Superconformal theories from Pseudoparticle Mechanics

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Abstract

We consider a one-dimensional $\mathrm{Osp}(N|2M)$ pseudoparticle mechanical model which may be written as a phase space gauge theory. We show how the pseudoparticle model naturally encodes and explains the two-dimensional zero curvature approach to finding extended conformal symmetries. We describe a procedure of partial gauge fixing of these theories which leads generally to theories with superconformally extended $\mathcal{W}$-algebras. The pseudoparticle model allows one to derive the finite transformations of the gauge and matter fields occurring in these theories with extended conformal symmetries. In particular, the partial gauge fixing of the $\mathrm{Osp}(N|2)$ pseudoparticle mechanical models results in theories with the $\mathrm{SO}(N)$ invariant N-extended superconformal symmetry algebra of Bershadsky and Knizhnik. These algebras are nonlinear for $N \geq 3$. We discuss in detail the cases of $N = 1$ and $N = 2$, giving two new derivations of the superschwarzian derivatives. Some comments are made in the $N = 2$ case on how twisted and topological theories represent a significant deformation of the original particle model. The particle model also allows one to interpret superconformal transformations as deformations of flags in super jet bundles over the associated super Riemann surface.

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1 Introduction

Extended conformal symmetries play a central role in many two-dimensional systems including string theories, two-dimensional gravity theories, statistical mechanical systems at phase transition points, as well as in integrable hierarchies of nonlinear differential equations.

There has been a great deal of research on the systematic construction and understanding of bosonic and/or fermionic extensions of the two-dimensional conformal symmetry algebra, or Virasoro algebra. As far back as 1976, Ademollo, et.al. [1], generalized the two-dimensional $N = 1$ superconformal algebra to N-extended $\mathrm{O}(N)$ superconformal algebras, for any N. These algebras are graded Lie algebras with a spin 2 stress tensor, N spin $\frac{3}{2}$ supersymmetry currents, $N(N-1)/2$ spin 1 $\mathrm{O}(N)$ Kac-Moody currents, and $2^N - 1 - N(N+1)/2$ additional generators.

An explosion of progress occurred after the paper of Belavin, Polyakov and Zamolodchikov [2] in which a method was developed for studying conformal symmetry through operator product expansions (OPE's) and conformal Ward identities. The semi-direct product of Virasoro with a Kac-Moody algebra of spin 1 currents was investigated in [3], and more generally the systematic extension of Virasoro by bosonic currents was initiated by Zamolodchikov and by Zamoldchikov and Fateev [4]. This program gives rise to the $\mathcal{W}$-algebras. For recent reviews on $\mathcal{W}$-algebras, see [5, 6].

Supersymmetric extensions were considered by Knizhnik [7] and by Bershadsky [8] from the OPE point of view, and the resulting algebras have important differences from those of Ademollo. In particular, Bershadsky and Knizhnik found $\mathrm{SO}(N)$ and $\mathrm{U}(N)$ invariant N-extended superconformal algebras containing only spin $\frac{3}{2}$ and an $\mathrm{SO}(N)$ or $\mathrm{U}(N)$ of spin 1 currents in addition to the spin 2 energy-momentum tensor. These algebras are not graded Lie algebras for $N \geq 3$, since the OPE of two spin $\frac{3}{2}$ supercurrents yields a term bilinear in the Kac Moody currents. In the case of the $\mathrm{SO}(N)$ invariant algebras, one has a spin 2 energy-momentum tensor, N spin $\frac{3}{2}$ fermionic stress tensors and $N(N-1)/2$ spin 1 currents forming a $\mathrm{SO}(N)$ current algebra.

A common feature of the $\mathcal{W}$-algebras and the $N \geq 3$ Bershadsky-Knizhnik $\mathrm{SO}(N)$ and $\mathrm{U}(N)$ superconformal algebras is that their OPE's are nonlinear, and therefore Lie algebraic techniques are not directly applicable. Two methods have enjoyed much success. Quantum hamiltonian reduction, the quantum version of the Drinfel'd-Sokolov reduction for coadjoint orbits, furthered understanding of extended conformal algebras by showing that they may be obtained from constrained Kac Moody current algebras. The Bershadsky-Knizhnik algebras may be obtained from reduction of $\mathrm{Osp}(N|2)$ current algebras [1]. Another method is the zero curvature approach [10, 13, 11] which gives a prescription for determining the infinitesimal transformations of gauge fields of extended conformal algebras from two-dimensional gauge theories, provided one identifies a spacetime derivative as the gauge variation operation. While these approaches have provided insight into extended conformal algebras, a complete understanding of the geometry associated with these symmetries is still lacking. A major shortcoming of quantum hamiltonian reduction and the zero curvature method is that they give only OPE's of the reduced algebras, or equivalently determine only the infinitesimal transformations of the symmetries. On the other hand, progress has been made in understanding $\mathcal{W}$-geometry in the work of Gerasimov, Levin, Marshakov [12], and of Bilal, et. al. [13], where it is shown that $\mathcal{W}$ transformations may be regarded as deformations of flags in jet bundles over the two-dimensional Riemann surfaces. Other approaches to $\mathcal{W}$-geometry have been considered by [14, 15, 16, 17, 18, 19, 20].

In this paper, we present one-dimensional pseudoparticle mechanical models which may be written as $\mathrm{Osp}(N|2M)$ phase space gauge theories. The partial gauge fixing of these theories results in theories with superconformal $\mathcal{W}$-algebras, and the resulting theories may be considered as chiral sectors of two-dimensional super $\mathcal{W}$-gravity theories. The particle model formulated as a gauge theory in phase space sheds light on the two-dimensional zero curvature prescription. Of particular interest are the $\mathrm{Osp}(N|2)$ pseudoparticle mechanical models whose partial gauge fixing results in the $\mathrm{SO}(N)$-invariant N-extended superconformal theories of Bershadsky and Knizhnik. Upon partial gauge fixing, one obtains chiral sectors of two-dimensional theories of conformal matter coupled to supergravity. This reduction is fundamentally different than the hamiltonian reduction of Wess-Zumino-Novikov-Witten (WZNW) models, as the particle model contains both gauge and matter fields which possess only canonical Dirac brackets, as opposed to the WZNW model which possesses only gauge currents with the Lie-Poisson Kac-Moody brackets. A method for obtaining the finite transformations for $\mathrm{SO}(N)$ extended superconformal algebras, valid also for $\mathcal{W}$-extensions of the these algebras, is presented. The nonsupersymmetric case is studied in [21]. In brief, the method is as follows. First one makes gauge field dependent redefinitions of the gauge parameters to put

[1] For a review and list of references, see [9].

the infinitesimal $\mathrm{Osp}(N|2)$ transformations of the matter and gauge fields into "standard" form, i.e. the form in which one may immediately recognize diffeomorphisms and supersymmetries. This step is of practical necessity since otherwise it would be difficult to recognize, for example, ordinary diffeomorphisms in the algebra after partial gauge fixing. At this stage the choice of partial gauge fixing condition becomes clear. Next one integrates the infinitesimal $\mathrm{Osp}(N|2)$ transformations and transforms the matter and gauge fields by successive finite transformations. Finally, one imposes the partial gauge fixing at the level of the finite transformations. This yields the finite transformations for the matter fields and non-gauge-fixed gauge fields. This method is illustrated by explicit calculations for the (albeit linear) cases of $N = 1$ and $N = 2$. The pseudoparticle model also facilitates the interpretation of superconformal transformations as deformations of flags in the super jet bundles over the associated super Riemann surfaces.

This paper is organized as follows. In section 2, we illustrate how an $\mathrm{Osp}(N|2M)$ pseudoparticle mechanical model can be formulated as a phase space gauge theory, and explain the connection with the zero curvature prescription. We comment on the partial gauge fixing procedure from the point of view of orbits of the group, and contrast the reduction of the pseudoparticle model with the hamiltonian reduction of a WZNW model. In section 3, we restrict our considerations to the $\mathrm{Osp}(N|2)$ model. We discuss the general method for obtaining the finite transformations for $\mathrm{SO}(N)$-invariant N-extended superconformal algebras. We discuss the closure of the infinitesimal gauge algebra and illustrate the partial gauge fixing of the model at the infinitesimal level explicitly for $N = 1$ and $N = 2$. In section 4, we carry out the gauge fixing at the level of finite transformations for $N = 1$. Finite transformations are given for matter and gauge fields, thus providing a derivation of the $N = 1$ superschwarzian. In section 5, we present the cases of $N = 1$ and $N = 2$ in superfield form. Here we give an alternate derivation of the superschwarzians by writing the matter equation of motions in superfield form and then demanding the covariance of the equation of motion under superconformal transformations. In section 6, we discuss the completely gauge fixed pseudoparticle model, which is invariant under super Möbius transformations. In section 7, we return to the infinitesimal $N = 2$ and discuss how twisted and topological theories represent a significant deformation of the original particle model. In section 8, we show how superconformal transformations may be understood as deformations of flags in the N-supersymmetrized 1-jet bundles over the super Riemann surfaces. We construct the flags in the super 1-jet bundle explicitly for $N = 1$ and for $N = 2$, and conjecture the result for general N. Section 9 contains the conclusions and some directions for further investigation.

2 Osp($N|2M$) Pseudoparticle Mechanical Models as Phase Space Gauge Theories

Consider a one-dimensional hamiltonian system with M bosonic and N fermionic dynamical coordinates

$$\begin{aligned} &(x_i(t)^\mu, p_i(t))_\mu) && i = 1, \ldots M \\ &(\psi_\alpha(t)^\mu, \pi_{\psi_\alpha}(t)_\mu) && \alpha = 1, \ldots N \end{aligned} \tag{2.1}$$

and einbein-like coordinates

$$(\lambda_A(t), \pi_{\lambda_A}(t)) \tag{2.2}$$

which will act as Lagrange multipliers to implement a set of constraints $T_A(x, p, \psi)$ on the dynamical phase space coordinates. The index μ runs over the d spacetime dimensions, where d is sufficiently large so the constraints do not collapse the theory. The metric $g_{\mu\nu}$ should be taken not as euclidean, but as diagonal 1's and -1's.

The canonical action is (summation convention assumed and spacetime indices suppressed)

$$S = \int dt \left[\dot{x}_i \cdot p_i + \frac{1}{2} \dot{\psi}_\alpha \cdot \psi_\alpha - \lambda_A \, T_A(x, p, \psi) \right]. \tag{2.3}$$

where the fermion momenta π_{ψ_α} have been eliminated by the second class constraints $\pi_{\psi_\alpha} - \frac{1}{2}\psi_\alpha = 0$. The fundamental Dirac-Poisson brackets of this model are

$$\{x_i^\mu, p_{\nu\, j}\}^* = g^\mu{}_\nu \delta_{ij} \qquad \{\psi_\alpha^\mu, \psi_\beta^\nu\}^* = -\delta_{\alpha\beta}\, g^{\mu\nu}. \tag{2.4}$$

This system has the primary constraints $\pi_{\lambda_A} \approx 0$ whose stability implies the secondary constraints $T^A(x, p, \psi) \approx 0$. In turn, the stability of the constraints $T_A \approx 0$ implies that (in suggestive notation)

$$\{T_i, T_j\} = f_{ij}{}^k T_k \approx 0$$

where the $f_{ij}{}^k$ could in general depend on the coordinates (x, p, ψ).

