SELFCONSISTENCY AND OBJECTIFICATION

PAVEL BÓNA

Department of Theoretical Physics, Comenius University
842 15 Bratislava, Slovakia
and
Department of Mathematics, Silesian University
746 01 Opava, Czech Republic

ABSTRACT

Three mathematical models of interaction of a "small" quantummechanical system with a "large" one are presented. It is discussed their possible rôle in solving interpretation problems of quantum mechanics (QM). The third model determines the ("unsharp") quantity measured in the described process as a consequence of the chosen interaction of the "object" with a macroscopic "apparatus". The fourth of the presented models introduces a natural nonlinearity into dynamics of QM; the nonlinear evolution allows to distinguish between the two kinds of mixed states: between the improper ones (called also "elementary mixtures"), and the genuine mixtures (called also "Gemenge"). A rôle of "selfconsistency" in trials of interpretation of QM as a universal theory is briefly discussed. The paper contains some remarks connecting the "objectification problem" of quantum measurement theory with the problem of universality of quantum mechanics.

1. General Framework

1.1. Some Philosophical Notes

We shall restrict present discussion to technical questions mainly, leaving the relevant but difficult philososphical considerations almost untouched. These notions have to offer to readers just very rough orientation in some relevant aspects of the philosophical position accepted by the present author.

Let us first mention that the notions like "object", "physical system", or "observer" are considered here as just names for specified sets of statistically correlated "events" (considered as elements of *our* knowledge; however, the concept *of ourselves* is also of this kind). Hence, we do not pursue here any "realistic" point of view. A physical theory is considered as a model (consisting of *formalism* and of its *interpretation*) which is to be compared with ever anew empirically selected phenomena. *Universal theory* contains (sub-) models for (potential) description of arbitrary phenomena (at least from a specified "Universe"). Theories are human (although not arbitrary) "constructs" which allow us increase probabilities of our realized empirical expectations. A key problem of a "working" application of a theory is the problem of *identification* of specific events with a certain classes (e.g. concepts) dealt with (resp. contained in) the theory. Expressed briefly, the problem often arises to decide "what is the same as..." in specified conditions.

If a universal theory includes a model of human mind (i.e. of its own "constructor"), and if one wants maintain in such a theory the man's "free will", the theory should contain possibility of more than one of different responses of a man to each of most of conceived states of his mind. This consideration alown (before taking into account any specific physical knowledge) seems to be sufficient to require an introduction of some irreducible statistical elements into that theory. This in turn seems to indicate that any "completely hidden variables theory" (in which all situations could be described by classical probabilities) cannot be universal: it makes conceivable fully deterministic behaviour of Universe.

1.2. On Universality and Objectification

In considerations on foundations of QM, we are faced with situation when an interpretation of any quantummechanical (quantal) "observable" is specified in terms of kinematics of classical mechanics (CM) reflecting our reproducible experience with macroscopic bodies. Hence, any interpretation of universality of QM requires a possibility of description of concepts of CM in framework of QM. This leads to a kind of selfreference of QM: Quantum theory should contain its "classical background" which itself is a prerequisite of any interpretation of QM. If inclusion of an "observer" into description of the process of measurement in QM would be inevitable, then universal QM should describe also a model of human mind in its framework.

As a decisive problem of interpretation of QM, the "objectification problem" is considered[1] often, posing the question whether and when a state of the system S (e.g. an apparatus or even of a microsystem) can be unambiguously considered as a "Genuine mixture" (resp. "Gemenge"), i.e. considered as a state seen "in all possible situations" as the same statistical ensemble of systems each of which "really possesses" definite values of certain properties (in the measurement process these properties should be in a certain way related to values of the measured quantity). To endow with an empirical meaning this vague specification of that concept, let us determine here the "objectification" as a physical process with the system S extended (into a larger system) by *any such system correlations of which with S could be measured in the considered framework* (the mentioned "considered framework" has to be specified, cf. also Secs.1.3 and 1.4); the state *of the whole extended system* reached at the end of this process should behave as the corresponding mixture with respect to *all possible measurements* (cf. Secs.1.3 and 1.4). In this way, any member of that statistical ensemble corresponds to a distinct superselection sector.[1−4] Single members of the corresponding convex decomposition of such a state of S need not represent pure states, neither should correspond to orthogonal projections in its Hilbert state-space. It could be seen from these considerations that a successful solution to the "objectification problem" requires a possibility of such a dynamical process in QM which brings into correlations (in course of measurement, e.g.) classically distinguished states with the corresponding "objectified" states of the considered system S (e.g. microscopic; S could be, for example, that "second" $\frac{1}{2}$-spin system which was not actually measured in the Bohm version of the EPR-like process). "State-vector collapse" comes from emergence of such correlations.

