renormalized theory. The label "mod" is to signal that we have to replace in the bare insertions (41a)-(41d) $R^0_i \to \tilde{R}^0_i = O(\hbar)$ for $i = 1, 4$ since the respective tree order is absent on the l.h.s.

We can write (122) in terms of proper vertex functions. Fourier transforming (122), using our conventions (37), (48), and employing the relations (72), (71) yields

$$(2\pi)^4\, \frac{\delta \Gamma^{0,\Lambda_0}(\underline{\Phi})}{\delta \underline{\bar{c}}^a(q)} \;=\; -\,\frac{q^2+\alpha m^2}{\sigma_{0,\Lambda_0}(q^2)}\, \underline{c}^a(-q) \,-\, i q_\mu \Gamma^{0,\Lambda_0}_{\gamma^a_\mu}(q;\underline{\Phi})|_{mod} \,-\, \alpha m \Gamma^{0,\Lambda_0}_{\gamma^a}(q;\underline{\Phi})|_{mod}\,. \tag{123}$$

The first term on the r.h.s. is the tree level 2-point function. Restricting (123) to its relevant part, $\sigma_{0,\Lambda_0}(q^2)$ is replaced by $\sigma_{0,\Lambda_0}(0)=1$ due to (21), the first term then provides the tree order of R_1 and R_4 excluded in the insertions as indicated by the label *mod*, cf. (122).

The proof of (123) or equivalently (122) consists in two steps of the same nature as those employed in the previous section. We may consider the (regularized) inserted functional

$$\Gamma_{c^a}^{\Lambda,\Lambda_0}(q\,;\underline{\Phi}) \;:=(2\pi)^4\,\frac{\delta\Gamma^{\Lambda,\Lambda_0}(\underline{\Phi})}{\delta\underline{\bar{c}}^a(q)}+\frac{q^2+\alpha m^2}{\sigma_{0,\Lambda_0}(q^2)}\,\underline{c}^a(-q)+iq_\mu\Gamma^{\Lambda,\Lambda_0}_{\gamma^a_\mu}(q;\underline{\Phi})|_{mod}+\alpha m\Gamma^{\Lambda,\Lambda_0}_{\gamma^a}(q\,;\underline{\Phi})|_{mod}\,. \tag{124}$$

In the mass expansion scheme it corresponds to an operator insertion of dimension 3, where we take into account also the momentum and mass factors in front of the last three terms. Since the flow equations for inserted functionals are linear, the new functional obeys again a linear flow equation obtained from those for the functionals on the r.h.s. by superposition. Note that the second term on the r.h.s., being a tree level contribution, does not flow.
If we can choose renormalization conditions such that all relevant contributions to $\Gamma_{c^a}^{\Lambda,\Lambda_0}(q\,;\underline{\Phi})$ vanish, we can prove by induction on the linear flow equation (the solution of which is unique for specified boundary conditions) that $\Gamma_{c^a}^{\Lambda,\Lambda_0}(q\,;\underline{\Phi})\equiv 0$. Note that for this functional there are no irrelevant boundary contributions at $\Lambda=\Lambda_0$, since such terms only appear in the first two terms on the r.h.s. at the tree level and cancel exactly. So the situation is simpler than that of the functional Γ_1 analysed in the previous section.

At the end of the next section it is shown explicitly that the relevant contributions to (124) can be made to vanish for suitable renormalization conditions so that *the equation of motion for the antighost (123) or (122) holds at the quantum level.*

5.3 Analysis of the relevant part of the Slavnov-Taylor Identities and of the equation for the antighost

We now require the relevant part of the functional Γ_1^{0,Λ_0} to vanish in accord with the VSTI (108). This requirement amounts to satisfy the 53 equations presented in the Appendix C. It is satisfied in the tree order. Noticing that the normalization constants of the BRS-insertions behave as $R_i=1+\mathcal{O}(\hbar), i=1,\cdots 7$, we first analyse the equations IX to $XXIX$, but take already into account the equations VII_d, $VIII_c$, the latter ones providing

$$r_2^{hBA}\;=\;r_2^{\bar{c}cA}\stackrel{!}{=}0\,. \tag{125}$$

In proceeding we use conditions determined before, if needed.
From XIV_b, XIV_e, XV_{1b}, $XXIII$ directly follow

$$r_1^{AA\bar{c}c}\;=\;r_2^{AA\bar{c}c}=r_1^{BB\bar{c}c}\;=\;r_2^{AABB}\stackrel{!}{=}0\,, \tag{126}$$

and than, from $XIV_{a+c}\,, XVII_b\,, XVIII_c\,, XXVIII, XXIX$,

$$r_2^{AAAA} \; = r^{hh\bar{c}c} \; = r^{\bar{c}c\bar{c}c} \; = r^{hB\bar{c}c} \; = r_2^{BB\bar{c}c} \overset{!}{=} 0\,. \tag{127}$$

$XVI_a\,, XVIII_a\,$, and XV_{2a} combined with $XVI_b\,$, respectively, require

$$R_2 \overset{!}{=} R_6 \overset{!}{=} R_7\,, \qquad\qquad R_3\, R_5 \overset{!}{=} (R_2)^2\,. \tag{128}$$

$$XIV_c: \qquad 2F_1^{AAAA}\, R_1 \overset{!}{=} -F^{AAA} g R_2 \tag{129}$$

$$XI: \qquad F^{\bar{c}cB\,(1)}\, R_5 \overset{!}{=} -F^{\bar{c}ch\,(1)} R_2\,. \tag{130}$$

