CONSTRUCTING NON-SELF-DUAL YANG-MILLS CONNECTIONS ON S^4 WITH ARBITRARY CHERN NUMBER

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A connection A on a principal bundle over a 4-manifold M is called *Yang-Mills* if it is a critical point of the Yang-Mills (YM) action

$$S(A) = \int_M |F_A|^2 dVol = \int_M -Tr(*F_A \wedge F_A), \tag{1}$$

where $F_A = dA + [A, A]$ is the curvature of the connection A and $*$ is the Hodge dual. Equivalently, Yang-Mills connections are solutions of the *Yang-Mills equations*,

$$d_A * F_A = 0, \tag{2}$$

where d_A denotes the covariant exterior derivative. These are the variational equations of the YM action, and constitute a system of second-order PDE's in A.

For a given second Chern number C_2, the YM action is bounded below by $8\pi^2|C_2|$. To see this, let

$$F_\pm = \frac{1}{2}\left(F_A \pm *F_A\right)$$

be the self-dual and anti-self-dual parts of the curvature. We can then express the action and the Chern number as

$$S(A) = \int_M |F_A|^2 = \int_M |F_+|^2 + |F_-|^2$$

$$C_2 = \frac{-1}{8\pi^2}\int_M Tr(F_A \wedge F_A) = \frac{1}{8\pi^2}\int_M |F_+|^2 - |F_-|^2.$$

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The absolute minima of the action, connections with action attaining the topological bound $8\pi^2|C_2|$, are thus characterized as having either self-dual curvature ($*F_A = F_A$, hence $F_- = 0$) or anti-self-dual curvature ($*F_A = -F_A$). These (anti) self-dual connections have been well-understood for some time. The first non-trivial example was the self-dual $SU(2)$ instanton on S^4, discovered in 1975 [BPST]. Three years later, all self-dual connections on S^4 were classified [ADHM], not only for $SU(2)$ but for all classical groups. The study of self-dual $SU(2)$ connections over arbitrary 4-manifolds led to spectacular progress in topology, including the discovery of exotic differentiable structures on $\mathbb{R}^4$ (see [FU] for an overview).

A natural question is whether any *non*-self-dual (NSD) Yang-Mills connections exist. Several classes of NSD YM connections on four-manifolds are known. One class, due to Itoh [I], consists of homogeneous connections on bundles over S^4 with several large structure groups, including $SU(4)$. Other solutions on S^4 have been constructed by 'twistor' methods. Buchdal [Bu] has produced solutions with the noncompact structure group $SL(2,\mathbb{C})$, and Manin [Ma] has produced solutions with various compact groups of very high dimension (as well as with supergroups). Parker [P] has constructed a solution on S^4 with a nonstandard Riemannian metric and structure group $SU(2)$. Some solutions are also known on other four-manifolds, namely $S^2 \times S^2$ [Ur], and $S^1 \times S^3$ [P, Ur].

Until recently it appeared that for $SU(2)$, NSD YM connections over the standard 4-sphere S^4 might not exist. Indeed, analogies with harmonic maps from S^2 to S^2 appeared to indicate that none exist [AJ], and NSD YM connections with certain simple symmetries were ruled out [T2, JT]. Moreover, it was shown that no *local* minima of the YM action exist [BLS, T1]; YM connections are either global minima (hence (anti) self-dual) or saddle points.

Sibner, Sibner and Uhlenback [SSU] recently showed that NSD YM connections on the trivial bundle $S^4 \times SU(2)$ do exist. As Lesley Sibner explained in this meeting, their construction involves using minmax theory to generate monopoles on hyperholic space $\mathbb{H}^3$, which correspond to Yang-Mills connections on $\mathbb{R}^4$ with a certain $U(1)$ symmetry.

In this talk we would like to explain an alternate and somewhat simpler method for constructing NSD YM connections. This work is described in the papers [SS], and is based extensively on the work of Urakawa [Ur] and the work of Bor and Montgomery [BoMo]. Our construction produces examples not only on the trivial bundle, but on all $SU(2)$ bundles over S^4, except those with second Chern number equal to ± 1. We still do not know whether any NSD YM connections exist with Chern number ± 1.