Now let us examine a system where the constraints are all quadratic combinations of the coordinates x_i, p_i, and ψ_α

$$\begin{array}{llll} T_{1ij} = \frac{1}{2}p_i p_j & T_{2ij} = -\frac{1}{2}x_i x_j & T_{3ij} = \frac{1}{2}p_i x_j & (i \leq j) \\ T_{4i\alpha} = \frac{1}{2}p_i \psi_\alpha & T_{5i\alpha} = \frac{1}{2}x_i \psi_\alpha & T_{6\alpha\beta} = \frac{1}{4}\psi_\alpha \psi_\beta & (\alpha < \beta). \end{array} \tag{2.5}$$

Using the fundamental Dirac-Poisson brackets of this theory one can check that the Poisson algebra formed by the constraints T_A is isomorphic to the Lie algebra $\mathrm{osp}(N|2M)$.

The action of this theory is

$$\begin{aligned} S &= \int dt \left[\dot{x}_i \cdot p_i + \frac{1}{2}\dot{\psi}_\alpha \cdot \psi_\alpha - (\lambda_1)_{ij}\frac{p_i p_j}{2} + (\lambda_2)_{ij}\frac{x_i x_j}{2} \right. \\ &\quad \left. -(\lambda_3)_{ij}\frac{x_i p_j}{2} - (\lambda_4)_{i\alpha}\frac{p_i \psi_\alpha}{2} - (\lambda_5)_{i\alpha}\frac{x_i \psi_\alpha}{2} - (\lambda_6)_{\alpha\beta}\frac{\psi_\alpha \psi_\beta}{4} \right]. \end{aligned} \tag{2.6}$$

Related particle models have been investigated by [22, 23, 24, 25, 26].

Following the insights of Kamimura [27] on reparametrization invariant theories, this model may be understood better by using matrix notation. In particular, we can put the action into standard Yang Mills form, with gauge group $\mathrm{Osp}(N|2M)$. Let us write the first order formalism coordinates in an $N + 2M$ vector Φ. It will be convenient to write also the bosonic and fermionic coordinates separately in $2M$ and N-dimensional vectors

$$\Phi = \begin{pmatrix} \phi \\ \psi \end{pmatrix}, \qquad \phi = \begin{pmatrix} x_1 \\ \vdots \\ x_M \\ p_1 \\ \vdots_M \\ p_M \end{pmatrix}, \qquad \psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix}. \tag{2.7}$$

The orthosymplectic group $\mathrm{Osp}(N|2M)$ consists of those elements leaving fixed the quadratic form

$$\xi^\top \eta \xi \qquad \text{where } \eta = \begin{pmatrix} J_{2M} & 0 \\ 0 & 1_N \end{pmatrix}$$

where J_{2M} is the $2M \times 2M$ symplectic matrix $J_{2M} = \begin{pmatrix} 0 & 1_M \\ -1_M & 0 \end{pmatrix}$ and 1_N is the $N \times N$ unit matrix. The conjugate to Φ is therefore given by

$$\bar{\Phi} = \Phi^\top \eta = \left(\phi^\top, \psi^\top\right)\eta = (-p_1, \ldots, -p_M, x_1, \ldots, x_M, \psi_1, \ldots, \psi_N).$$

The Lagrange multiplier gauge fields may be assembled into the form of a most general osp$(N|2M)$ matrix [2]

$$\Lambda = \begin{pmatrix} A & \Omega \\ \Omega^\top J_{2M} & B \end{pmatrix} \tag{2.8}$$

where A is a $2M \times 2M$ symplectic matrix, Ω is a $N \times 2M$ matrix of fermionic entries, and B is an $N \times N$ antisymmetric matrix. The matrices A, Ω and B may be written explicitly in terms of the gauge fields. The matrix A is an sp$(2M)$ matrix

$$A = \begin{pmatrix} \frac{1}{2}\lambda_3 & \lambda_1 \\ \lambda_2 & -\frac{1}{2}\lambda_3^\top \end{pmatrix} \tag{2.9}$$

where the components of the $M \times M$ matrices $(\lambda_1)_{ij}, (\lambda_2)_{ij}, (\lambda_3)_{ij}$ are the Lagrange multiplier fields appearing in the action above. The fermionic gauge fields are arranged as

$$\Omega = \frac{1}{2} \begin{pmatrix} \lambda_4 \\ -\lambda_5 \end{pmatrix}, \tag{2.10}$$

where the $M \times N$ matrices $(\lambda_4)_{i\alpha}$ and $(\lambda_5)_{i\alpha}$ appear in the action. Finally, B is the $N \times N$ antisymmetric matrix of Lagrange multipliers

$$B_{\alpha\beta} = -\frac{1}{2}(\lambda_6)_{\alpha\beta}, \tag{2.11}$$

which implement the O(N) rotations among the Grassmann variables.

Using this matrix notation, the canonical action of the constrained pseudoparticle model may be written simply as a phase space gauge theory

$$S = -\frac{1}{2}\int dt\, \bar{\Phi}\mathcal{D}\Phi, \tag{2.12}$$

where $\mathcal{D}$ is the covariant derivative

$$\mathcal{D} = \frac{d}{dt} - \Lambda.$$

[2]Recall the transpose of a supermatrix with bosonic blocks b_1 and b_2 and fermionic blocks f_1 and f_2 is given by

$$\begin{pmatrix} b_1 & f_1 \\ f_2 & b_2 \end{pmatrix}^\top = \begin{pmatrix} b_1^\top & f_2^\top \\ -f_1^\top & b_2^\top \end{pmatrix}.$$

The constrained pseudoparticle action is just an $\mathrm{Osp}(N|2M)$ gauge theory with no kinetic term for the Yang Mills gauge fields.

We are now in the position to make the connection with the zero curvature method. In this prescription, one considers a two dimensional theory with gauge fields $A_z = A_z(z, \bar{z})$, $A_{\bar{z}} = A_{\bar{z}}(z, \bar{z})$. Making a gauge fixing ansätz for A_z and keeping the form of $A_{\bar{z}}$ general, one solves the zero curvature equation, i.e.

$$F_{z\bar{z}} = [\partial - A_z, \bar{\partial} - A_{\bar{z}}] \equiv 0$$

to eliminate some elements of $A_{\bar{z}}$ and to determine the $\bar{\partial}$ derivatives of the non-gauge-fixed components of A_z. If one makes the identification

$$\bar{\partial} \leftrightarrow \delta$$

then, given a suitable ansätz for A_z, the resulting equations for the nonconstrained elements give their infinitesimal transformations under the residual extended conformal symmetry.

Let us now return to the pseudoparticle phase space gauge theory. Under infinitesimal $\mathrm{osp}(N|2M)$ gauge transformations, the matter and gauge fields transform in the usual way

$$\begin{aligned} \delta_\epsilon \Phi &= \epsilon \Phi \\ \delta_\epsilon \Lambda &= \dot{\epsilon} - [\Lambda, \epsilon] , \end{aligned} \tag{2.13}$$

where ϵ is an $\mathrm{osp}(N|2M)$ matrix of gauge parameters. The equation of motion for the matter fields is

$$\mathcal{D}\Phi = \dot{\Phi} - \Lambda\Phi = 0. \tag{2.14}$$

The compatibility condition of the two linear equations for the matter fields is nothing but the gauge transformation equation for the gauge fields. Explicitly, the compatibility equation is

$$0 = \left[\delta_\epsilon - \epsilon, \frac{d}{dt} - \Lambda\right] \Phi = \underbrace{(-\dot{\epsilon} + \delta_\epsilon \Lambda + [\Lambda, \epsilon])}_{\text{gauge field transformation}} \Phi. \tag{2.15}$$

This relation continues to hold when a partial gauge fixing condition is imposed. Thus, the one dimensional particle model allows one to make sense of the the two-dimensional zero curvature approach. The infinitesimal gauge transformation equation and equation of motion for the matter in the model provide the two linear operators $\partial - A_z$ and $\bar{\partial} - A_{\bar{z}}$ appearing in the zero curvature approach, while the gauge field transformation equation is equivalent to $F_{z\bar{z}} = 0$. ¿From the particle model point of view there is no need to make the curious identification $\bar{\partial} \leftrightarrow \delta$. ¿From the point of view of the particle model, the zero curvature condition translates into a statement of the gauge symmetry and the identification $\bar{\partial} \leftrightarrow \delta$ is not required.

Before closing this section, we comment on the partial gauge fixing of this Yang Mills type action. The finite gauge transformations are

$$\begin{aligned} \Phi' &= g\Phi \\ \Lambda' &= g^{-1}\Lambda g - g^{-1}\dot{g} = Ad_g^*\Lambda \end{aligned} \tag{2.16}$$

where $g \in \mathrm{Osp}(N|2M)$. The gauge transformed fields Λ' sweep out the coadjoint orbit of the point Λ. Let Λ_c be the gauge field matrix with some entries constrained to be ones or zeros. If we demand that the gauge transformed matrices Λ_c' leave these constraints intact, then the reduced orbit will describe finite transformations of the gauge fields under the residual gauge symmetry. In general the relations among the parameters will be gauge field dependent and the symmetry will be a quasigroup [28]. With a suitable choice of partial gauge fixing, the resulting equations for the unconstrained elements of Λ_c give finite transformations for the gauge fields under extended conformal algebras.

Before discussing a technical obstruction to obtaining the finite transformations directly from the group orbit, we contrast the reduction of the pseudoparticle model with the corresponding hamiltonian reduction of a WZNW model. In the latter case, the original phase space that one reduces is that of a chiral sector of WZNW currents. A point in the phase space is specified by $(J(z), k)$ where $J(z) = J^a(z)T_a$ is a mapping from the circle to the algebra (which is $\mathrm{osp}(N|\,2)$ for the Bershadsky-Knizhnik superconformal algebras) and k is a number. The $J^a(z)$ have Poisson brackets which are isomorphic to the affine Lie algebra. The coadjoint action on a phase space point is $Ad_g^*\,(J(z), k) = \left(g^{-1}J(z)g + kg^{-1}\frac{d}{dz}, k\right)$. In hamiltonian reduction, or the Drinfel'd-Sokolov reduction for coadjoint orbits, one constrains some of the $J^a(z)$'s (generally fewer entries are constrained than in the relevant pseudoparticle model reduction), and then mods out by the coadjoint action of the subgroup which preserves these constraints. The reduced orbits are then parametrized by new phase space coordinates which are polynomials of the original phase space coordinates and their derivatives. In contrast to the pseudoparticle model, the orbit equation is used to determine the new coordinates instead of the transformations. The original Poisson brackets are then used to obtain the Poisson brackets of the extended conformal symmetry. Thus, the reduction of the pseudoparticle model is markedly different from the hamiltonian reduction of WZNW in two major ways. The former includes both matter and Lagrange multiplier gauge fields while the latter has gauge fields only with affine Lie-Poisson brackets. The former employs the one-dimensional gauge theory coadjoint orbit to determine the finite transformations, while the latter uses the affine coadjoint action to determine the new phase space coordinates.