From X , XX , XIX , IX follow for the self-coupling of the scalar field

$$8\, F^{BBBB}\, R_4 \;\overset{!}{=}\; F^{BBh(1)}\, gR_3\,, \tag{131}$$

$$4\, F^{BBhh}\, R_4 \;\overset{!}{=}\; F^{BBh(1)}\, gR_5\,, \tag{132}$$

$$8\, F^{hhhh}\, R_4 R_3 \;\overset{!}{=}\; F^{BBh(1)}\, g(R_5)^2\,, \tag{133}$$

$$F^{hhh\,(1)}\, R_3 \;\overset{!}{=}\; F^{BBh(1)}\, R_5\,, \tag{134}$$

and from XVI_b , $XVII_a$, XXI , $XIII_2$ for the scalar-vector coupling

$$2\, F^{BBA}\, R_5 \;\overset{!}{=}\; -\, F_1^{hBA}\, R_2\,, \tag{135}$$

$$4\, F^{AAhh}\, R_1 \;\overset{!}{=}\; F_1^{hBA}\, gR_5\,, \tag{136}$$

$$4\, F_1^{AABB}\, R_1 \;\overset{!}{=}\; F_1^{hBA}\, gR_3\,, \tag{137}$$

$$F^{AAh(1)}\, R_1 \;\overset{!}{=}\; F_1^{hBA}\, R_4\,. \tag{138}$$

One easily verifies that the remaining equations of IX to $XXIX$ are satisfied due to these conditions (125)-(138).

At this stage, all those relevant couplings with $|n| = 3, 4$ not appearing already in the tree order are required to vanish: (125)-(127). All other couplings involving four fields are determined by particular couplings with $|n| = 3$: (129), (131)-(133), (136),(137). In addition, there are 4 conditions relating couplings with $|n| = 3$: (130), (134), (135) and (138). Moreover, the normalization constants of the BRS-insertions are required to satisfy the three conditions (128).

There are still $18 - 2$ equations among I to $VIII$ to be considered. They contain the relevant parameters of Γ^{0,Λ_0} with $|\,n| = 1, 2, 3$, except F^{hhh}, together with the normalization constants of the BRS-insertions. Since 2 of these parameters have been fixed before, (125), there remain 26 to be dealt with. (F^{hhh} will then be determined by (134).) These parameters in addition have to obey the conditions derived before: We first observe that the condition

(138) is identical to equation VI_b. There remain the 5 conditions to be satisfied: 3 conditions (128), together with (130), (135). All these conditions generate 4 linear relations among the equations still to be considered: denoting by $\{X\}$ the content of the bracket $\{\cdots\}$ appearing in equation X, we find [M, (4.94-97)]

$$\begin{aligned} 0 &= \alpha^{-1}\{VIII_b\} + gR_2\{I_b\} + R_1\big(\{III_a\} + \{III_b\}\big)\,, && (139)\\ 0 &= gR_2\{II_b\} - \{VIII_b\} + R_1\{IV_b\} - 2R_4\{V\}\,, && (140)\\ 0 &= R_2\{IV_a\} - R_3\big(\{VI_a\} - \{VI_b\}\big)\,, && (141)\\ 0 &= R_2\{V\} - R_3\{VII_c\}\,. && (142) \end{aligned}$$

Hence, the 26 parameters in question are constrained by $16 + 5 - 4 = 17$ equations. As *renormalization conditions* we then fix $\kappa^{(3)} = 0$ and let

$$\Sigma_{\text{trans}}\,, \Sigma_{\text{long}}\,, \Sigma^{AB(1)}, \dot{\Sigma}^{\bar{c}c}, \dot{\Sigma}^{BB}, F^{AAA}, F^{BBh(1)}, R_3 \qquad (143)$$

be chosen freely. These parameters correspond to the number of wave function renormalizations (including one for the BRS sector) and coupling constant renormalizations of the theory. Thus, there are $26 - 9$ parameters left, together with 17 equations. These parameters are now determined successively in terms of (143) and possibly parameters determined before in proceeding. We list them in this order, writing in bracket the particular equation fulfilled:

$$\begin{gathered} R_1(I_b)\,, R_4(II_b)\,, R_2(III_b) \to R_6, R_7, R_5 \quad \text{due to} \quad (128)\,,\\ F_1^{\bar{c}cA}(III_a)\,, F^{BBA}(V) \to F_1^{hBA} \quad \text{due to} \quad (135)\,,\\ F^{AAh(1)}(VI_b)\,,\, F^{\bar{c}cB(1)}(IV_a) \to F^{\bar{c}ch(1)} \quad \text{due to} \quad (130)\,,\\ \Sigma^{\bar{c}c(2)}(VIII_a)\,,\, \Sigma^{BB(2)}(II_a)\,,\, \delta m^2_{(2)}(I_a)\,,\, \Sigma^{hh(2)}(VII_a)\,,\, \dot{\Sigma}^{hh}(VII_{b+c})\,. \qquad (144) \end{gathered}$$

Now all parameters are determined, without using the equations $IV_b\,, VI_a\,, VII_c\,, VIII_b$. These equations, however, are satisfied because of the relations (139)-(142). Finally, the relevant couplings with $|n| = 4$, as well as $F^{hhh(1)}$, then are explicitely given by (129), (131)-(134), (136) and (137).