1.3. On C^*-algebraic Approach

The C^*-algebraic (quasilocal) description of systems with infinite number of degrees of freedom offers mathematically well formulated models of QM-systems containing a CM-subsystem: The classical quantities $\mathcal{C}_{cl}$ form a coveniently chosen subalgebra of the commutative subalgebra $\mathcal{Z}$ ($\equiv$"the center of the second topological dual $\mathcal{A}^{**}$ of the quasilocal C^*-algebra $\mathcal{A}$"). The choice of $\mathcal{C}_{cl} \subset \mathcal{Z}$ ($\mathcal{Z}$ is too large to be useful whole at once) can be done with a help of dynamics of the system described (locally) by $\mathcal{A}$, as it is the case of the mean-field models of Hepp and Lieb[6−8]. These models lead also to an effectively nonlinear evolution of each elementary subsystem of the infinite system in translationally invariant states. Such a nonlinear dynamics can be described, however, as an autonomous (formally classical Hamiltonian) dynamics on the (infinite dimensional) quantum state space of an elementary constituent subsystem without any reference to the "mean-field background" produced by the infinite colection of such systems (cf.Sec.3).[11]

The algebra $\mathcal{A}$ can be faithfully represented as an algebra of bounded operators on a (possibly nonseparable) Hilbert space $\mathcal{H}$ which is strictly smaller than the algebra $\mathcal{L}(\mathcal{H})$ of all bounded operators. The whole algebra $\mathcal{L}(\mathcal{H})$ is traditionally considered, however, to be an "algebra of observables" of closed finite quantal system (with separable $\mathcal{H}$, and without superselection rules).[1,4] Hence, in the C^*-algebraic approach to (infinitely) large systems, it is used an (effective) *restriction* of the set of observables: It is assumed that only observables of (arbitrarily large, but) finite subsystems are accepted as measurable quantities. This approach leads to a natural appearance of superselection rules determined by classical "observables at infinity" belonging to $\mathcal{Z}$. The mentioned addition of the global quantities $\mathcal{C}_{cl}$ from $\mathcal{Z}$ is a mathematically independent theoretical procedure made without generally accepted principles. Quasilocal C^*-algebras are used in Secs.2.1 and 2.3.

1.4. Other Restrictions to the Sets of Observables

If one wants to avoid a use of infinite systems in QM, and if one wants to have a universal (version of) QM, then one needs some, at least effective introduction of superselection rules.[1] An idea how to do this consists in proposal of detailed consideration of what a quantal measurement is, to obtain a subsequent restriction to those quantities which could be "really measured". We expect that the empirically found restriction to the known kinds of interactions will lead to restricted possibility of constructing measuring apparatuses, hence, perhaps, also to some superselection rules. Any general theory of this kind of selection of observable quantities does not exist yet. It might be interesting in this context to follow the effort of experimental physicists to measure some multiparticle observables. It might be seen that it is very difficult to measure even three particle quantities.[9] We shall discuss here briefly only a simple specific proposal how to formulate a restriction of the set of observables to the set of at most k-particle quantities in an N-particle system (with very large N).

Let the large system be described by tensor-product Hilbert space $\mathcal{H}^N := \otimes_{p=1}^N \mathcal{H}_p \ni \Phi, \Psi;\ \Phi := \otimes_{p=1}^N \varphi_p,\ \Psi := \otimes_{p=1}^N \psi_p$, with φ_p and $\psi_p \in \mathcal{H}_p$. Assume that

$\|\varphi_p\| = \|\psi_p\| = 1$, and $|\langle\varphi_p|\psi_p\rangle| \leq \lambda < 1$ for all p. Let all the observables be of the form

$$A := \sum_{p_1,p_2,\dots p_k} X^1_{p_1} \otimes X^2_{p_2} \otimes \dots \otimes X^k_{p_k} \otimes I_{\{N\setminus\{p_1,p_2,\dots p_k\}\}} \tag{1.1}$$

with X^j_p being selfadjoint in $\mathcal{L}(\mathcal{H}_p)$ of the norm ≤ 1. Let $k(\ll N)$ be fixed. Then, for $N \to \infty$, one has[3]

$$|\langle\Psi|A\Phi\rangle| < N^k \lambda^{N-k} \approx O(e^{-\alpha N}) \tag{1.2}$$

for some $\alpha > 0$. This presents a kind of "effective disjoitness" of the states described by Ψ and Φ with respect to the chosen set of observables of the form (1.1).