Returning to the pseudoparticle model, consider the residual gauge transformations arising from the gauge orbit equation 2.17 by directly plugging in constraints on Λ. Unfortunately, the transformations obtained in this manner will not be directly recognizable as corresponding to extended conformal algebras because the gauge parameters would not correspond in a simple way to "standard" transformations such as diffeomorphisms or supersymmetries for example. To put expressions in a recognizable form would require gauge field dependent parameter redefinitions which from a practical point of view is unfeasible. Instead we will take a different approach, whose first step involves making field redefinitions first at the infinitesimal level to get "standard" transformations. For arbitrary N and $M = 1$ there is no appreciable difficulty in finding the standard form of the transformations, whereas for $M \geq 2$ where the $\mathcal{W}$-algebras appear, the standard transformations are not so well established. The nonsupersymmetric case has been investigated in [21]. In the remainder of this paper, we will restrict to $M = 1$ to focus on the N-extended superconformal algebras, which are nonlinear for $N \geq 3$.

3 Superconformal theories from osp($N|2$) pseudoparticle model

We now restrict to pseudoparticle mechanical models which have as dynamical coordinates a single bosonic coordinate $x^\mu(t)$ and its momentum $p_\mu(t)$ and N fermionic coordinates $\psi^\mu_\alpha(t)$ with $\alpha = 1, \ldots N$. The constraints

$$\begin{array}{llll} T_1 = \frac{1}{2}p^2 & T_2 = -\frac{1}{2}x^2 & T_3 = \frac{1}{2}px & \\ T_{4\alpha} = \frac{1}{2}p\psi_\alpha & T_{5\alpha} = \frac{1}{2}x\psi_\alpha & T_{6\alpha\beta} = \frac{1}{4}\psi_\alpha\psi_\beta & (\alpha < \beta) \end{array} \tag{3.1}$$

form an osp($N|$ 2) algebra, with T_1, T_2 and T_3 forming an sl(2,R) subalgebra.

We pass to the lagrangian form, integrating out the momentum

$$
\begin{aligned}
S &= \int dt \left[\frac{1}{2\lambda_1}(\dot{x} - \frac{1}{2}\lambda_3 x - \frac{1}{2}\lambda_{4\alpha}\psi_\alpha)^2 + \frac{1}{2}\dot{\psi}_\alpha\psi_\alpha + \lambda_2\frac{x^2}{2}\right. \\
&\left. -\lambda_{5\alpha}\frac{x\psi_\alpha}{2} - \lambda_{6\alpha\beta}\frac{\psi_\alpha\psi_\beta}{4}\right]. \qquad (3.2)
\end{aligned}
$$

This action describes a relativistic spinning particle moving in a d-dimensional spacetime and interacting with gauge fields λ_i which subject the particle to an osp$(N|\ 2)$ algebra of constraints. This action has been considered by Mårtensson [25]. Denoting the pullback of the momentum by

$$
\kappa \equiv FL^* p = \frac{1}{\lambda_1}(\dot{x} - \frac{1}{2}\lambda_3 x - \frac{1}{2}\lambda_{4\alpha}\psi_\alpha) \qquad (3.3)
$$

the equations of motion are

$$
\begin{aligned}
S_x &= \lambda_2 x - \frac{1}{2}\lambda_3\kappa - \frac{1}{2}\lambda_{5\alpha}\psi_\alpha - \dot{\kappa} \\
S_{\psi_\alpha} &= -\dot{\psi}_\alpha + \frac{1}{2}\kappa\lambda_{4\alpha} + \frac{1}{2}x\lambda_{5\alpha} - \frac{1}{2}\lambda_{6\alpha\beta}\psi_\beta. \qquad (3.4)
\end{aligned}
$$

Our method for obtaining N-extended superconformal theories from the pseudoparticle models is composed of the following steps:

1. Put Osp$(N|2)$ infinitesimal transformations in "standard" form by gauge field dependent redefinitions of gauge parameters. Determine the partial gauge fixing condition.

2. Integrate the linear osp$(N|\ 2)$ algebra to get the finite transformations, and transform the matter and gauge fields by successive finite transformations.

3. Perform partial gauge fixing at the finite level, thus obtaining finite transformations for N-extended superconformal transformations.

This is a very general prescription which will work for particle models with linear gauge symmetries. The partial gauge fixing of Osp$(N|2M)$ will in general result in superconformal $\mathcal{W}$-algebras. It is unwise to attempt to reverse the order of integration and partial gauge fixing since in general the residual infinitesimal gauge symmetries will be nonlinear and therefore difficult or impossible to integrate. Partial gauge fixing and integration do not in general commute, and therefore will not in general produce the same finite transformations for the residual symmetries of the gauge fixed model (the transformations will be related by gauge field dependent redefinitions of parameters). This is clear is we consider the orbit equation that we are solving. If we do a partial gauge fixing Λ_c at the infinitesimal level, then solving $0 = \delta\Lambda = ad^*_\epsilon \Lambda$ implies some gauge field dependent relations along the gauge parameters, giving a matrix ϵ_c. Integrating this gives $\Lambda'_c = Ad^*_{e^{\epsilon_c}}\Lambda_c$ for the finite transformations. On the other hand, partial gauge fixing at the finite level means that we solve $\Lambda'_c = Ad^*_g \Lambda_c$, which will give in general different gauge field dependent relations among the gauge parameters.

If we work in first order formalism, the gauge algebra on the phase space coordinates x, p, ψ_α and on the gauge fields λ_A closes, i.e. $[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}] = \delta_{\varepsilon_*}$. If we instead work in the Lagrangian formalism, then the transformation is the pullback of the former transformation, and generally one can expect the equations of motions of the coordinates to appear in the commutator of the gauge transformations. The transformations of the gauge fields are unaffected since p does not enter into the expressions. Explicitly, the gauge variations of the coordinates are

$$\delta_\varepsilon x = \varepsilon_1 \kappa + \frac{1}{2}\varepsilon_3 x + \frac{1}{2}\varepsilon_{4\alpha}\psi_\alpha \tag{3.5}$$

$$\delta_\varepsilon \psi_\alpha = \frac{1}{2}\varepsilon_{4\alpha}\kappa + \frac{1}{2}\varepsilon_{5\alpha} x - \frac{1}{2}\varepsilon_{6\alpha\beta}\psi_\beta. \tag{3.6}$$

The pullback of the gauge variation of the momentum and the gauge variation of the pullback of the momentum do not coincide

$$\begin{aligned} FL^*\delta_\varepsilon p &= \varepsilon_2 x - \frac{1}{2}\varepsilon_3\kappa - \frac{1}{2}\varepsilon_{5\alpha}\psi_\alpha \\ \delta_\varepsilon FL^* p &= \varepsilon_2 x - \frac{1}{2}\varepsilon_3\kappa - \frac{1}{2}\varepsilon_{5\alpha}\psi_\alpha - \varepsilon_1\frac{S_x}{\lambda_1} - \varepsilon_{4\alpha}\frac{S_{\psi_\alpha}}{2\lambda_1}. \end{aligned} \tag{3.7}$$

The noncoincidence of the momentum and its pullback is an indication that the gauge algebra in the Lagrangian formulation may not be closed. A calculation reveals

$$\begin{aligned} [\delta_\eta, \delta_\varepsilon]\, x &= \delta_{[\varepsilon,\eta]} x + (\varepsilon_1\eta_{4\alpha} - \eta_1\varepsilon_{4\alpha})\frac{-S_{\psi_\alpha}}{2\lambda_1} \\ [\delta_\eta, \delta_\varepsilon]\, \psi_\alpha &= \delta_{[\varepsilon,\eta]}\psi_\alpha + (\varepsilon_1\eta_{4\alpha} - \eta_1\varepsilon_{4\alpha})\frac{S_x}{2\lambda_1} + (\eta_{4\beta}\varepsilon_{4\alpha} + \eta_{4\alpha}\varepsilon_{4\beta})\frac{S_{\psi_\beta}}{4\lambda_1}. \end{aligned} \tag{3.8}$$

At this point, it is useful to change from the parameters occurring naturally in the gauge theory to parameters which represent "standard" transformations. In the following formulas, we now restrict further to the case of

$N \leq 2$. For $N = 2$, we write the single $\lambda_{6\alpha\beta}$ field as λ_6 and its associated parameter as ϵ_6. These are otherwise $(N = 0, 1)$ absent. The procedure for arbitrary N is carried out in exactly the same manner, with the inclusion of a total of $\frac{N(N-1)}{2}$ $\lambda_{6\alpha\beta}$ gauge fields and their associated gauge parameters. Let us make the following change of variables

$$\begin{aligned} \varepsilon_1 &= \lambda_1 \xi \\ \varepsilon_2 &= \lambda_2 \xi + h \\ \varepsilon_3 &= \lambda_3 \xi + \sigma \\ \varepsilon_{4\alpha} &= \lambda_{4\alpha} \xi + 2\omega_\alpha \\ \varepsilon_{5\alpha} &= \lambda_{5\alpha} \xi + \chi_\alpha \\ \varepsilon_6 &= \lambda_6 \xi + R, \end{aligned} \tag{3.9}$$

where the only real modification is in the reparametrization. By inspection it is clear that the transformations are as follows: reparametrization ξ, scale σ, shift transformation h, supersymmetries ω_α, further fermionic symmetries χ_α, and in the case of $N = 2$, we have O(2) rotations parametrized by R.