We have not yet implemented the field equation of the antighost (123). Performing the mass scaling as before and then extracting the local content $|n| + |w| + \nu \leq 4$ leads to the relations

$$
\begin{aligned}
1+\dot{\Sigma}^{\bar{c}c} &= R_1\,, &(145)\\
\alpha+\Sigma^{\bar{c}c(2)} &= \alpha R_4\,, &(146)\\
F_1^{\bar{c}cA} &= gR_2\,, &(147)\\
F^{\bar{c}cB(1)} &= \frac{\alpha}{2}\,gR_6\,, &(148)\\
F^{\bar{c}ch(1)} &= -\frac{\alpha}{2}\,gR_5\,. &(149)
\end{aligned}
$$

Fixing now the hitherto free renormalization constant Σ_{long} at the particular value $\Sigma_{\text{long}}=0$, we claim these relations to be satisfied: (145) and (147) follow at once from I_b and III_{a+b}, respectively; (148) follows from $2\{IV_a\}-\{IV_b\}$, due to (147) and (128); and herefrom follow (149) due to (130), and (146) because of $VIII_a$, thus establishing the claim.
Given these additional relations (145)-(149) we can adjust the procedure (144) choosing now a *reduced set of free renormalization conditions (143) in which* Σ_{long} *is excluded.* Proceeding similarly as before we find

$$
I_b:\quad \Sigma_{\text{long}}\,=\,0\,,\qquad II_a:\quad \Sigma^{BB(2)}\,=\,0\,, \tag{150}
$$

$$
III_b:\qquad gR_2\,=\,-\,2F^{AAA}\,\frac{1+\dot{\Sigma}^{\bar{c}c}}{1+\Sigma_{\text{trans}}}\longrightarrow\ R_6,\ R_7,\ R_5\quad \text{due to}\quad (128), \tag{151}
$$

$$
II_b:\qquad R_4\,=\,\frac{1+\dot{\Sigma}^{\bar{c}c}}{1+\dot{\Sigma}^{BB}}\Big(1+\Sigma^{AB(1)}\Big)\,, \tag{152}
$$

$$
I_a:\qquad 1+\delta m^2_{(2)}\,=\,\frac{1}{1+\dot{\Sigma}^{BB}}\Big(1+\Sigma^{AB(1)}\Big)^2\,, \tag{153}
$$

$$
V:\quad 2\,F^{BBA}\,=\,F^{AAA}\frac{1+\dot{\Sigma}^{BB}}{1+\Sigma_{\text{trans}}}\ \longrightarrow F_1^{hBA}\ \longrightarrow\ F^{AAh(1)}\quad \text{due to}\quad (135),(138), \tag{154}
$$

$$
VII_a:\qquad \Big(\frac{M}{m}\Big)^2+\Sigma^{hh(2)}\,=\,\frac{4}{g}\,F^{BBh(1)}\,\frac{R_4}{R_3}\,, \tag{155}
$$

$$
VII_{b+c}:\qquad 1+\dot{\Sigma}^{hh}\,=\,(1+\dot{\Sigma}^{BB})\,\frac{R_5}{R_3}\,. \tag{156}
$$

Resuming the following task has been achieved: we first treated the functional Γ^{0,Λ_0} and its ancillary functionals $\Gamma^{0,\Lambda_0}_{\gamma_\tau},\Gamma^{0,\Lambda_0}_{\omega}$ with a BRS-insertion, disregarding the STI. There appear 37 + 7 relevant parameters. Fixing among these parameters a priori $\kappa=0$ (no tadpoles) and $\Sigma_{\text{long}}=0$ (due to the field equation of the antighost), and regarding the set (143) without Σ_{long}, as *renormalization constants to be chosen freely*, we can uniquely determine the remaining relevant parameters upon requiring the relevant part of the functional

Γ_1^{0,Λ_0} to vanish, (118), on account of the VSTI (108). Finally, since the relevant part of the functional Γ_1^{0,Λ_0} vanishes , due to Proposition 3, (119), its irrelevant part vanishes in the limit $\Lambda_0 \to \infty$, too. Thus perturbatively the functional $\Gamma^{0,\infty}$ and its ancillary funtionals $\Gamma_{\gamma_\tau}^{0,\infty}, \Gamma_{\omega}^{0,\infty}$ are finite and satisfy the STI, i.e. equation (108) for $\Lambda_0 \to \infty$ with the r.h.s. vanishing.

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Appendix A

The bare functional L^{Λ_0,Λ_0} and the relevant part of the generating functional Γ^{0,Λ_0} for the proper vertex functions have the same general form. We present the latter and give the tree order explicitly. At the end we state the modification to obtain the bare functional L^{Λ_0,Λ_0}. Writing

$$\Gamma^{0,\Lambda_0}(\underline{A},\underline{h},\underline{B},\underline{\bar{c}},\underline{c}) = \sum_{|n|=1}^{4} \Gamma_{|n|} + \Gamma_{(|n|>4)}\,,$$

where $|n|$ counts the number of fields, we extracted the relevant part, i.e. its local field content with mass dimension not greater than four. Moreover, in the sequel we do not underline the field variables though all arguments in the Γ- functional should appear underlined, of course.

1) One-point function

$$\Gamma_1 = \kappa \hat{h}(0).$$

2) Two-point functions

$$\Gamma_2 = \int_p \Big\{ \frac{1}{2}\, A_\mu^a(p) A_\nu^a(-p) \Gamma_{\mu\nu}^{AA}(p) + \frac{1}{2}\, h(p) h(-p) \Gamma^{hh}(p) + \frac{1}{2}\, B^a(p) B^a(-p) \Gamma^{BB}(p)$$
$$- \bar{c}^a(p) c^a(-p) \Gamma^{\bar{c}c}(p) + A_\mu^a(p) B^a(-p) \Gamma_\mu^{AB}(p) \Big\},$$

$$\Gamma_{\mu\nu}^{AA}(p) = \delta_{\mu\nu}(m^2 + \delta m^2) + (p^2 \delta_{\mu\nu} - p_\mu p_\nu)(1 + \Sigma_{\text{trans}}(p^2)) + \frac{1}{\alpha} p_\mu p_\nu (1 + \Sigma_{\text{long}}(p^2)),$$

$$\begin{aligned} \Gamma^{hh}(p) &= p^2 + M^2 + \Sigma^{hh}(p^2), \quad \Gamma^{BB}(p) = p^2 + \alpha m^2 + \Sigma^{BB}(p^2), \\ \Gamma^{\bar{c}c}(p) &= p^2 + \alpha m^2 + \Sigma^{\bar{c}c}(p^2), \quad \Gamma_\mu^{AB}(p) = i p_\mu \Sigma^{AB}(p^2). \end{aligned}$$