The strategy is as follows:

1) Pick a symmetry on S^4. This reduces all calculations from a 4-dimensional

space, S^4, to a much smaller space, the space of group orbits. In our case the symmetry group is the rotation group $SO(3)$ (and its cover $SU(2)$), and the space of orbits is isomorphic to the interval $[0, \pi/3] \subset \mathbb{R}$.

2) Consider $SU(2)$ connections on S^4 that are equivariant under this symmetry. Since connections live on bundles, we need to lift the symmetry group action from the base manifold S^4 to $SU(2)$ bundles over S^4. The equivariant connections fall into distinct classes, corresponding to different lifts of the group action. In our case these classes are indexed by two positive odd integers $n_\pm$. The bundle corresponding to (n_+, n_-) has second Chern number $C_2 = (n_+^2 - n_-^2)/8$.

3) In each class of equivariant connections, look for *minima* of the action. By the principle of symmetric criticality [Pal], such minima must be stationary points of the action in the space of all connections, i.e. must by Yang-Mills. However, they need not be minima in the space of all connections, as the second variation in some non-equivariant directions may be negative. In our case, we show that minima exist for all classes with $n_+ \neq 1$, $n_- \neq 1$.

4) Finally, show that some classes do not contain (anti) self-dual connections. In our case we find that self-dual connections can only exist for $n_- = 1$ and anti-self-dual connections only exist for $n_+ = 1$.

These results give NSD YM connections in every class (n_+, n_-) with $n_\pm \geq 3$. Since every integer N except ± 1 can be written as $N = (n_+^2 - n_-^2)/8$ with $n_\pm \geq 3$ in at least one way, this gives examples with every Chern number except ± 1. For some Chern numbers we get several solutions (e.g. $5 = (7^2 - 3^2)/8 = (11^2 - 9^2)/8$), and for the trivial bundle we have a countably infinite number of solutions (just take $n_+ = n_-$).

1. The Symmetry

Let $V \simeq \mathbb{R}^5$ be the space of symmetric, traceless, real 3×3 matrices Q, with inner product $\langle Q, Q' \rangle = \frac{1}{2} Tr(QQ')$. It's convenient to work with an explicit orthonormal basis

$$Q_0 = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}; \qquad Q_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \qquad Q_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix};$$

$$Q_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \qquad Q_4 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

We let $SO(3)$ act on V by conjugation, $g(Q) = gQg^{-1}$. Restricting ourselves to the unit sphere in V, this gives an action of $SO(3)$ on S^4. Since $SU(2)$ is the double cover of $SO(3)$, this also gives an action of $SU(2)$ on S^4.

Since all matrices in V are diagonalizable, it's not hard to check that

Proposition 1. *Every $Q \in S^4$ is related by the group action to a unique $Q_\theta = \cos(\theta)Q_0 + \sin(\theta)Q_3$ with $0 \leq \theta \leq \pi/3$. The orbits of Q_0 and $Q_{\pi/3}$ are two-dimensional, while all other orbits are three-dimensional.*

As a result, equivariant connections forms are determined by their values on the path Q_θ, $0 \leq \theta \leq \pi/3$. We put coordinates (θ, y^1, y^2, y^3) on a neighborhood of this path by

$$(\theta, \vec{y}) \mapsto \exp(\vec{y} \cdot \vec{\sigma})\,(Q_\theta)\,,$$

where σ_1, σ_2, and σ_3 are the usual (antihermitian) generators of $SU(2)$, and the action of $SU(2)$ on Q_θ is as above. On the path the tangent vectors $\partial_\theta \equiv \partial/\partial\theta$ and $\partial_i \equiv \partial/\partial y^i$ are orthogonal but not orthonormal. The vector ∂_θ is normalized, but the length of the vector ∂_i at Q_θ is $f_i(\theta)$, where

$$f_1(\theta) = 2\sin(\pi/3 + \theta); \qquad f_2(\theta) = 2\sin(\pi/3 - \theta); \qquad f_3(\theta) = 2\sin(\theta).$$

Note that f_3 vanishes at $\theta = 0$, as Q_0 is invariant under rotations about the z-axis. Similarly, f_2 vanishes at $\pi/3$.