Upon examination of $\delta\psi_\alpha$, we see that to get standard reparametrization, we must add a term to the variation of ψ_α

$$\delta\psi_\alpha \to \delta\psi_\alpha - \xi S_{\psi_\alpha}.$$

The infinitesimal forms of the gauge transformations are

$$\begin{aligned} \delta x &= \xi \dot{x} + \frac{1}{2}\sigma x + \omega_\alpha \psi_\alpha \\ \delta\psi_\alpha &= \xi \dot{\psi}_\alpha + \omega_\alpha \kappa + \frac{1}{2}\chi_\alpha x - \frac{1}{2} R \epsilon_{\alpha\beta} \psi_\beta \\ \delta\lambda_1 &= \frac{d}{dt}(\xi\lambda_1) + \sigma\lambda_1 + \omega_\alpha \lambda_{4\alpha} \\ \delta\lambda_2 &= \frac{d}{dt}(\xi\lambda_2) + \dot{h} + h\lambda_3 - \sigma\lambda_2 - \frac{1}{2}\chi_\alpha \lambda_{5\alpha} \\ \delta\lambda_3 &= \frac{d}{dt}(\xi\lambda_3) + \dot{\sigma} - 2h\lambda_1 + \omega_\alpha \lambda_{5\alpha} + \frac{1}{2}\chi_\alpha \lambda_{4\alpha} \\ \delta\lambda_{4\alpha} &= \frac{d}{dt}(\xi\lambda_{4\alpha}) + 2\dot{\omega}_\alpha + \chi_\alpha \lambda_1 + \frac{1}{2}\sigma\lambda_{4\alpha} - \omega_\alpha \lambda_3 + \epsilon_{\alpha\beta}\omega\lambda_6 \\ &\quad -\frac{1}{2} R \epsilon_{\alpha\beta} \lambda_{4\beta} \\ \delta\lambda_{5\alpha} &= \frac{d}{dt}(\xi\lambda_{5\alpha}) - h\lambda_{4\alpha} + 2\omega_\alpha \lambda_2 - \frac{1}{2}\sigma\lambda_{5\alpha} + \frac{1}{2}\chi_\alpha \lambda_3 + \frac{1}{2}\epsilon_{\alpha\beta}\chi_\beta \lambda_6 \end{aligned}$$

$$
\begin{aligned}
& -\frac{1}{2}R\epsilon_{\alpha\beta}\lambda_{5\beta}+\dot{\chi}_\alpha \\
\delta\lambda_6 &= \xi\frac{d}{dt}(\xi\lambda_6)+\dot{R}+\epsilon_{\alpha\beta}\omega_\alpha\lambda_{5\beta}+\frac{1}{2}\epsilon_{\alpha\beta}\chi_\beta\lambda_{4\alpha} \qquad (3.10)
\end{aligned}
$$

The gauge algebra for x and for all the λ_i closes as $[\delta_A,\delta_B]=\delta_*$ with the starred parameters given below

$$
\begin{aligned}
\xi^* &= \xi^B\dot{\xi}^A-\xi^A\dot{\xi}^B-\frac{2}{\lambda_1}\omega^A_\alpha\omega^B_\alpha \\
\sigma^* &= \xi^B\dot{\sigma}^A-\xi^A\dot{\sigma}^B+\frac{2}{\lambda_1}\omega^A_\alpha\omega^B_\alpha\lambda_3+\chi^B_\alpha\omega^A_\alpha-\chi^A_\alpha\omega^B_\alpha \\
\omega^*_\alpha &= \xi^B\dot{\omega}^A_\alpha-\xi^A\dot{\omega}^B_\alpha+\frac{1}{\lambda_1}\omega^A_\beta\omega^B_\beta\lambda_{4\alpha}+\frac{1}{2}(\sigma^B\omega^A_\alpha-\sigma^A\omega^B_\alpha) \\
&\quad +\frac{1}{2}\epsilon_{\alpha\beta}(R^A\omega^B_\beta-R^B\omega^A_\beta) \\
h^* &= \xi^B\dot{h}^A-\xi^A\dot{h}^B+\frac{2}{\lambda_1}\omega^A_\alpha\omega^B_\alpha\lambda_2+h^B\sigma^A-h^A\sigma^A+\frac{1}{2}\chi^A_\alpha\chi^B_\alpha \\
\chi^*_\alpha &= \xi^B\dot{\chi}^A_\alpha-\xi^A\dot{\chi}^B_\alpha+\frac{2}{\lambda_1}\omega^A_\beta\omega^B_\beta\lambda_{5\alpha}-2(h^B\omega^A_\alpha-h^A\omega^B_\alpha) \\
&\quad -\frac{1}{2}(\sigma^B\chi^A_\alpha-\sigma^A\chi^B_\alpha)+\frac{1}{2}\epsilon_{\alpha\beta}(R^A\chi^B_\beta-R^B\chi^A_\beta) \\
R^* &= \xi^B\dot{R}^A-\xi^A\dot{R}^B+\frac{2}{\lambda_1}\omega^A_\alpha\omega^B_\alpha\lambda_6+\epsilon_{\alpha\beta}(\omega^B_\alpha\chi^A_\beta-\omega^A_\alpha\chi^B_\beta). \qquad (3.11)
\end{aligned}
$$

The supersymmetry part of the gauge algebra on ψ_α is still not closed for $N=2$ (and will not be for $N\geq 2$)

$$
\begin{aligned}
[\delta_A,\delta_B]\psi_\alpha &= \xi^*\dot{\psi}_\alpha+\omega^*_\alpha\kappa+\frac{1}{2}\chi^*_\alpha x+\frac{1}{2}R^*\epsilon_{\alpha\beta}\psi_\beta-\frac{2}{\lambda_1}\omega^A_\beta\omega^B_\beta S_{\psi_\alpha} \\
&\quad +\frac{1}{\lambda_1}(\omega^A_\beta\omega^B_\alpha-\omega^B_\beta\omega^A_\alpha)S_{\psi_\beta} \qquad (3.12)
\end{aligned}
$$

In order to close the algebra for $N=2$ and still maintain the the standard form of reparametrization for ψ_α, one must introduce an auxiliary field F in the following way. The transformation for ψ_α is further modified as

$$
\delta\psi_\alpha \to \delta\psi_\alpha+\epsilon_{\alpha\beta}\omega_\beta F
$$

and F must transform as

$$
\delta F=\xi\dot{F}-\frac{1}{2}\sigma F+\frac{1}{\lambda_1}\epsilon_{\alpha\beta}\omega_\alpha S_{\psi_\beta}-\frac{1}{2\lambda_1}\omega_\beta\lambda_{4\beta}F. \qquad (3.13)
$$

The action will be modified by a term

$$\delta S = \frac{1}{2}\int dt\ \lambda_1\ F^2. \tag{3.14}$$

With this auxiliary field, the gauge algebra closes on ψ_α with the starred parameters listed above. More generally, auxiliary fields should be added to exactly compensate for the lack of noncommutativity of the pullback and the gauge variation. For example, by inspection the case of $N=3$ requires three bosonic auxiliary fields to close the algebra.

With the infinitesimal transformations in standard form, we are a position to determine the partial gauge fixing choice. Reparametrizations should be generated by the $p^2/2$ term in the lagrangian, and therefore the $\varepsilon_1\kappa$ term in $\delta_\varepsilon x$ should give rise to reparametrizations of x. ¿From the equations 3.5 and 3.3 (valid for all N)

$$\delta_\varepsilon x \supset \frac{\varepsilon_1}{\lambda_1}(\dot{x} - \frac{1}{2}\lambda_3 x - \frac{1}{2}\lambda_{4\alpha}\psi_\alpha),$$

it is clear that for any N the partial gauge fixing choice is $\lambda_1 = 1$, $\lambda_3 = 0$ and $\lambda_{4\alpha} = 0$. Note that the number of gauge field degrees of freedom is originally $3 + 2N + N(N-1)/2$. After imposing the $2+N$ gauge fixing conditions, we have $1 + N + N(N-1)/2$ gauge fields remaining, which is precisely the number we expect for the SO(N)-invariant N-extended superconformal algebra. The gauge fixed lagrangian is

$$S = \int dt \left[\frac{1}{2}\dot{x}^2 + \frac{1}{2}\dot{\psi}_\alpha\psi_\alpha + \lambda_2\frac{x^2}{2} - \lambda_{5\alpha}\frac{x\psi_\alpha}{2} - \lambda_6\epsilon_{\alpha\beta}\frac{\psi_\alpha\psi_\beta}{4} + \frac{1}{2}F^2\right]. \tag{3.15}$$

This manifestly conformally invariant lagrangian without the auxiliary field has been studied by Siegel [24].

While we will eventually do the gauge fixing at the finite level, it is enlightening to consider the partial gauge fixing at the infinitesimal level as well. Imposing the constraints gives

$$\left\{\begin{array}{rl} \delta(\lambda_1 - 1) & = 0 = \dot{\xi} + \sigma \\ \delta\lambda_3 & = 0 = \dot{\sigma} - 2h + \omega_\alpha\lambda_{5\alpha} \\ \delta\lambda_{4\alpha} & = 0 = 2\dot{\omega}_\alpha + \chi_\alpha + \epsilon_{\alpha\beta}\omega_\beta\lambda_6 \end{array}\right. \tag{3.16}$$

which may be solved to obtain

$$\Rightarrow \qquad \left\{\begin{array}{rl} \sigma & = -\dot{\xi} \\ h & = \frac{1}{2}\omega_\alpha\lambda_{5\alpha} - \frac{1}{2}\ddot{\xi} \\ \chi_\alpha & = -2\dot{\omega}_\alpha - \epsilon_{\alpha\beta}\omega_\beta\lambda_6. \end{array}\right. \tag{3.17}$$

Notice there is a field-dependence in the parameters in the case of $N=2$. Plugging these into the infinitesimal transformations gives

$$\begin{aligned}
\delta x &= \xi \dot{x} - \frac{1}{2}\dot{\xi} x + \omega_\alpha \psi_\alpha && (3.18)\\
\delta \psi_\alpha &= \xi \dot{\psi}_\alpha + \omega_\alpha \dot{x} - \dot{\omega}_\alpha x + \epsilon_{\alpha\beta}\omega_\beta \left(F - \frac{1}{2}\lambda_6 x\right) - \frac{1}{2} R \epsilon_{\alpha\beta}\psi_\beta && (3.19)\\
\delta \lambda_2 &= 2\dot{\xi}\lambda_2 + \xi\dot{\lambda}_2 - \frac{1}{2}\dddot{\xi} + \frac{3}{2}\dot{\omega}_\alpha \lambda_{5\alpha} + \frac{1}{2}\omega_\alpha \dot{\lambda}_{5\alpha} + \frac{1}{2}\epsilon_{\alpha\beta}\omega_\beta \lambda_6 \lambda_{5\alpha} && (3.20)\\
\delta \lambda_{5\alpha} &= \frac{3}{2}\dot{\xi}\lambda_{5\alpha} + \xi \dot{\lambda}_{5\alpha} - 2\ddot{\omega}_\alpha + 2\omega_\alpha \left(\lambda_2 + \frac{1}{4}\lambda_6^2\right) - \epsilon_{\alpha\beta}\omega_\beta \dot{\lambda}_6 && (3.21)\\
&\quad -2\epsilon_{\alpha\beta}\dot{\omega}_\beta \lambda_6 - \frac{1}{2}\epsilon_{\alpha\beta} R \lambda_{5\beta} &&\\
\delta \lambda_6 &= \dot{\xi}\lambda_6 + \xi \dot{\lambda}_6 + \dot{R} + \epsilon_{\alpha\beta}\omega_\alpha \lambda_{5\beta} && (3.22)\\
\delta F &= \frac{1}{2}\dot{\xi} F + \xi \dot{F} - \epsilon_{\alpha\beta}\omega_\alpha \dot{\psi}_\beta + \frac{1}{2}\epsilon_{\alpha\beta}\omega_\alpha \lambda_{5\beta} x + \frac{1}{2}\omega_\alpha \psi_\alpha \lambda_6 . && (3.23)
\end{aligned}$$