Besides the unregularized tree order explicitly stated, there emerge 10 relevant parameters from the various self-energies:

$$\delta m^2\,,\Sigma_{\rm trans}(0)\,,\Sigma_{\rm long}(0)\,,\Sigma^{hh}(0)\,,\dot{\Sigma}^{hh}(0)\,,\Sigma^{BB}(0)\,,\dot{\Sigma}^{BB}(0)\,,\Sigma^{\bar{c}c}(0)\,,\dot{\Sigma}^{\bar{c}c}(0)\,,\Sigma^{AB}(0)\,,$$

where the notation $\dot{\Sigma}(0) \equiv (\partial_{p^2}\Sigma)(0)$ has been used. We note that because of the regularization, the inverse of the regularized propagators (22) actually appears as the tree order ($l = 0$) of the 2-point functions. Due to the property (21), however, the regularizing factor $(\sigma_{0,\Lambda_0}(p^2))^{-1}$ does not contribute to the relevant part.

3) Three-point functions

We only present the relevant part explicitly. A relevant parameter vanishing in the tree order is denoted by $r \in \mathcal{O}(\hbar)$, otherwise it is denoted by F. Moreover, we indicate an irrelevant part by a symbol $\mathcal{O}_n,\ n \in \mathbf{N}$, reminding that this part vanishes like an n-th power of a momentum when all momenta tend to zero homogeneously.

$$\begin{aligned}
\Gamma_3 \;=\; & \int_p\int_q \big\{\epsilon^{rst}A^r_\mu(p)A^s_\nu(q)A^t_\lambda(-p-q)\Gamma^{AAA}_{\mu\nu\lambda}(p,q) \\
+\; & A^r_\mu(p)A^r_\nu(q)h(-p-q)\Gamma^{AAh}_{\mu\nu}(p,q) \\
+\; & \epsilon^{rst}B^r(p)B^s(q)A^t_\mu(-p-q)\Gamma^{BBA}_\mu(p,q) \\
+\; & h(p)B^r(q)A^r_\mu(-p-q)\Gamma^{hBA}_\mu(p,q) + \epsilon^{rst}\bar{c}^r(p)c^s(q)A^t_\mu(-p-q)\Gamma^{\bar{c}cA}_\mu(p,q) \\
+\; & B^r(p)B^r(q)h(-p-q)\Gamma^{BBh}(p,q) + h(p)h(q)h(-p-q)\Gamma^{hhh}(p,q) \\
+\; & \bar{c}^r(p)c^r(q)h(-p-q)\Gamma^{\bar{c}ch}(p,q) + \epsilon^{rst}\bar{c}^r(p)c^s(q)B^t(-p-q)\Gamma^{\bar{c}cB}(p,q)\big\},
\end{aligned}$$

$$\begin{array}{lcllcl}
\Gamma^{AAA}_{\mu\nu\lambda}(p,q) &=& \delta_{\mu\nu}i(p-q)_\lambda F^{AAA} + \mathcal{O}_3, & F^{AAA} &=& -\frac{1}{2}g + r^{AAA}, \\
\Gamma^{AAh}_{\mu\nu}(p,q) &=& \delta_{\mu\nu}F^{AAh} + \mathcal{O}_2, & F^{AAh} &=& \frac{1}{2}mg + r^{AAh}, \\
\Gamma^{BBA}_{\mu}(p,q) &=& i(p-q)_\mu F^{BBA} + \mathcal{O}_3, & F^{BBA} &=& -\frac{1}{4}g + r^{BBA}, \\
\Gamma^{hBA}_{\mu}(p,q) &=& i(p-q)_\mu F^{hBA}_1 & F^{hBA}_1 &=& \frac{1}{2}g + r^{hBA}_1, \\
 & & +i(p+q)_\mu r^{hBA}_2 + \mathcal{O}_3, & & & \\
\Gamma^{\bar{c}cA}_{\mu}(p,q) &=& ip_\mu F^{\bar{c}cA}_1 + iq_\mu r^{\bar{c}cA}_2 + \mathcal{O}_3, & F^{\bar{c}cA}_1 &=& g + r^{\bar{c}cA}_1, \\
\Gamma^{BBh}(p,q) &=& F^{BBh} + \mathcal{O}_2, & F^{BBh} &=& \frac{1}{4}g\frac{M^2}{m} + r^{BBh}, \\
\Gamma^{hhh}(p,q) &=& F^{hhh} + \mathcal{O}_2, & F^{hhh} &=& \frac{1}{4}g\frac{M^2}{m} + r^{hhh}, \\
\Gamma^{\bar{c}ch}(p,q) &=& F^{\bar{c}ch} + \mathcal{O}_2, & F^{\bar{c}ch} &=& -\frac{1}{2}\alpha gm + r^{\bar{c}ch}, \\
\Gamma^{\bar{c}cB}(p,q) &=& F^{\bar{c}cB} + \mathcal{O}_2, & F^{\bar{c}cB} &=& \frac{1}{2}\alpha gm + r^{\bar{c}cB}.
\end{array}$$

The 3-point functions AAB and BBB have no relevant local content.