¿From this it is easy to see how the Hodge dual operator $*$ acts on 2-forms:

$$*(d\theta \wedge dy^i) = \frac{f_j f_k}{f_i} dy^j \wedge dy^k, \tag{3}$$

where (i, j, k) are cyclic permutations of $(1, 2, 3)$. To simplify the notation we define the functions

$$G_1 = \frac{f_2 f_3}{f_1}; \qquad G_2 = \frac{f_1 f_3}{f_2}; \qquad G_3 = \frac{f_1 f_2}{f_3}.$$

G_1 and G_2 have zeroes at $\theta = 0$, while G_3 has a pole. Similarly, G_1 and G_3 have zeroes at $\theta = \pi/3$, while G_2 has a pole.

2. Equivariant Connections

We next look at $SU(2)$ connections on S^4 that are equivariant under the above action of $SU(2)$. Such equivariant connections appeared in a study on non-Abelian Berry's phase [ASSS], and were classified by Bor and Montgomery [BoMo], who took Urakawa's general theory of equivariant connections with one-dimensional orbit spaces [Ur] and applied it to this particular symmetry. Much of this section is due to [BoMo].

An equivariant connection is of course determined by its values on the path $\{Q_\theta\}$. The most general Lie-algebra valued one-form on the path is

$$A = -\sum_{i,j=1}^{3} \alpha_{ij}(\theta) dy^i \otimes \sigma^j - \sum_{i=1}^{3} \beta_i(\theta) d\theta \otimes \sigma^i,$$

where α_{ij} and β_i are real-valued functions. However, while rotations by 180° about the x, y, or z axes send Q_θ to itself, they do not preserve a general A of this form. For example, rotation by 180° about the z-axis flips the signs of dy^1, dy^2, σ^1, and σ^2, but not the signs of dy^3, $d\theta$, and σ^3. As a result, for an equivariant connection the coefficients α_{13}, α_{23}, α_{31}, α_{32}, β_1, and β_2 must be identically zero. Similarly, invariance under 180° rotations about the x-axis forces α_{12}, α_{21} and β_3 to be zero. Thus equivariant connections may be described by only three real-valued functions, $a_i = \alpha_{ii}$, and we write

$$A = -\sum_{i=1}^{3} a_i(\theta) dy^i \otimes \sigma^i.$$

We call such a triplet of functions $a = (a_1, a_2, a_3)$ a *reduced connection.*

Given an equivariant connection A, the curvature F_A is easily computed:

$$F_A = \big((a_1 + a_2 a_3) dy^2 \wedge dy^3 - a_1' d\theta \wedge dy^1\big) \otimes \sigma^1 + (cyclic),$$

where $'$ denotes $d/d\theta$, and $(cyclic)$ denotes the other cyclic permutations of the indices $(1, 2, 3)$. By equation (3) the (anti) self-duality equations are then

$$-a_1' = \pm \frac{(a_1 + a_2 a_3)}{G_1}, \qquad -a_2' = \pm \frac{(a_2 + a_1 a_3)}{G_2}, \qquad -a_3' = \pm \frac{(a_3 + a_1 a_2)}{G_3}, \tag{4}$$

where $+$ denotes self-duality and $-$ denotes anti-self-duality.

¿From F_A and $*$ we compute the action (1). The result is

$$\begin{aligned} S(A) = \pi^2 \int_0^{\pi/3} & d\theta \Big[(a_1')^2 G_1 + (a_2')^2 G_2 + (a_3')^2 G_3 \\ & + \frac{(a_1 + a_2 a_3)^2}{G_1} + \frac{(a_2 + a_1 a_3)^2}{G_2} + \frac{(a_3 + a_1 a_2)^2}{G_3} \Big]. \end{aligned} \tag{5}$$

Finite action connections must have well-defined boundary values $r = a_3(0)$ and $t = a_2(\pi/3)$. Also, since G_1 and G_2 have zeroes at $\theta = 0$, they must have

$$a_1(0) + a_2(0) a_3(0) = a_2(0) + a_1(0) a_3(0) = 0.$$

If $r \neq \pm 1$, then these conditions imply that both $a_1(0)$ and $a_2(0)$ equal zero. Similarly, if $t \neq \pm 1$ then $a_1(\pi/3) = a_2(\pi/3) = 0$.