These infinitesimal transformations have previously been obtained by Siegel [24]. The case of $N=1$ is straightforward (here the auxiliary field F is not present, nor is the gauge field λ_6 and its associated parameter R). The bosonic and fermionic stress tensor correspond to λ_2 and λ_5 and there are matter fields x and ψ of weight -1/2 and 0. On the other hand, the infinitesimal algebra appears to be nonlinear for $N=2$! The field dependence of the gauge parameters will generally introduce terms quadratic in the gauge and matter fields. It is easy to dismiss this illusion. Firstly, we expect the matter be arranged in a supermultiplet. This indicates that the auxiliary field is not F, but is

$$\hat{F} \equiv F - \frac{1}{2}\lambda_6 x. \qquad (3.24)$$

Furthermore the terms

$$\begin{aligned}
\delta \lambda_2 &\supset \frac{1}{2}\epsilon_{\alpha\beta}\omega_\beta \lambda_{5\alpha}\lambda_6\\
\delta \lambda_6 &\supset -\frac{1}{2}\epsilon_{\alpha\beta}\omega_\beta \lambda_{5\alpha}
\end{aligned}$$

suggest that we take the combination $\lambda_2 + \frac{1}{4}\lambda_6^2$ to be the spin two stress tensor for $N=2$. With these identifications, the expressions above become linear. Note that if the F^2 term in the action is rewritten in terms of the primary field $\hat{F}$, the shift in the stress tensor becomes apparent

$$\lambda_2 x^2 + F^2 = \underbrace{(\lambda_2 + \frac{1}{4}\lambda_6^2)}\, x^2 + \hat{F}^2 + \hat{F}\lambda_6 x.$$

4 Finite Superconformal Transformations

Let us proceed with partial gauge fixing at the level of the finite Osp$(N|2)$ transformations. The finite transformations can be obtained by exponentiation. A recent discussion on finite gauge transformations may be found in [30]. The symmetries of the action in finite form are

- Reparametrizations

$$\begin{aligned} x'(t) &= x(f(t)) \\ \psi'_\alpha(t) &= \psi_\alpha(f(t)) \\ F'(t) &= F(f(t)) \\ \lambda'_A(t) &= \dot{f}(t)\lambda_A(f(t)) \qquad \forall\, A \end{aligned}$$

- Scale Transformations

$$\begin{array}{ll} x' = e^{\sigma/2}\, x & \lambda'_1 = e^{\sigma}\,\lambda_1 \\ \psi'_\alpha = \psi_\alpha & \lambda'_2 = e^{-\sigma}\,\lambda_2 \\ F' = e^{-\sigma/2}\, F & \lambda'_3 = \lambda_3 + \dot{\sigma} \\ & \lambda'_{4\alpha} = e^{\sigma/2}\,\lambda_{4\alpha} \\ & \lambda'_{5\alpha} = e^{-\sigma/2}\,\lambda_{5\alpha} \\ & \lambda'_6 = \lambda_6 \end{array}$$

- Shift Transformations

$$\begin{array}{ll} x' = x & \lambda'_1 = \lambda_1 \\ \psi'_\alpha = \psi_\alpha & \lambda'_2 = \lambda_2 + \dot{h} + h\lambda_3 - h^2\lambda_1 \\ F' = F & \lambda'_3 = \lambda_3 - 2h\lambda_1 \\ & \lambda'_{4\alpha} = \lambda_{4\alpha} \\ & \lambda'_{5\alpha} = \lambda_{5\alpha} - h\lambda_{4\alpha} \\ & \lambda'_6 = \lambda_6 \end{array}$$

- Supersymmetries

$$\begin{aligned} x' &= x + \omega_\alpha\psi_\alpha + \frac{1}{2}\epsilon_{\alpha\beta}\omega_\alpha\omega_\beta F \\ \psi'_\alpha &= \psi_\alpha + \omega_\alpha\kappa + \epsilon_{\alpha\beta}\omega F - \frac{1}{\lambda_1}\omega_\alpha\omega_\beta S_{\psi_\beta} - \frac{F\lambda_{4\gamma}}{4\lambda_1}\left(\epsilon_{\beta\gamma}\omega_\alpha\omega_\beta - \epsilon_{\alpha\beta}\omega_\gamma\omega_\beta\right) \\ F' &= F + \frac{1}{\lambda_1}\epsilon_{\alpha\beta}\omega_\alpha S_{\psi_\beta} - \frac{1}{2\lambda_1}\omega_\alpha\lambda_{4\alpha}F + \frac{1}{2\lambda_1}\epsilon_{\alpha\beta}\omega_\alpha\omega_\beta S_x \end{aligned}$$

$$-\frac{1}{4\lambda_1^2}\epsilon_{\alpha\beta}\omega_\alpha\omega_\beta\lambda_{4\gamma}S_{\psi_\gamma} + \frac{1}{4\lambda_1^2}\left(\omega_\alpha\lambda_{4\alpha}\right)^2$$

$$\begin{aligned}
\lambda_1' &= \lambda_1 + \omega_\alpha\lambda_{4\alpha} + \omega_\alpha\dot{\omega}_\alpha + \frac{1}{2}\epsilon_{\alpha\beta}\omega_\alpha\omega_\beta\lambda_6 \\
\lambda_3' &= \lambda_3 + \omega_\alpha\lambda_{5\alpha} \\
\lambda_{4\alpha}' &= \lambda_{4\alpha} + 2\dot{\lambda}_\alpha - \omega_\alpha\lambda_3 + \epsilon_{\alpha\beta}\omega_\beta\lambda_6 - \omega_\alpha\omega_\beta\lambda_{5\beta} \\
\lambda_{5\alpha}' &= \lambda_{5\alpha} + 2\omega_\alpha\lambda_2 \\
\lambda_6' &= \lambda_6 + \epsilon_{\alpha\beta}\omega_\alpha\lambda_{5\beta} + \epsilon_{\alpha\beta}\omega_\alpha\omega_\beta\lambda_2
\end{aligned}$$

- Additional Fermionic Symmetries

$$\begin{aligned}
x' &= x \\
\psi_\alpha' &= \psi_\alpha + \frac{1}{2}x\chi_\alpha \\
F' &= F \\
\lambda_1 &= \lambda_1 \\
\lambda_2' &= \lambda_2 - \frac{1}{2}\chi_\alpha\lambda_{5\alpha} - \frac{1}{8}\epsilon_{\alpha\beta}\chi_\alpha\chi_\beta\lambda_6 - \frac{1}{4}\chi_\alpha\dot{\chi}_\alpha \\
\lambda_3' &= \lambda_3 + \frac{1}{2}\chi_\alpha\lambda_{4\alpha} \\
\lambda_{4\alpha}' &= \lambda_{4\alpha} + \chi_\alpha\lambda_1 \\
\lambda_{5\alpha}' &= \lambda_{5\alpha} + \frac{1}{2}\chi_\alpha\lambda_3 + \frac{1}{2}\epsilon_{\alpha\beta}\chi_\beta\lambda_6 + \dot{\chi}_\alpha + \frac{1}{4}\chi_\alpha\chi_\beta\lambda_{4\beta} \\
\lambda_6' &= \lambda_6 + \frac{1}{2}\epsilon_{\alpha\beta}\chi_\beta\lambda_{4\alpha} - \frac{1}{4}\epsilon_{\alpha\beta}\chi_\alpha\chi_\beta\lambda_1
\end{aligned}$$

- o(2) Rotations

$$\begin{aligned}
x' &= x \\
\psi_\alpha' &= (\cos\frac{R}{2}\delta_{\alpha\beta} + \sin\frac{R}{2}\epsilon_{\alpha\beta})\psi_\beta \\
F' &= F \\
\lambda_1' &= \lambda_1 \\
\lambda_2' &= \lambda_2 \\
\lambda_3' &= \lambda_3 \\
\lambda_{4\alpha}' &= \left(\cos\frac{R}{2}\delta_{\alpha\beta} + \sin\frac{R}{2}\epsilon_{\alpha\beta}\right)\lambda_{4\beta} \\
\lambda_{5\alpha}' &= \left(\cos\frac{R}{2}\delta_{\alpha\beta} + \sin\frac{R}{2}\epsilon_{\alpha\beta}\right)\lambda_{5\beta}
\end{aligned}$$

$$\lambda_6' \;=\; \lambda_6 + \dot{R}$$

Perform successive finite transformations on each field $\mathcal{F}$ in the following (arbitrary but consistent order) manner:

$$\mathcal{F} \xrightarrow{\sigma} \Box \xrightarrow{h} \Box \xrightarrow{\chi_\alpha} \Box \xrightarrow{R} \Box \xrightarrow{\omega_\alpha} \Box \xrightarrow{f} \tilde{\mathcal{F}}. \tag{4.1}$$

For $N = 1$, we obtain the following finite Osp(1|2) transformed fields. These are the active transformations of the fields. The fields on the left hand side with tildes are functions of t, while the fields and parameters on the right hand side are functions of $f(t)$.

$$\tilde{x}(t) \;=\; e^{\sigma/2}\left(x + \omega\psi\right) \tag{4.2}$$

$$\tilde{\psi}(t) \;=\; \psi + \frac{\omega}{\lambda_1}\left(\frac{\dot{x}}{\dot{f}} - \frac{\lambda_3 x}{2} - \frac{\lambda_4\psi}{2}\right) + \frac{x}{2}\chi + \frac{\omega\psi}{2}\chi \tag{4.3}$$

$$\tilde{\lambda_1}(t) \;=\; e^{\sigma}\dot{f}\left(\lambda_1 + \omega\lambda_4 + \frac{\omega\dot{\omega}}{\dot{f}}\right) \tag{4.4}$$

$$\begin{aligned}\tilde{\lambda_2}(t) \;=\; & e^{-\sigma}\dot{f}\left(\lambda_2 + \frac{1}{2}\lambda_5\chi + \omega\lambda_2\chi - \frac{\chi\dot{\chi}}{4\dot{f}} + \frac{\dot{h}}{\dot{f}} + h\lambda_3 + h\omega\lambda_5\right.\\ & \left.-\frac{1}{2}h\lambda_4\chi - \frac{h}{\dot{f}}\dot{\omega}\chi + \frac{1}{2}h\lambda_3\omega\chi - h^2\lambda_1 - h^2\omega\lambda_4 - h^2\frac{\omega\dot{\omega}}{\dot{f}}\right)\end{aligned} \tag{4.5}$$

$$\begin{aligned}\tilde{\lambda_3}(t) \;=\; & \dot{f}\left(\lambda_3 + \frac{\dot{\sigma}}{\dot{f}} - 2h\lambda_1 + \omega\lambda_5 - \frac{1}{2}\lambda_4\chi - \frac{\dot{\omega}}{\dot{f}}\chi + \frac{1}{2}\omega\lambda_3\chi - 2h\omega\lambda_4\right.\\ & \left.-2h\frac{\omega\dot{\omega}}{\dot{f}}\right)\end{aligned} \tag{4.6}$$

$$\tilde{\lambda_4}(t) \;=\; e^{\sigma/2}\dot{f}\left(\lambda_4 + 2\frac{\dot{\omega}}{\dot{f}} - \omega\lambda_3 + \lambda_1\chi + \omega\lambda_4\chi + \frac{\omega\dot{\omega}\chi}{\dot{f}}\right) \tag{4.7}$$

$$\begin{aligned}\tilde{\lambda_5}(t) \;=\; & e^{-\sigma/2}\dot{f}\left(\lambda_5 + 2\omega\lambda_2 + \frac{\dot{\chi}}{\dot{f}} + \frac{1}{2}\lambda_3\chi + \frac{1}{2}\omega\lambda_5\chi - h\lambda_4 - 2h\frac{\dot{\omega}}{\dot{f}}\right.\\ & \left.+h\omega\lambda_3 - h\lambda_1\chi - h\omega\lambda_4\chi - h\frac{\omega\dot{\omega}}{\dot{f}}\chi\right).\end{aligned} \tag{4.8}$$