4) Four-point functions

Defining as before parameters r and F, then

$$
\begin{aligned}
\Gamma_4|_{\rm rel} \;=\; & \int_k\int_p\int_q \big\{\epsilon^{abc}\epsilon^{ars}A^b_\mu(k)A^c_\nu(p)A^r_\mu(q)A^s_\nu(-k-p-q)F_1^{AAAA} \\
& +A^r_\mu(k)A^r_\mu(p)A^s_\nu(q)A^s_\nu(-k-p-q)r_2^{AAAA} \\
& +A^a_\mu(k)A^b_\mu(p)\bar{c}^r(q)c^s(-k-p-q)(\delta^{ab}\delta^{rs}r_1^{AA\bar{c}c}+\delta^{ar}\delta^{bs}r_2^{AA\bar{c}c}) \\
& +A^a_\mu(k)A^b_\mu(p)B^r(q)B^s(-k-p-q)(\delta^{ab}\delta^{rs}F_1^{AABB}+\delta^{ar}\delta^{bs}r_2^{AABB}) \\
& +B^a(k)B^b(p)\bar{c}^r(q)c^s(-k-p-q)(\delta^{ab}\delta^{rs}r_1^{BB\bar{c}c}+\delta^{ar}\delta^{bs}r_2^{BB\bar{c}c}) \\
& +h(k)h(p)h(q)h(-k-p-q)F^{hhhh} \\
& +B^r(k)B^r(p)h(q)h(-k-p-q)F^{BBhh} \\
& +B^r(k)B^r(p)B^s(q)B^s(-k-p-q)F^{BBBB} \\
& +A^r_\mu(k)A^r_\mu(p)h(q)h(-k-p-q)F^{AAhh} \\
& +h(k)h(p)\bar{c}^r(q)c^r(-k-p-q)r^{hh\bar{c}c} \\
& +\bar{c}^a(k)c^a(p)\bar{c}^r(q)c^r(-k-p-q)r^{\bar{c}c\bar{c}c} \\
& +\epsilon^{rst}h(k)B^r(p)\bar{c}^s(q)c^t(-k-p-q)r^{hB\bar{c}c}\big\},
\end{aligned}
$$

$$
\begin{array}{lcllcl}
F_1^{AAAA} & = & \frac{1}{4}g^2+r_1^{AAAA}, & F_1^{AABB} & = & \frac{1}{8}g^2+r_1^{AABB}, \\
F^{hhhh} & = & \frac{1}{32}g^2\left(\frac{M}{m}\right)^2+r^{hhhh}, & F^{BBhh} & = & \frac{1}{16}g^2\left(\frac{M}{m}\right)^2+r^{BBhh}, \\
F^{BBBB} & = & \frac{1}{32}g^2\left(\frac{M}{m}\right)^2+r^{BBBB}, & F^{AAhh} & = & \frac{1}{8}g^2+r^{AAhh}.
\end{array}
$$

Hence, Γ^{0,Λ_0} in total involves $1+10+11+15=37$ relevant parameters.
We now obtain the form of the bare functional L^{Λ_0,Λ_0}, together with its order $l=0$ explicitly given, upon deleting in the two-point functions the contributions of the order $l=0$, i.e. keeping there only the 10 parameters which appear in the various self-energies.

Appendix B

Analysing the STI, vertex functions (72) with one operator insertion, generated by the BRS-variations, have to be considered, too. These insertions have mass dimension $D=2$. We remind the notation (47) and (48) of the corresponding Fourier-transform, presenting the respective relevant part of these four vertex functions with one insertion,

$$
\begin{aligned}
\hat{\Gamma}^{0,\Lambda_0}_{\gamma^a_\mu}(q,\underline{\Phi})|_{\rm rel} &= -\,iq_\mu\,\underline{c}^a(-q)\,R_1+\epsilon^{arb}\int_k \underline{A}^r_\mu(k)\underline{c}^b(-q-k)gR_2\,,\\
\hat{\Gamma}^{0,\Lambda_0}_{\gamma}(q;\underline{\Phi})|_{\rm rel} &= -\frac{1}{2}\,g\int_k \underline{B}^r(k)\,\underline{c}^r(-q-k)R_3\,,\\
\hat{\Gamma}^{0,\Lambda_0}_{\gamma^a}(q;\underline{\Phi})|_{\rm rel} &= m\underline{c}^a(-q)R_4\\
&\quad+\int_k \underline{h}(k)c^a(-q-k)\frac{1}{2}\,gR_5+\epsilon^{arb}\int_k \underline{B}^r(k)\,\underline{c}^b(-q-k)\frac{1}{2}\,gR_6\,,\\
\hat{\Gamma}^{0,\Lambda_0}_{\omega^a}(q;\underline{\Phi})|_{\rm rel} &= \epsilon^{ars}\int_k \underline{c}^r(k)\,\underline{c}^s(-q-k)\frac{1}{2}\,gR_7.
\end{aligned}
$$

There appear 7 relevant parameters

$$R_i=1+r_i\,,\qquad r_i=\mathcal{O}(\hbar),\qquad i=1,...,7.$$

All the other two-point functions, and the higher ones, of course, are of irrelevant type.