1.5. On Relevance of Nonlinear Evolution

An effectively nonlinear evolution of the constituent subsystem of an infinite system appeares in models of large systems with mean-field type long-range interaction.[7,10] Such a time evolution can be described also as autonomous evolution of the subsystems.[11] Whether such an evolution will be observed or not, we want to point out that nonlinear motions only could distinguish between "elementary mixtures" and "Gemenge". Any "Gemenge" evolves, from its definition, in such a way that its density matrix ϱ expressed as the convex combination

$$\varrho := \sum_j \lambda_j \varrho_j \tag{1.4}$$

of density matrices ϱ_j of the single systems included into the statistical ensemble desribed by this "Gemenge" evolves with time t as the same convex combination of independently time-evolved density operators ϱ_j:

$$\varrho_t \equiv \sum_j \lambda_j \varrho_{jt}. \tag{1.5}$$

Such an evolution corresponds to linear QM. In the case of nonlinear evolutions one cannot keep Eq.(1.5) to stay valid for any ϱ and for each of its decompositions of the form (1.4).

Nonlinear evolution can be considered as an evolution with Hamiltonian depending on state of the system.[14,11] Let us assume that the considered microsystem evolves interacting with its "purely macroscopic" environment, i.e. it has "its own" Hamiltonian depending on an actual state of the environment. If, moreover, the (macroscopic) state of the environment depends on the state of the microsystem,[15] we arrive at nonlinear evolution. Such a situation will occur at the process of quantal measurement, if there would be some additional (up to now, perhaps, unknown) long range interaction between the macrosystems and microsystems.[7,11]

2. Models of Quantum Measurement

In the first two models of this section the "measured microsystem" is a $\frac{1}{2}$-spin with $n = 0$ lying at the very beginning of a linear chain consisting of such spins. In the third model a particle is detected by such a chain.

2.1. An Unstable Infinite Spin Chain

Let $a_n^{(*)}$ $(n \in \mathbf{Z}_+)$ be the spin creation - annihilation operators, and let

$$H := H_\infty := \sum_{n \in \mathbf{Z}_+} a_n^* a_n (a_{n+1} + a_{n+1}^*) a_{n+2} a_{n+2}^* \tag{2.1}$$

be formal expression for the sequence of local Hamiltonians defining time evolution as a one-parameter group τ_t $(t \in \mathbf{R})$ of automorphisms of the quasilocal C^*-algebra $\mathcal{A}$ generated by the set $\{a_n, a_n^* : n = 0, 1, 2, \ldots\}$. Let the initial state ω_0 (i.e. the expectation functional) of the whole infinite chain (including the site $n = 0$ representing the "measured system") be given by

$$\omega_0(a_n^* a_n) := 0 \quad \text{for } n > 0, \quad \omega_0(a_0^* a_0) := 1. \tag{2.2}$$

An explicit solution of this model[5] gives for any $n \geq 0$ (the state-vector Ω corresponds to the state with all the spins of the compound system pointing down):

$$\omega_t(a_n^* a_n) := \omega_0(\tau_t(a_n^* a_n)) \tag{2.3a}$$

$$= 1 - \sum_{m=0}^{n-1} \langle a_0^* \Omega | e^{itH} \textstyle\prod_{j=0}^m a_j^* \Omega \rangle^2 \tag{2.3b}$$

$$= 1 - \sum_{m=1}^{n} (-1)^{m+1} \frac{m^2}{t^2} \left[J_m(2t)\right]^2 \tag{2.3c}$$

$$= 1 - O(t^{-3}), \qquad for \quad t \to \infty. \tag{2.3d}$$

Here J_m are the Bessel functions. Hence, the (weak-*) limit of ω_t is the state ω_1 determined by the relation:

$$\omega_1(a_n^* a_n) = 1 \qquad \text{for all } n \geq 0. \tag{2.4}$$

This shows that if the stationary state of the chain with all spins pointing down is perturbed by turning up just the first spin (i.e. the measured microsystem) then the whole chain would evolve into a new stationary state with all spins pointing up. But these two states, the initial and final ones, are mutually disjoint. So, the state described by a (coherent) linear combination of states differing initially just in the state of the first spin of the chain (pointing down, or up) evolves as $t \to \infty$ into a mixture of two macroscopically distinguishable states.