The gauge fixing conditions give the relations

$$\left\{\begin{array}{ll} \sigma & = -\ln\dot{f} - \frac{\omega\dot{\omega}}{\dot{f}} \\ \chi & = -\frac{2\dot{\omega}}{\dot{f}} \\ h & = \frac{1}{2}\omega\lambda_5 + \omega\dot{\omega}\frac{\ddot{f}}{\dot{f}^3} - \omega\ddot{\omega}\frac{1}{2\dot{f}^2} - \frac{\ddot{f}}{2\dot{f}^2} \end{array}\right. \tag{4.9}$$

The resulting finite transformations of the matter fields x and ψ and the gauge fields $T_B \equiv \lambda_2$ and $T_F \equiv \frac{1}{2}\lambda_5$ under the residual symmetry are

$$\begin{aligned}
\tilde{x}(t) &= \dot{f}^{-\frac{1}{2}}\left(x + \omega\psi - \frac{1}{2}\frac{\omega\dot{\omega}}{\dot{f}}x\right) && (4.10)\\
\tilde{\psi}(t) &= \psi + \frac{\omega}{\dot{f}}\dot{x} - \frac{\dot{\omega}}{\dot{f}}x + \frac{\omega\dot{\omega}}{\dot{f}}\psi && (4.11)\\
\tilde{T}_B(t) &= \dot{f}^2\left[T_B - \frac{\omega\dot{\omega}}{\dot{f}}T_B + \frac{\omega}{\dot{f}}\dot{T}_F + 3\frac{\dot{\omega}}{\dot{f}}T_F\right] - \frac{1}{2}\left(\frac{\dddot{f}}{\dot{f}} - \frac{3\ddot{f}^2}{2\dot{f}^2}\right) && (4.12)\\
&\quad + \frac{1}{2}\omega\dot{\omega}\frac{\dddot{f}}{\dot{f}^2} - \frac{\omega\dddot{\omega}}{2\dot{f}} - \frac{3\ddot{f}^2}{2\dot{f}^3}\omega\dot{\omega} + \frac{3\ddot{f}}{2\dot{f}^2}\omega\ddot{\omega} - \frac{3\dot{\omega}\ddot{\omega}}{2\dot{f}}\\
\tilde{T}_F(t) &= \dot{f}^{\frac{3}{2}}\left[T_F + \omega T_B + \frac{3\omega\dot{\omega}}{2\dot{f}}T_F\right] - \frac{1}{\sqrt{\dot{f}}}\left(\ddot{\omega} - \dot{\omega}\frac{\ddot{f}}{\dot{f}} + \frac{\omega\dot{\omega}\ddot{\omega}}{2\dot{f}}\right). && (4.13)
\end{aligned}$$

These equations give the most general $N = 1$ superconformal finite transformations of the matter and gauge fields. This procedure may unambiguously be carried out for any N using the steps outlined here for $N = 1$. We will not write out the expressions for $N = 2$ here, but instead make use of the superfield formulation to give an alternate derivation of the $N = 2$ superschwarzian.

5 Superfield Formulation for $N = 1, 2$ and Alternate Derivation of the Superschwarzian

5.1 $N = 1$ Superfield Formulation

Examination of the results in the previous section for $N = 1$ show that the matter fields and the gauge fields form supermultiplets

$$\begin{aligned}
\phi(Z) &= x(Z) + \theta\psi(Z)\\
T(Z) &= T_F(Z) + \theta T_B(Z) && (5.1)
\end{aligned}$$

where $Z = (z, \theta)$ denotes the supercoordinates [31]. Here we make a notational switch from t in the 1-dimensional particle model to z to suggest a chiral sector of a two-dimensional conformal theory. The component transformations given in the previous section are equivalent to the superfield transformations

$$\begin{aligned} \phi(Z) &= \phi'(Z')\,(D\theta')^{2h} \quad \text{where } h = -\frac{1}{2} \\ T(Z) &= T'(Z')\,(D\theta')^3 + \frac{\hat{c}}{4}\,S(Z,Z') \quad \text{where } \hat{c} = -4 \end{aligned} \tag{5.2}$$

where the superconformal derivative is

$$D = \frac{\partial}{\partial\theta} + \theta\frac{\partial}{\partial z}$$

and $S(Z,Z')$ is the superfield superschwarzian, which we will address momentarily. Superconformal transformations are those which induce a homogeneous transformation law for the superconformal derivative. ¿From $D = (D\theta')D' + (Dz' - \theta' D\theta')\partial_z'$, we deduce that the superconformal transformations are

$$\begin{aligned} z'(z,\theta) &= f(z) + \theta\omega(z)\sqrt{\partial_z f(z)} \\ \theta'(z,\theta) &= \omega(z) + \theta\sqrt{\partial_z f(z) + \omega(z)\partial_z\omega(z)}. \end{aligned}$$

Using the superfield formulation, we may give an alternate derivation of the superschwarzian in terms of superfields rather than components, which is far simpler than any other method. The equations of motion for the matter fields in the partially gauge fixed pseudoparticle model are

$$\begin{aligned} S_x &= -\partial_z^2 x + T_B x - T_F\psi \\ S_\psi &= -\partial_z\psi + T_F x. \end{aligned} \tag{5.3}$$

These may be expressed as a superfield equation

$$\Big(D^3 - T(Z)\Big)\,\phi(Z) = 0. \tag{5.4}$$

The schwarzian may be derived by demanding that this equation maintain covariance under superconformal transformations. ¿From the infinitesimal transformation equation 3.20 for λ_2, we see that $c = -6$ or in superfield notation $\hat{c} = \frac{2}{3}c = -4$. The knowledge of the central charge allows us to normalize the super schwarzian. We write the anomalous term in the superfield transformation of the super stress tensor as an apriori unknown function $\mathcal{S}(Z,Z')$. Writing the passive transformations

$$\begin{aligned} \phi'(Z') &= (D\theta')\phi(Z) \\ D' &= (D\theta')^{-1}D \\ T'(Z') &= (D\theta')^{-3}(T(Z) + \mathcal{S}(Z,Z')) \end{aligned}$$

and plugging the expressions into the equation of motion, we obtain

$$\Big((D')^3 - T'(Z')\Big)\,\phi'(Z') = (D\theta')^{-2}\Big(D^3 - T(Z)\Big)\,\phi(Z) \tag{5.5}$$

where

$$\mathcal{S}(Z,Z') = \frac{D^4\theta'}{D\theta'} - 2\frac{D^3\theta'}{D\theta'}\frac{D^2\theta'}{D\theta'}$$

in agreement with Friedan [31].

5.2 $N=2$ Superfield Formulation

We may repeat the above discussion for $N=2$. Using complex notation and the identifications

$$\begin{aligned} T_B &= \lambda_2 + \frac{1}{4}\lambda_6^2 && (5.6)\\ G^+ &= \frac{1}{2}\lambda_{5+} && (5.7)\\ G^- &= -\frac{1}{2}\lambda_{5-} && (5.8)\\ H &= \frac{1}{2}i\lambda_6 && (5.9) \end{aligned}$$

the superfields are, recalling that the correct matter auxiliary field is $\hat{F} = F - \frac{1}{2}\lambda_6 x$

$$\begin{aligned} \phi(Z) &= x(Z) + \theta^+\psi^-(Z) + \theta^-\psi^+(Z) + \theta^+\theta^-\hat{F}(Z)\\ T(Z) &= H(Z) + \theta^+G^-(Z) + \theta^-G^+(Z) - \theta^+\theta^-T_B(Z), \qquad (5.10) \end{aligned}$$

where $Z = (z,\theta^+,\theta^-)$ denotes the supercoordinates. The equations of motion in components are

$$\begin{aligned} S_x &= -\partial_z^2 x + \left(T_B + \frac{1}{4}\lambda_6^2\right)x - G^+\psi^- + G^-\psi^+ && (5.11)\\ S_{\psi_\pm} &= -\partial_z\psi_\pm + xG^\pm \pm H\psi^\pm && (5.12)\\ S_{\hat{F}} &= \hat{F} + \frac{1}{2}\lambda_6 x && (5.13) \end{aligned}$$

(the last equation is just equivalent to the equation $F=0$). Using the superconformal derivatives

$$D^\pm = \frac{\partial}{\partial\theta_\mp} + \theta^\pm\partial_z,$$

the pseudoparticle equations of motion can be written as the superfield equation

$$\left[\frac{1}{2}\left(D^+D^- - D^-D^+\right) + T(Z)\right]\phi(Z) = 0. \tag{5.14}$$

The $N = 2$ super derivative transforms as

$$D^{\pm} = (D^{\pm}\tilde{\theta}^-)\tilde{D}^+ + (D^{\pm}\tilde{\theta}^+)\tilde{D}^- + \left(D^{\pm}\tilde{z} - \tilde{\theta}^+D^{\pm}\tilde{\theta}^- - \tilde{\theta}^-D^{\pm}\tilde{\theta}^+\right)\partial_{\tilde{z}},$$

so that superconformal transformations must satisfy $D^{\pm}\tilde{z} - \tilde{\theta}^+D^{\pm}\tilde{\theta}^- - \tilde{\theta}^-D^{\pm}\tilde{\theta}^+ = 0$ and either $D^{\pm}\tilde{\theta}^{\mp} = 0$ or $D^{\pm}\tilde{\theta}^{\pm} = 0$. Choosing the superconformal condition

$$D^{\pm}\tilde{\theta}^{\pm} = 0$$

leads to the transformations in terms of $f(z)$, $\omega^{\pm}(z)$ and $R(z)$

$$\begin{aligned}
\tilde{z}(z,\theta^+,\theta^-) &= f + \theta^+\omega^-e^{-\frac{iR}{2}}\sqrt{\partial_z f + \omega^+\partial_z\omega^- + \omega^-\partial_z\omega^+} \\
&\quad + \theta^-\omega^+e^{\frac{iR}{2}}\sqrt{\partial_z f + \omega^+\partial_z\omega^- + \omega^-\partial_z\omega^+} + \theta^+\theta^-\partial_z(\omega^+\omega^-) \\
\tilde{\theta}^+(z,\theta^+,\theta^-) &= \omega^+ + \theta^+\sqrt{\partial_z f + \omega^+\partial_z\omega^- + \omega^-\partial_z\omega^+}\,e^{-i\frac{R}{2}} + \theta^+\theta^-\partial_z\omega^+ \\
\tilde{\theta}^-(z,\theta^+,\theta^-) &= \omega^- + \theta^-\sqrt{\partial_z f + \omega^+\partial_z\omega^- + \omega^-\partial_z\omega^+}\,e^{i\frac{R}{2}} + \theta^-\theta^+\partial_z\omega^-.
\end{aligned}$$