Not all finite-action reduced connections correspond to connections on all of S^4. A holonomy condition for infinitesimal paths around Q_0 forces $r \equiv -1 \pmod 4$, and a similar condition at $Q_{\pi/3}$ forces $t \equiv -1 \pmod 4$. If these conditions are met we define the positive odd integers $n_+ = |r|$, $n_- = |t|$. (If the holonomy conditions are not met, then our reduced connection corresponds to a singular connection with non-integer Chern number [FHP, SiSi], with the singularities occurring at the orbits of Q_0 and $Q_{\pi/3}$.)

Finally, we compute the Chern number of a connection. On our path the Chern form is

$$\begin{aligned}\frac{-1}{8\pi^2}Tr(F_A \wedge F_A) =& \frac{-1}{8\pi^2}(a_1'(a_2 + a_2 a_3) + cyclic) d\theta \wedge dy^1 \wedge dy^2 \wedge dy^3 \\ =& \frac{-1}{16\pi^2} d(a_1^2 + a_2^2 + a_3^2 + 2a_1 a_2 a_3) \wedge dy^1 \wedge dy^2 \wedge dy^3,\end{aligned}$$

which we integrate, first over the group and then over $[0, \pi/3]$, to get a Chern number of $(r^2 - t^2)/8 = (n_+^2 - n_-^2)/8$.

3. Nonexistence of Self-Dual Connections

Before showing that Yang-Mills connections do exist, we would like to prove that in certain classes (anti) self-dual connections do not exist. We prove this not only for the non-singular classes (n_+, n_-) with $n_\pm \geq 3$, but also for a large number of singular classes (r, t). Specifically,

Theorem 2. *There are no finite-action self-dual reduced connections with $|t| > 1$. There are no finite-action anti-self-dual reduced connections with $|r| > 1$.*

We prove the second statement, the first being similar. An anti-self-dual connection has non-positive Chern number, so $|t| \geq |r| > 1$. Since $|r|$ and $|t|$ both differ from 1, finite action implies that $a_1(0) = a_2(0) = a_1(\pi/3) = a_3(\pi/3) = 0$. We will show that a solution to the anti-self-dual equations with $a_1(0) = a_2(0) = 0$ must have $a_3(\pi/3) \neq 0$, contradicting this.

Suppose $r > 1$ (the case $r < -1$ is similar). Then a_3 is positive and greater than 1 on a neighborhood $I_\epsilon = (0, \epsilon)$. If at some point in this neighborhood both a_1 and a_2 are non-negative, then by the anti-self-duality equations (4) all three derivatives will be non-negative, and the signs will persist. In particular, $a_3(\pi/3)$ will be positive, not zero. Similarly, if at some point in I_ϵ both a_1 and a_2 are non-positive, then $a_1', a_2' \leq 0 \leq a_3'$ and again the signs persist. Thus it suffices to find a single point $\theta \in I_\epsilon$ at which a_1 and a_2 have the same sign (or where one is zero).

Suppose there is no such point, so a_1 and a_2 have opposite signs on all of I_ϵ. Then $|a_1 - a_2| < |a_1 + a_2|$. Equations (4) yield

$$\begin{aligned}\frac{d(a_1 - a_2)^2}{d\theta} &= 2(a_1 - a_2)(a_1 - a_2)' \\ &= -(\frac{1}{G_1} + \frac{1}{G_2})(a_3 - 1)(a_1 - a_2)^2 + (\frac{1}{G_1} - \frac{1}{G_2})(a_3 + 1)(a_1 - a_2)(a_1 + a_2).\end{aligned}$$

The first term dominates in a neighborhood of $\theta = 0$, since $G_1^{-1} + G_2^{-1}$ has a pole at zero, while $G_1^{-1} - G_2^{-1}$ does not, and since $(a_3 - 1)$ is bounded away from zero on I_ϵ. Thus $[(a_1 - a_2)^2]'$ is strictly negative, and $(a_1 - a_2)^2$ is a strictly decreasing non-negative function. However, at $\theta = 0$ we have $(a_1 - a_2)^2 = (0 - 0)^2 = 0$, and cannot decrease further. We have a contradiction, and so are done.