Appendix C

As a consequence of the expansion in the mass parameters the conditions following from the fact that the relevant part of the functional Γ_1 should vanish

$$\Gamma_1(\underline{A},\underline{h},\underline{B},\underline{\bar{c}},\underline{c})|_{\dim\le 5}\ \overset{!}{=}\ 0.$$

can be reordered according to the value of ν which appears. We get contributions for $0\le\nu\le 3$. The value of ν in the various relevant couplings is indicated as a superscript in parentheses *if* $\nu>0$. We explicitly indicate the momentum and the power of m in front of each STI. The power of m indicates the value of ν in the corresponding contribution to Γ_1.

Two fields

I) $\delta_{A^a_\mu(q)}\delta_{c^r(k)}\Gamma_1|_0$

a) $0\ \overset{!}{=}\ m^2\,q_\mu\left\{-(1+\delta m^2_{(2)})R_1+\sum^{AB(1)}R_4+1+\frac{1}{\alpha}\sum^{\bar{c}c(2)}\right\}$,

b) $0\ \overset{!}{=}\ q^2q_\mu\Big\{-\frac{1}{\alpha}(1+\sum_{\rm long})R_1+\frac{1}{\alpha}(1+\dot{\sum}^{\bar{c}c})\Big\}$.

II) $\delta_{B^a(q)}\delta_{c^r(k)}\Gamma_1|_0$

a) $0\ \overset{!}{=}\ m^3\left\{(\alpha+\sum^{BB(2)})R_4-(\alpha+\sum^{\bar{c}c(2)})-\frac{g}{2}\,\kappa^{(3)}R_3\right\}$.

b) $0 \overset{!}{=} m\,q^2\Big\{-\sum^{AB(1)} R_1 + (1+\dot{\sum}^{BB})R_4 - (1+\dot{\sum}^{\bar{c}c})\Big\}.$

Three fields

III) $\delta_{A^r_\mu(p)}\delta_{A^s_\nu(q)}\delta_{c^t(k)}\Gamma_1|_0$

a) $0 \overset{!}{=} (p_\mu p_\nu - q_\mu q_\nu)\Big\{-2F^{AAA}R_1 - \frac{1}{\alpha}(F_1^{\bar{c}cA} - r_2^{\bar{c}cA}) + \Big[\frac{1}{\alpha}(1+\sum_{\text{long}}) - (1+\sum_{\text{trans}})\Big]\, gR_2\Big\},$

b) $0 \overset{!}{=} (p^2 - q^2)\delta_{\mu\nu}\,\{2F^{AAA}R_1 + (1+\sum_{\text{trans}})gR_2\},$

IV) $\delta_{A^r_\mu(p)}\delta_{B^s(q)}\delta_{c^t(k)}\Gamma_1|_0$

a) $0 \overset{!}{=} m\,p_\mu\Big\{2F^{BBA}R_4 + \frac{1}{2}g\sum^{AB(1)} R_6 + \frac{1}{\alpha}F^{\bar{c}cB,(1)} - r_2^{\bar{c}cA}\Big\},$

b) $0 \overset{!}{=} m\,q_\mu\Big\{g\sum^{AB(1)} R_2 + 4F^{BBA}R_4 + (F_1^{\bar{c}cA} - r_2^{\bar{c}cA})\Big\},$

V) $\delta_{B^r(p)}\delta_{B^s(q)}\delta_{c^t(k)}\Gamma_1|_0$

$0 \overset{!}{=} (p^2 - q^2)\Big\{2R_1F^{BBA} + (1+\dot{\sum}^{BB})\frac{g}{2}R_6\Big\},$

VI) $\delta_{A^r_\mu(p)}\delta_{h(q)}\delta_{c^t(k)}\Gamma_1|_0$

a) $0 \overset{!}{=} m\,p_\mu\Big\{-2R_1F^{AAh(1)} + R_4(F_1^{hBA} - r_2^{hBA}) + \sum^{AB(1)}\frac{1}{2}gR_5 - \frac{1}{\alpha}F^{\bar{c}ch(1)}\Big\},$

b) $0 \overset{!}{=} m\,q_\mu\,\{-2R_1F^{AAh(1)} + 2R_4F_1^{hBA}\},$

VII) $\delta_{h(p)}\delta_{B^s(q)}\delta_{c^t(k)}\Gamma_1|_0$

a) $0 \overset{!}{=} m^2\Big\{(\frac{M^2}{m^2} + \sum^{hh(2)})(-\frac{1}{2}gR_3) + 2F^{BBh(1)}R_4 + F^{\bar{c}ch(1)} + (\alpha + \sum^{BB(2)})\frac{1}{2}gR_5\Big\},$

b) $0 \overset{!}{=} p^2\Big\{F_1^{hBA}R_1 - (1+\dot{\sum}^{hh})\frac{1}{2}gR_3\Big\},$

c) $0 \overset{!}{=} q^2\Big\{-F_1^{hBA}R_1 + (1+\dot{\sum}^{BB})\frac{1}{2}gR_5\Big\},$

d) $0 \overset{!}{=} k^2\,\{r_2^{hBA}R_1\},$

VIII) $\delta_{c^t(q)}\delta_{c^s(p)}\delta_{\bar{c}^r(k)}\Gamma_1|_0$

a) $0 \overset{!}{=} m^2\Big\{2F^{\bar{c}cB(1)}R_4 - (\alpha + \sum^{\bar{c}c(2)})gR_7\Big\},$

b) $0 \overset{!}{=} k^2\Big\{F_1^{\bar{c}cA}R_1 - r_2^{\bar{c}cA}R_1 - (1+\dot{\sum}^{\bar{c}c})gR_7\Big\},$

c) $0 \overset{!}{=} (p^2 + q^2)\Big\{r_2^{\bar{c}cA}R_1\Big\}.$

Four fields

IX) $\delta_{h(p)}\delta_{h(q)}\delta_{B^1(k)}\delta_{c^1(l)}\Gamma_1|_0$

$$0 \stackrel{!}{=} m\,\Big\{6F^{hhh,(1)}(-\tfrac{1}{2}gR_3)+4F^{BBhh}R_4+2F^{BBh,(1)}gR_5+2r^{hh\bar{c}c}\Big\}.$$