¿From the infinitesimal gauge fixing, the conformal weights of ϕ and T are determined, as well as the central charge of the theory $\hat{c} = 1$. We write passive transformation of the super derivatives and fields as

$$\begin{aligned}
\tilde{D}^+ &= (D^+\tilde{\theta}^-)^{-1}D^+ \\
\tilde{D}^- &= (D^-\tilde{\theta}^+)^{-1}D^- \\
\tilde{\phi}(\tilde{Z}) &= (D^+\tilde{\theta}^-)^{\frac{1}{2}}(D^-\tilde{\theta}^+)^{\frac{1}{2}}\phi(Z) \\
\tilde{T}(\tilde{Z}) &= (D^+\tilde{\theta}^-)^{-1}(D^-\tilde{\theta}^+)^{-1}\left[T(Z) - \mathcal{S}(Z,\tilde{Z})\right]
\end{aligned}$$

where $\mathcal{S}(Z,\tilde{Z})$ is apriori an unknown function. By demanding covariance of the superfield equation of motion

$$\begin{aligned}
&\left[\frac{1}{2}\left(\tilde{D}^+\tilde{D}^- - \tilde{D}^-\tilde{D}^+\right) + \tilde{T}(\tilde{Z})\right]\tilde{\phi}(\tilde{Z}) \qquad (5.15)\\
&= (D^+\tilde{\theta}^-)^{-\frac{1}{2}}(D^-\tilde{\theta}^+)^{-\frac{1}{2}}\left[\frac{1}{2}\left(D^+D^- - D^-D^+\right) + T(Z)\right]\phi(Z)
\end{aligned}$$

one obtains the $N = 2$ superschwarzian

$$\mathcal{S} = \frac{\partial_z D^-\tilde{\theta}^+}{D^-\tilde{\theta}^+} - \frac{\partial_z D^+\tilde{\theta}^-}{D^+\tilde{\theta}^-} + 2\frac{\partial_z\tilde{\theta}^+}{D^-\tilde{\theta}^+}\frac{\partial_z\tilde{\theta}^-}{D^+\tilde{\theta}^-} \tag{5.16}$$

in agreement with the result of Cohn [29].

6 Complete gauge fixing of the pseudoparticle model

Finally we perform the complete gauge fixing of the model, i.e. in addition to setting $\lambda_1 = 1$ and $\lambda_3 = \lambda_{4\alpha} = 0$, we further fix $\lambda_2 = 0$, $\lambda_{5\alpha} = 0$ and $\lambda_6 = 0$. Upon complete gauge fixing, the lagrangian reduces to

$$S = \int dt \left[\frac{1}{2}\dot{x}^2 + \frac{1}{2}\dot{\psi}_\alpha \psi_\alpha + \frac{1}{2}F^2\right]. \tag{6.1}$$

Consequently the resulting pseudoparticle model will be governed by the equations of motion

$$\ddot{x} = 0 \qquad \dot{\psi}_\alpha = 0 \qquad F = 0 \tag{6.2}$$

which are invariant under the super Möbius transformations. These are the transformations which are compatible with putting the gauge field variations equal to zero, i.e. those transformations for which the superschwarzian vanishes. At the infinitesimal level for $N = 1$, we have $\delta\lambda_2 = -\frac{1}{2}\dddot{\xi}$ and $\delta\lambda_5 = -2\ddot{\omega}$ so that $\xi(t) = A + tB + t^2 C$ and $\omega(t) = \alpha + t\beta$, where A, B, C and α, β are bosonic and fermionic constants. At finite level, we have

$$\begin{aligned} f(t) &= \frac{at+b}{ct+d} \qquad \text{where } ad - bc = 1 \\ \omega(t) &= \frac{\gamma t + \delta}{ct+d} = (\gamma t + \delta)\sqrt{\dot{f}} \end{aligned} \tag{6.3}$$

or in terms of superfields (using $t'(t,\theta) = f(t) + \theta\omega(t)\sqrt{\dot{f}(t)}$ and $\theta'(t,\theta) = \omega(t) + \theta\sqrt{\dot{f}(t) + \omega(t)\dot{\omega}(t)}$)

$$t'(t,\theta) = \frac{at+b}{ct+d} + \theta\frac{\gamma t + \delta}{(ct+d)^2} \qquad \theta'(t,\theta) = \frac{\gamma t+\delta}{ct+d} + \theta\frac{1 + \delta\gamma/2}{ct+d}. \tag{6.4}$$

Similarly, for $N = 2$, the infinitesimal transformations are $\delta\lambda_2 = -\frac{1}{2}\dddot{\xi}$, $\delta\lambda_{5\alpha} = -2\ddot{\omega}_\alpha$ and $\delta\lambda_6 = \dot{R}$ so that $\xi(t) = A + tB + t^2C$, $\omega^\pm(t) = \alpha^\pm + t\beta^\pm$ and $R(t) = R_0$ is a constant. At the finite level, we find

$$\begin{aligned} f(t) &= \frac{at+b}{ct+d} \qquad \text{where } ad - bc = 1 \\ \omega^\pm(t) &= \frac{\gamma^\pm t + \delta^\pm}{ct+d} = (\gamma^\pm t + \delta^\pm)\sqrt{\dot{f}} \\ R(t) &= R_0. \end{aligned} \tag{6.5}$$

The superfield expressions for the $N = 2$ super Möbius transformations may be obtained by plugging the above into the expressions in the previous section for general superconformal transformations.

7 $N = 2$ twisted and topological theories

In this section, we comment on twisted and topological theories in relation to the $N = 2$ pseudoparticle model. ¿From the pseudoparticle model point of view, the twisting of the stress-tensor is a significant deformation of the untwisted theory. Before elaborating on this point, we return to the infinitesimal gauge fixing of the $N = 2$ theory to show the emergence of the twisted and topological theories. ¿From the infinitesimal transformation equations 3.20 and 3.22, one may define a twisted stress tensor

$$T_B = \lambda_2 + \frac{1}{4}\lambda_6^2 + i\alpha_0\dot{\lambda}_6, \tag{7.1}$$

with which one may obtain a closed and consistent twisted $N = 2$ algebra. It will also prove useful to allow a shift of the parameter R

$$R = i\varphi\dot{\xi} + r. \tag{7.2}$$

The transformations for gauge fields and the matter fields are

$$\begin{aligned}
\delta T_B &= 2T_B\dot{\xi} + \dot{T}_B\xi - (\frac{1}{2} + \alpha_0\varphi)\dddot{\xi} + i\alpha_0\ddot{r}(\frac{3}{2} - \alpha_0)\dot{\omega}_+\lambda_{5-} && (7.3)\\
&\quad +(\frac{3}{2} + \alpha_0)\dot{\omega}_-\lambda_{5+} + (\frac{1}{2} - \alpha_0)\omega_+\dot{\lambda}_{5-} + (\frac{1}{2} + \alpha_0)\omega_-\dot{\lambda}_{5+} \\
&\quad + \left(\frac{1}{2}\dot{r} + i(\alpha_0 + \frac{\varphi}{2})\ddot{\xi}\right)\lambda_6 \\
\delta\lambda_{5\pm} &= \left(\frac{3}{2} \mp \frac{\varphi}{2}\right)\lambda_{5\pm}\dot{\xi} + \dot{\lambda}_{5\pm}\xi \pm \frac{i}{2}r\lambda_{5\pm} + 2\omega_\pm T_B - 2\ddot{\omega}_\pm && (7.4)\\
&\quad \pm i\,(1 \mp 2\alpha_0)\,\omega_\pm\dot{\lambda}_6 \pm 2\,i\,\dot{\omega}_\pm\lambda_6 \\
\delta\lambda_6 &= \dot{\lambda}_6\xi + \lambda_6\dot{\xi} + \dot{r} + i\varphi\ddot{\xi} + i\,(\omega_+\lambda_{5-} - \omega_-\lambda_{5+}) && (7.5)\\
\delta x &= \xi\dot{x} - \frac{1}{2}\dot{\xi}x + \omega_-\psi_+ + \omega_+\psi_- && (7.6)\\
\delta\psi_\pm &= \xi\dot{\psi}_\pm \mp \frac{1}{2}\varphi\dot{\xi}\psi_\pm + \omega_\pm\dot{x} - \dot{\omega}_\pm x \mp i\,\omega_\pm\hat{F} \pm \frac{i}{2}r\psi_\pm && (7.7)\\
\delta\hat{F} &= \xi\dot{\hat{F}} + \frac{1}{2}\dot{\xi}\hat{F} - \frac{i\varphi}{2}\ddot{\xi}x - \frac{\dot{r}}{2}x + i(\omega_-\dot{\psi}_+ - \omega_+\dot{\psi}_-). && (7.8)
\end{aligned}$$

¿From the transformation law for T_B, it is evident that for ordinary diffeomorphisms one should take $\varphi = -2\alpha_0$. For generic α_0, the matter fields x, $\psi_\pm$ and the fermionic gauge fields $\lambda_{5\pm}$ transform as primary fields under diffeomorphisms with weights $-\frac{1}{2}$, $\pm\alpha_0$ and $\frac{3}{2} \pm \alpha_0$ respectively. The gauge fields T_B, λ_6 and the auxiliary field are quasi-primary with weights 2, 1 and $\frac{1}{2}$. The classical central charge of this theory is $c = -6 + 24\alpha_0^2$.

Of particular interest is the point $\alpha_0^2 = \frac{1}{4}$ where the central charge vanishes. This is the $N = 2$ topological theory. With $\alpha_0 = -\frac{1}{2}$, we find

$$\delta T_B = 2T_B\dot{\xi} + \dot{T}_B\xi - i\,\ddot{r}\omega_+\lambda_{5-} + \frac{1}{2}\dot{r}\lambda_6 + 2\dot{\omega}_+\lambda_{5-} + \dot{\omega}_-\lambda_{5+} + \omega_+\dot{\lambda}_{5-} \tag{7.9}$$

$$\delta\lambda_{5+} = \lambda_{5+}\dot{\xi} + \dot{\lambda}_{5+}\xi + \frac{i}{2}r\lambda_{5+} + 2\omega_+T_B - 2\ddot{\omega}_+ + 2i\dot{\omega}_+\lambda_6 \tag{7.10}$$

$$\delta\lambda_{5-} = 2\lambda_{5-}\dot{\xi} + \dot{\lambda}_{5-}\xi - \frac{i}{2}r\lambda_{5-} + 2\omega_-T_B - 2\ddot{\omega}_- - 2i\dot{\omega}_-\lambda_6 + 2i\omega_-\dot{\lambda}_6 \tag{7.11}$$

$$\delta\lambda_6 = \dot{\lambda}_6\xi + \lambda_6\dot{\xi} + \dot{r} + i\ddot{\xi} + i\left(\omega_+\lambda_{5-} - \omega_-\lambda_{5+}\right) \tag{7.12}$$

$$\delta x = \xi\dot{x} - \frac{1}{2}\dot{\xi}x + \omega_-\psi_+ + \omega_+\psi_- \tag{7.13}$$

$$\delta\psi_\pm = \xi\dot{\psi}_\pm \mp \frac{1}{2}\dot{\xi}\psi_\pm + \omega_\pm\dot{x} - \dot{\omega}_\pm x \mp i\,\omega_\pm\hat{F} \pm \frac{i}{2}r\psi_\pm \tag{7.14}$$

$$\delta\hat{F} = \xi\dot{\hat{F}} + \frac{1}{2}\dot{\xi}\hat{F} - \frac{i}{2}\ddot{\xi}x - \frac{\dot{r}}{2}x + i(\omega_-\dot{\psi}_+ - \omega_+\dot{\psi}_-). \tag{7.15}$$

Thus T_B, λ_{5+}, λ_{5-}, x, ψ_+ and ψ_- transform as primary fields of weights 2,1,2,$\frac{1}{2}$, $-\frac{1}{2}$ and $\frac{1}{2}$ respectively. The U(1) current λ_6 and the auxiliary field are quasiprimary with weights 1 and $\frac{1}{2}$ respectively.