4. Existence of Minima

What remains is to show that in each class (n_+, n_-) the action achieves its minimum. This is to be expected [BoMo], since the symmetry should prevent any bubbling-off phenomena, as in the equivariant Sobolev theorems of Parker [P]. By symmetry, such bubbling would have to occur on a complete orbit. But each orbit contains an infinite number of points, and Uhlenbeck's theorem only allows bubbling at a finite number of points.

This is in fact true, and as before, we prove our result for both singular and non-singular connections.

Theorem 3. *On each class (r, t) with $|r| > 1$, $|t| > 1$ the action achieves its minimum.*

The proof, which is only sketched here (the details being rather grungy), is by the direct method of the calculus of variations. We first define a Hilbert space $\mathcal{H}$ with norm

$$\|a\|^2 = \int_0^{\pi/3} d\theta \Big[(a_1')^2 G_1 + (a_2')^2 G_2 + (a_3')^2 G_3 + \frac{(a_1)^2}{G_1} + \frac{(a_2)^2}{G_2} + \frac{(a_3)^2}{G_3} \Big].$$

This norm resembles the action, only with the cubic and quartic terms removed. $\mathcal{H}$ is the direct sum of three weighted Sobolev spaces, one for each a_i.

We next show that, for fixed (r, t), sets of bounded action have bounded norm. This implies that any minimizing sequence lies in a finite radius ball in $\mathcal{H}$, which is weakly compact. Finally we show that the action is weakly lower-semicontinuous, hence that the weak limit of a minimizing sequence achieves the minimum action.

The difficulty is in showing that bounded action implies bounded norm. Away from the boundaries we have no problem, but near 0 and $\pi/3$ various functions G_i and G_i^{-1} diverge, complicating the analysis. The biggest difficulty is in bounding $a_1^2/G_1 + a_2^2/G_2$ near $\theta = 0$ (with a similar problem at $\theta = \pi/3$). This is done using the fact that both G_1 and G_2 go as $1/\theta$, and noting that

$$(a_1 + a_2 a_3)^2 + (a_2 + a_1 a_3)^2 = (a_3 + 1)^2 (a_1 + a_2)^2 + (a_3 - 1)^2 (a_1 - a_2)^2 \geq 2(|a_3| - 1)^2 (a_1^2 + a_2^2).$$

For $|r| > 1$ we can bound $|a_3| - 1$ away from zero in some neighborhood of $\theta = 0$, and so can bound $a_1^2/G_1 + a_2^2/G_2$ by a multiple of $(a_1 + a_2 a_3)^2/G_1 + (a_2 + a_1 a_3)^2/G_2$.

This whole approach breaks down for $r = \pm 1$ or $t = \pm 1$. In those cases bounded action does *not* imply bounded norm, and we have no proof that the minimum is achieved.

5. Regularity

For each pair (r, t) with $|r| > 1$ and $|t| > 1$ we have found solutions to the one-dimensional variational problem that do not satisfy the (anti) self-duality equations. These correspond to equivariant non-self-dual Yang-Mills connections on S^4 with the two exceptional orbits removed. The only remaining question is whether these connections may be extended smoothly across these two orbits.

For $r \not\equiv -1 \pmod 4$ they cannot be extended across the orbit of Q_0 due to a holonomy obstruction, and for $t \not\equiv -1 \pmod 4$ they cannot be extended across the orbit of $Q_{\pi/3}$. For r and t congruent to $-1 \pmod 4$, however, the extension is possible. The proof is straightforward but lengthly, and we only sketch the main ideas here.

We first choose a particular connection (call it B), which is known to be smooth by the theorems of [BoMo]. We then show that the difference between our finite-action connection A and the reference connection B approaches zero as $\theta \to 0$, and hence that A can be extended *continuously* across the orbit of Q_0.

We then compute the L_1^2 norm of $A - B$ in a particular gauge and show it to be finite. Since B is smooth, this means that A is in L_1^2. By Uhlenbeck's regularity theorem [Uh], there exists a gauge in which $d * \tilde{A} = 0$, where $\tilde{A}$ is gauge equivalent to A. The Yang-Mills equations together with this gauge condition form an elliptic system of equations, which implies that $\tilde{A}$ is smooth. Finally, since A was continuous to begin with, the gauge transformation must be C^1, and so cannot change the topology of the bundle.

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