X) $\delta_{B^1(k)}\delta_{B^1(p)}\delta_{B^2(q)}\delta_{c^2(l)}\Gamma_1|_0$

$$0 \stackrel{!}{=} m\,\Big\{-F^{BBh,(1)}gR_3+8F^{BBBB}R_4+\big(2r_1^{BB\bar{c}c}+r_2^{BB\bar{c}c}\big)\Big\}.$$

XI) $\delta_{h(l)}\delta_{\bar{c}^3(k)}\delta_{c^1(p)}\delta_{c^2(q)}\Gamma_1|_0$

$$0 \stackrel{!}{=} m\,\Big\{2r^{hB\bar{c}c}R_4+F^{\bar{c}cB(1)}gR_5+F^{\bar{c}ch,(1)}gR_7\Big\}.$$

XII) $\delta_{c^2(k)}\delta_{\bar{c}^2(l)}\delta_{c^1(p)}\delta_{B^1(q)}\Gamma_1|_0$

$$0 \stackrel{!}{=} m\,\Big\{F^{\bar{c}ch(1)}(-\tfrac{1}{2}gR_3)+(2r_1^{BB\bar{c}c}-r_2^{BB\bar{c}c})R_4+F^{\bar{c}cB(1)}(\tfrac{1}{2}gR_6-gR_7)+2r^{\bar{c}c\bar{c}c}\Big\}.$$

XIII)$_1$ $\delta_{A^1_\mu(k)}\delta_{A^2_\nu(p)}\delta_{B^1(q)}\delta_{c^2(l)}\Gamma_1|_0$

$$0 \stackrel{!}{=} 2r_2^{AABB}R_4+r_2^{AA\bar{c}c}.$$

XIII)$_2$ $\delta_{A^1_\mu(k)}\delta_{A^1_\nu(p)}\delta_{B^2(q)}\delta_{c^2(l)}\Gamma_1|_0$

$$0 \stackrel{!}{=} m\,\Big\{-F^{AAh(1)}gR_3+4F_1^{AABB}R_4+2r_1^{AA\bar{c}c}\Big\}.$$

XIV) $\delta_{A^1_\mu(p)}\delta_{A^1_\nu(q)}\delta_{A^2_\rho(k)}\delta_{c^2(l)}\Gamma_1|_0$

a) $0 \stackrel{!}{=} 2\delta_{\mu\nu}l_\rho\Big\{4(F_1^{AAAA}+r_2^{AAAA})R_1+2F^{AAA}gR_2+\frac{1}{\alpha}r_1^{AA\bar{c}c}\Big\},$

b) $0 \stackrel{!}{=} \delta_{\mu\nu}(p_\rho+q_\rho)\,\big\{\frac{2}{\alpha}r_1^{AA\bar{c}c}\big\},$

c) $0 \stackrel{!}{=} (\delta_{\mu\rho}l_\nu+\delta_{\nu\rho}l_\mu)\,\big\{-4F_1^{AAAA}R_1-2F^{AAA}gR_2\big\},$

d) $0 \stackrel{!}{=} (\delta_{\mu\rho}p_\nu+\delta_{\nu\rho}q_\mu)\,\{0\},$

e) $0 \stackrel{!}{=} (\delta_{\mu\rho}q_\nu+\delta_{\nu\rho}p_\mu)\,\big\{-\frac{1}{\alpha}r_2^{AA\bar{c}c}\big\}.$

XV)$_1$ $\delta_{B^1(p)}\delta_{B^1(q)}\delta_{A^2_\mu(k)}\delta_{c^2(l)}\Gamma_1|_0$

a) $0 \stackrel{!}{=} l_\mu\Big\{4F_1^{AABB}R_1+2F^{BBA}gR_6\Big\},$

b) $0 \stackrel{!}{=} k_\mu\,\big\{r_1^{BB\bar{c}c}\big\},$

XV)$_2$ $\delta_{B^1(p)}\delta_{B^2(q)}\delta_{A^1_\mu(k)}\delta_{c^2(l)}\Gamma_1|_0$

$$\text{a)}\ 0 \overset{!}{=} p_\mu \left\{-2r_2^{AABB}R_1 + 2F^{BBA}gR_2 + F_1^{hBA}gR_3\right\},$$

$$\text{b)}\ 0 \overset{!}{=} q_\mu \left\{-2r_2^{AABB}R_1 - 2F^{BBA}gR_2 + 2F^{BBA}gR_6\right\},$$

$$\text{c)}\ 0 \overset{!}{=} k_\mu \Big\{ -2r_2^{AABB}R_1 + F_1^{hBA}\tfrac{1}{2}gR_3 + r_2^{hBA}\tfrac{1}{2}gR_3 + F^{BBA}gR_6 - \tfrac{1}{\alpha}r_2^{BB\bar{c}c}\Big\},$$

XVI) $\delta_{h(p)}\delta_{A^1_\mu(k)}\delta_{B^2(q)}\delta_{c^3(l)}\Gamma_1|_0$

$$\text{a)}\ 0 \overset{!}{=} p_\mu \left\{F_1^{hBA}g(R_6 - R_2) - r_2^{hBA}gR_2\right\},$$

$$\text{b)}\ 0 \overset{!}{=} q_\mu \left\{F_1^{hBA}gR_2 - r_2^{hBA}gR_2 + 2F^{BBA}gR_5\right\},$$

$$\text{c)}\ 0 \overset{!}{=} k_\mu \Big\{F_1^{hBA}\tfrac{1}{2}gR_6 - r_2^{hBA}\tfrac{1}{2}gR_6 + F^{BBA}gR_5 - \tfrac{1}{\alpha}r^{hB\bar{c}c}\Big\},$$