2.2. Radiating Finite-Size Apparatus

Let us take now the finite initial segment of the previously considered chain of spins of the lenght N with Hamiltonian

$$H_N := \sum_{n=1}^{N-2} a_n^* a_n (a_{n+1} + a_{n+1}^*) a_{n+2} a_{n+2}^* \tag{2.5}$$

(We shal use previously introduced notation, except of redefinition of the total system Hamiltonian H.) Let this spin chain interacts with a scalar Fermi field described by linear ascription of creation operators $b^*(\varphi)$ to one particle wave functions $\varphi \in L^2(\mathbf{R}, dx)$. The annihilation operators are $b(\varphi) = (b^*(\varphi))^*$. Let $d\Gamma(\hat{p}^2)$ be the second quantized[4] one particle Hamiltonian $\hat{p}^2$, with $\hat{p} := -i\partial_x$. The interaction is defined by

$$V := \kappa a_{N-1}^* a_{N-1} (a_N^* b^*(\sigma) + a_N b(\sigma)), \quad \kappa > 0,\ \sigma \in L^2(\mathbf{R}, dx),\ \|\sigma\| = 1. \tag{2.6}$$

Let the total Hamiltonian of our system in the common Fock representation (i.e. the Fermi field is in the finite particle number sector) be

$$H := H_N - \bar{\varepsilon} a_N^* a_N + d\Gamma(\hat{p}^2) + V. \tag{2.7}$$

One can show that the common vacuum (i.e. no particles, and all spins "down") is stationary. Let us initially turn up just the first spin from this common vacuum. Then one obtains an "almost exponential decay" into the stationary state where all spins are pointing up, and the Fermi field is again in its vacuum.[12] For the chosen initial state ω_0 the dynamics given by Eq. (2.7) leads, e.g., to

$$\omega_t(a_N^* a_N) = 1 - \mathrm{o}(t^{-m}) \qquad \text{for } t \to \infty, \text{ for all } m \in \mathbf{Z}_+. \tag{2.8}$$

2.3 Unsharp Measurement

In this subsection we introduce a model for nonideal measurement. The measured quantity could be defined from the interaction of the apparatus (it is again the infinite spin chain from Sec. 2.1) with the measured "microsystem" (this is now a scalar nonrelativistic particle interacting with *first two* spins in the chain). The microsystem (particle) is described by the Fermi field from Sec. 2.2. The total Hamiltonian is now

$$H := H_\infty + d\Gamma(\hat{p}^2) + W, \tag{2.9}$$

where the interaction W is defined by

$$W := \gamma(a_1^* + a_1) a_2 a_2^* \otimes b^*(\sigma) b(\sigma), \tag{2.10}$$

with $\gamma \in \mathbf{R}$ and $\sigma \in L^2(\mathbf{R}, dx)$, $\|\sigma\| = 1$. Let the chain be again at $t = 0$ in its "vacuum". Let the initial state of the particle be described by an arbitrary normalized wave function $\varphi_0 \in L^2(\mathbf{R}, dx)$. (The "incoming" asymptotics for $t \to -\infty$ of the scattering theory is not used here.) Denoting by ω_t the state of the total system in time t, and by A an arbitrary observable of that composed system, one can obtain[13] that

$$\bar{\omega}(A) := \lim_{t\to\infty} \omega_t(A) \tag{2.11}$$

exists, and one arrives for suitable parameters of W at:

$$\bar{\omega} = (w\omega_1^{chain} + (1-w)\omega_0^{chain}) \otimes \omega_0^{field}, \tag{2.12}$$

where ω_j^{chain} $(j = 0, 1)$ denotes the state of the chain with all spins pointing down, resp. pointing up (the spin with $n = 0$ is excluded now). The field state ω_0^{field} is the Fermi vacuum. We again obtained a mixture of two disjoint states.

The probability w of switching the apparatus can be expressed explicitly in terms of dynamics of the chain, as well as of the particle-chain interaction parameters.[13] It can be written in the form

$$w = \langle\varphi_0|\Lambda\varphi_0\rangle. \tag{2.13}$$

Here Λ is a positive operator of the norm $\|\Lambda\| \leq 1$ acting on the Hilbert space $L^2(\mathbf{R}, dx)$. This operator represents the measured (unsharp) observable of the particle. In engineer's terms, Λ can represent also a "characteristic" of apparatus.