XVII) $\delta_{h(p)}\delta_{h(q)}\delta_{A^1_\mu(k)}\delta_{c^1(l)}\Gamma_1|_0$

$$\text{a)}\ 0 \overset{!}{=} l_\mu \left\{4F^{AAhh}R_1 - F_1^{hBA}gR_5\right\},$$

$$\text{b)}\ 0 \overset{!}{=} k_\mu \left\{r_2^{hBA}gR_5 + \tfrac{2}{\alpha}r^{hh\bar{c}c}\right\}.$$

XVIII) $\delta_{A^2_\mu(k)}\delta_{c^2(p)}\delta_{c^1(q)}\delta_{\bar{c}^1(l)}\Gamma_1|_0$

$$\text{a)}\ 0 \overset{!}{=} l_\mu \left\{F_1^{\bar{c}cA}g(R_2 - R_7) + \tfrac{2}{\alpha}r^{\bar{c}c\bar{c}c}\right\},$$

$$\text{b)}\ 0 \overset{!}{=} p_\mu \Big\{2r_1^{AA\bar{c}c}R_1 + r_2^{\bar{c}cA}g(R_2 - R_7) + \tfrac{2}{\alpha}r^{\bar{c}c\bar{c}c}\Big\},$$

$$\text{c)}\ 0 \overset{!}{=} q_\mu \Big\{ - r_2^{AA\bar{c}c}R_1 - r_2^{\bar{c}cA}gR_7 + \tfrac{2}{\alpha}r^{\bar{c}c\bar{c}c}\Big\}.$$

Five fields

XIX) $\delta_{h(p)}\delta_{h(q)}\delta_{h(k)}\delta_{B^1(l)}\delta_{c^1(l')}\Gamma_1|_0$

$$0 \overset{!}{=} -2F^{hhhh}R_3 + F^{hhBB}R_5.$$

XX) $\delta_{h(p)}\delta_{B^1(q)}\delta_{B^1(k)}\delta_{B^2(l)}\delta_{c^2(l')}\Gamma_1|_0$

$$0 \overset{!}{=} -F^{BBhh}R_3 + 2F^{BBBB}R_5.$$

XXI) $\delta_{A^1_\mu(k)}\delta_{A^1_\nu(p)}\delta_{h(k)}\delta_{B^2(l)}\delta_{c^2(l')}\Gamma_1|_0$

$$0 \overset{!}{=} -F^{AAhh}R_3 + F_1^{AABB}R_5.$$

XXII) $\delta_{A^1_\mu(k)}\delta_{B^1(p)}\delta_{c^1(l')}\delta_{A^2_\nu(q)}\delta_{B^3(l)}\Gamma_1|_0$

$$0 \overset{!}{=} r_2^{AABB}(R_6 - 2R_2).$$

XXIII) $\delta_{A^1_\mu(k)}\delta_{B^1(q)}\delta_{A^2_\nu(p)}\delta_{c^2(l')}\delta_{h(l)}\Gamma_1|_0$

$$0 \overset{!}{=} r_2^{AABB}R_5.$$

XXIV) $\delta_{A^3_\mu(k)}\delta_{A^3_\nu(p)}\delta_{\bar{c}^2(q)}\delta_{c^3(l)}\delta_{c^1(l')}\Gamma_1|_0$

$$0 \overset{!}{=} r_2^{AA\bar{c}c}R_2 + r_1^{AA\bar{c}c}R_7.$$

XXV) $\delta_{A^3_\mu(k)}\delta_{\bar{c}^3(q)}\delta_{A^2_\nu(p)}\delta_{c^3(l)}\delta_{c^1(l')}\Gamma_1|_0$

$$0 \overset{!}{=} r_2^{AA\bar{c}c}(3R_2 - R_7).$$

XXVI) $\delta_{B^1(p)}\delta_{B^1(q)}\delta_{\bar{c}^1(k)}\delta_{c^2(l)}\delta_{c^3(l')}\Gamma_1|_0$

$$0 \overset{!}{=} r_2^{BB\bar{c}c}(R_6 - R_7) - r_1^{BB\bar{c}c}R_7.$$

XXVII) $\delta_{B^1(p)}\delta_{\bar{c}^1(k)}\delta_{B^2(q)}\delta_{c^3(l)}\delta_{c^1(l')}\Gamma_1|_0$

$$0 \overset{!}{=} -r^{hB\bar{c}c}R_3 + r_2^{BB\bar{c}c}(3R_6 - 2R_7).$$

XXVIII) $\delta_{h(p)}\delta_{h(q)}\delta_{\bar{c}^1(k)}\delta_{c^2(l)}\delta_{c^3(l')}\Gamma_1|_0$

$$0 \overset{!}{=} r^{hB\bar{c}c}R_5 + r^{hh\bar{c}c}R_7.$$

XXIX) $\delta_{h(p)}\delta_{B^1(q)}\delta_{c^1(l)}\delta_{\bar{c}^2(k)}\delta_{c^2(l')}\Gamma_1|_0$

$$0 \overset{!}{=} 2r^{hh\bar{c}c}R_3 - 2r_1^{BB\bar{c}c}R_5 + r_2^{BB\bar{c}c}R_5 + r^{hB\bar{c}c}(-R_6 + 2R_7).$$

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