3. Nonlinear Quantum Mechanics

Let us now consider QM of a finite (closed, or in external field) system. The sets of its mutually equivalent vector states are in bijective correspondence with points of the projective Hilbert space $P(\mathcal{H})$ elements of which are one-dimensional projections in $\mathcal{H}$ (e.g., $\mathcal{H} := L^2(\mathbf{R}, dx)$). Similarly, each "mixed state" in QM is represented by a density operator ϱ. As a quantal analogy with phase space of Hamiltonian CM, we shall consider the set $\mathcal{S}_*$ of all density matrices. (We do not use the larger set of states $\mathcal{S}$ containing, e.g., states with sharp values of continuous quantities.) We shall define Poisson brackets on $\mathcal{S}_*$. Let for all "sufficiently nice" real-valued functions $f,\ h, \dots$ the symbols $d_\varrho f,\ d_\nu h \dots$ denote the (Frèchet) differentials at the points $\varrho,\ \nu, \dots$ of $\mathcal{S}_*$. We can consider these differentials as unambigously defined bouned operators in $\mathcal{L}(\mathcal{H})$. Then we define the Poisson bracket of f and h by[11]

$$\{f, h\}(\nu) := i\,Tr(\nu[d_\nu f, d_\nu h]), \tag{3.1}$$

where $[\cdot,\cdot]$ means the commutator of bounded operators in $\mathcal{L}(\mathcal{H})$, and Tr is the trace in the space $\mathcal{T}(\mathcal{H})$, $\mathcal{L}(\mathcal{H}) \supset \mathcal{T}(\mathcal{H}) \supset \mathcal{S}_*$, of trace class operators.

Time evolution given by the Hamiltonian function h is "essentially" determined now in the usual way (except of some complications with infinite dimensionality of $\mathcal{H}$, and occurence of unbounded operators)[11] from Eq. (3.1). It can be shown[11] that the evolution of density matrices can be described by the 'cocycle' of unitary operators $u_h(t,\nu)$ (with $t \in \mathbf{R},\ \nu \in \mathcal{S}_*$) satisfying Schrödinger-like (in general nonlinear) equation with the 'Hamilton operator' $d_\varrho h$:

$$i\,\frac{d}{dt}u_h(t,\nu) = d_{\nu(t)}h\ u_h(t,\nu). \tag{3.2}$$

Here $\nu(t) := u_h(t,\nu)\,\nu\,u_h^*(t,\nu)$, if $\nu(0) := \nu$; this determines time evolution of any $\nu \in \mathcal{S}_*$ under the Hamiltonian dynamics corresponding to Poisson brackets (3.1). If $h(\nu) \equiv Tr(\nu H)$ for some ν-independent operator H, then the Eq. (3.2) converts into the usual Schrödinger equation with the Hamiltonian H; the dynamics is linear in the usual sense exactly in this last mentioned case.

Such a nonlinear quantum mechanics of a finite system arises naturally[7] from an automorphism group of an algebra of infinite collection of the same systems with usual kinematics in $\mathcal{L}(\mathcal{H})$, interacting mutually via a long range very weak (*linear*) interactions introduced in Ref. 6; i.e., it is a *reduced dynamics*. Its 'Hamilton operator' expresses the influence of the classical "mean-field background".[7,8]

Let us note that this theory contains much more observables and states than linear QM. Probability measures on $\mathcal{S}_*$ could describe "Gemenge". A consistent statistical interpretation of this theory (different from the Weinberg's one)[14] is also proposed.[11]

4. References

1. P. Busch, P. J. Lahti and P. Mittelstaedt, *The Quantum Theory of Measurement*, (Springer, Berlin, 1992).
2. R. Haag and D. Kastler, *J. Math. Phys.* **5** (1964) 848.
3. K. Hepp, *Helv. Phys. Acta* **45** (1972) 237.
4. O. Bratteli and D. W. Robinson, *Operator Algebras and Quantum Statistical Mechanics* (Springer, New York - Berlin, 1979 and 1981), Vols. I and II.
5. P. Bóna, *acta phys. slov.* **27** (1977) 101.
6. K. Hepp and E. H. Lieb, *Helv. Phys. Acta* **46** (1973) 573.
7. P. Bóna, *J. Math. Phys.* **29** (1988) 2223.
8. P. Bóna, *Czech. J. Phys.* **B37** (1987) 482.
9. D. M. Greenberger, M. A. Horne and A. Zeilinger, *Physics Today* **46** (August 1993) 22.
10. P. Bóna, *J. Math. Phys.* **30** (1989) 2994.
11. P. Bóna, *Quantum Mechanics With Mean-Field Background* (Comenius University, Bratislava 1991), preprint Ph10-91.
12. P. Bóna, *Unpublished Notes* (Bratislava 1993)
13. P. Bóna, *ACTA F.R.N. Univ.Comen. PHYSICA,* **XX** (1980) 65.
14. S. Weinberg, *Ann. Phys.* **194** (1989) 336.
15. W. H. Zurek, *Phys. Rev.* **D 26** (1982) 1862.