Finally, we comment on these twisted models as related to the original pseudoparticle models. The key observation is that adding the twisting term $\alpha_0\dot{\lambda}_6$ to the stress tensor is changing the pseudoparticle model in a dramatic way, since the added term implies that λ_6 no longer acts simply as a Lagrange multiplier field but now has some dynamical term in the action.

Furthermore, if we do a complete gauge fixing of the twisted theories, then the action and matter equations of motion remain as in the previous section, but the residual symmetries of the model are different. ¿From the central terms in equations 3.18-3.23, we have

$$\begin{cases} 0 & = (\frac{1}{2} - 2\alpha_0^2)\dddot{\xi} \\ 0 & = \omega_\alpha \\ 0 & = \dot{r} - 2i\,\alpha_0\ddot{\xi}. \end{cases}$$

For nonzero $\alpha_0 \neq -\frac{1}{2}$, the residual infinitesimal transformations are $\xi = A + Bt + Ct^2$, $\omega_\alpha = \gamma + \delta t$ and $r = 4\,i\alpha_0 Ct + r_0$. For the topological theory where $\alpha_0 = -\frac{1}{2}$, then $\xi = A + Bt$, $\omega_\alpha = \gamma + \delta t$ and $r = r_0$. Thus the pseudoparticle model allows one to see that the $N = 2$ topological theory is in some sense disconnected from the non-topological $N = 2$ theory.

8 Flag bundle interpretation

In this section we describe how one may interpret, via the particle model or the zero curvature equation, the superconformal transformations as deformations of flags in superjet bundles over super Riemann surfaces. This discussion will follow the general line of thought of Gerasimov, Levin and Marshakov [12] where the case of $\mathcal{W}_3$ was investigated, and of Bilal, Fock and Kogan [13]. Let us now explicitly construct the flags in the superjet bundle for the cases of $N = 1$ and $N = 2$. We will see that the pseudoparticle model can serve as a guideline for how to perform this construction.

8.1 $N = 1$ Super Flag

Consider the $-2h$ power of the canonical super line bundle over a super Riemann surface, where $-2h = 1$ is the factor appearing in the matter superfield transformation law. A section $\mathbf{f} = \mathbf{f}(Z)$ of this super bundle is just a $-2h = 1$ super differential. Under superconformal transformations, $\mathbf{f}$ will be transformed into an expression containing $\mathbf{f}$, $D\mathbf{f}$ and $D^2\mathbf{f}$. For this reason, we should consider the $N = 1$ super 1-jet bundle [3] (two bosonic dimensions and one Grassmann dimension) over the super Riemann surface. A prolongation of the section $\mathbf{f}$ into a section of the super 1-jet bundle, using a canonical basis is just

$$\hat{\mathbf{f}} = \mathbf{f}e_0 + (D\mathbf{f})e_1 + (D^2\mathbf{f})e_2. \tag{8.1}$$

We may alternately choose to express the section $\mathbf{f}$ using a different basis, determined by the solutions of the completely gauge fixed pseudoparticle superfield matter equation

$$D^3\phi(Z) = 0.$$

Working with 1-super differentials the solutions are

$$1dZ \qquad zdZ \qquad \theta dZ. \tag{8.2}$$

In a general coordinate system, the basis elements are defined by the solutions to

$$\left(D^3 - T\right)\phi(Z) = 0$$

and are thus

$$D\tilde{\theta}dZ \qquad \tilde{z}D\tilde{\theta}dZ \qquad \tilde{\theta}D\tilde{\theta}dZ. \tag{8.3}$$

[3] The term $N = 1$ super 1-jet is used since the highest bosonic derivative is ∂_z, or equivalently, this is the $N = 1$ supersymmetrization of an ordinary 1-jet.

One may reexpress this latter "dynamical" basis in terms of the canonical basis. Writing

$$\begin{aligned}
\eta_0 &= D\tilde{\theta} \\
\eta_{\frac{1}{2}} &= \tilde{\theta} D\tilde{\theta} \\
\eta_1 &= \tilde{z} D\tilde{\theta}
\end{aligned}$$

the dynamical basis in terms of the canonical basis is given by

$$\begin{pmatrix} \hat{\eta}_0 \\ \hat{\eta}_{\frac{1}{2}} \\ \hat{\eta}_1 \end{pmatrix} = \begin{pmatrix} \eta_0 & D\eta_0 & D^2\eta_0 \\ \eta_{\frac{1}{2}} & D\eta_{\frac{1}{2}} & D^2\eta_{\frac{1}{2}} \\ \eta_1 & D\eta_1 & D^2\eta_1 \end{pmatrix} \begin{pmatrix} e_0 \\ e_1 \\ e_2 \end{pmatrix}. \tag{8.4}$$

Writing the section of the super 1-jet bundle in the two different bases

$$\hat{\mathbf{f}} = \mathbf{f} e_0 + (D\mathbf{f}) e_1 + (D^2\mathbf{f}) e_2 = \mathbf{f}_0 \eta_0 + \mathbf{f}_{\frac{1}{2}} \eta_{\frac{1}{2}} + \mathbf{f}_1 \eta_1 \tag{8.5}$$

and then reexpressing $\mathbf{f}, D\mathbf{f}, D^2\mathbf{f}$ in terms of the $\mathbf{f}_0, \mathbf{f}_{\frac{1}{2}}, \mathbf{f}_1$, we find the following

$$\mathbf{f} = D\tilde{\theta}\mathcal{X} \tag{8.6}$$

$$D\mathbf{f} = D^2\tilde{\theta}\mathcal{X} + \left(D\tilde{\theta}^2 - 2\tilde{\theta}D^2\tilde{\theta}\right)\mathcal{Y} \tag{8.7}$$

$$D^2\mathbf{f} = D^3\tilde{\theta}\mathcal{X} + D\tilde{\theta}D^2\tilde{\theta}\mathcal{Y} + \left(D\tilde{\theta}^3 - 2\tilde{\theta}D\tilde{\theta}D^2\tilde{\theta}\right)\mathcal{Z} \tag{8.8}$$

where

$$\mathcal{X} = \mathbf{f}_0 + \mathbf{f}_{\frac{1}{2}}\tilde{\theta} + \mathbf{f}_1\tilde{z} \tag{8.9}$$

$$\mathcal{Y} = \mathbf{f}_{\frac{1}{2}} + \mathbf{f}_1\tilde{\theta} \tag{8.10}$$

$$\mathcal{Z} = \mathbf{f}_1. \tag{8.11}$$

Equations 8.6-8.8 describe a super flag in the $N = 1$ super 1-jet space

$$\mathcal{F}_{(1|0)} \quad \subset \quad \mathcal{F}_{(1|1)} \quad \subset \quad \mathcal{F}_{(2|1)} \tag{8.12}$$

where

$$\begin{aligned}
\mathcal{F}_{(1|0)} &= \{\mathcal{X}\} \\
\mathcal{F}_{(1|1)} &= \{\mathcal{X}, \mathcal{Y}\} \\
\mathcal{F}_{(2|1)} &= \{\mathcal{X}, \mathcal{Y}, \mathcal{Z}\}.
\end{aligned}$$

Superconformal transformations generate deformations of the flag

$$\begin{pmatrix} 1 & -\tilde{\theta} & \tilde{z} \\ 0 & 1 & \tilde{\theta} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{f}_0 \\ \mathbf{f}_{\frac{1}{2}} \\ \mathbf{f}_1 \end{pmatrix} = \begin{pmatrix} \mathbf{f}_0 + \mathbf{f}_{\frac{1}{2}}\tilde{\theta} + \tilde{z}\mathbf{f}_1 \\ \mathbf{f}_{\frac{1}{2}} + \tilde{\theta}\mathbf{f}_1 \\ \mathbf{f}_1 \end{pmatrix}. \tag{8.13}$$

8.2 $N = 2$ Super Flag

The particle model or zero curvature condition dictates that one should consider a $-(h, q) = (\frac{1}{2}, 0)$ superdifferential $\mathbf{f}$, i.e. the matter superfield transforms as a $(-\frac{1}{2}, 0)$ superfield. Under $N = 2$ superconformal transformations, this section $\mathbf{f}$ will be transformed into an expression containing $\mathbf{f}$, $D^{\pm}\mathbf{f}$ and $\frac{1}{2}\{D^+, D^-\}\mathbf{f}$. The $N = 2$ super 1-jet bundle is two-bosonic two-fermionic dimensional space over the super Riemann surface. The prolongation of the section $\mathbf{f}$ into a section of the $N = 2$ super 1-jet bundle may be written in the canonical basis or the dynamical basis. The latter is defined by the matter equation of motion in the complete gauge fixed case. The solutions of

$$\frac{1}{2}\left(D^+D^- - D^-D^+\right)\phi(Z) = 0 \tag{8.14}$$

are just 1, $\tilde{z}$, $\tilde{\theta}^+$ and $\tilde{\theta}^-$. In an arbitrary coordinate system, the basis differentials are

$$\begin{aligned} \eta_0 dZ &= (D^+\tilde{\theta}^-)^{\frac{1}{2}}(D^-\tilde{\theta}^+)^{\frac{1}{2}}dZ && (8.15) \\ \eta_+ dZ &= \tilde{\theta}^+(D^+\tilde{\theta}^-)^{\frac{1}{2}}(D^-\tilde{\theta}^+)^{\frac{1}{2}}dZ && (8.16) \\ \eta_- dZ &= \tilde{\theta}^-(D^+\tilde{\theta}^-)^{\frac{1}{2}}(D^-\tilde{\theta}^+)^{\frac{1}{2}}dZ && (8.17) \\ \eta_1 dZ &= \tilde{z}(D^+\tilde{\theta}^-)^{\frac{1}{2}}(D^-\tilde{\theta}^+)^{\frac{1}{2}}dZ, && (8.18) \end{aligned}$$

and as with $N = 1$, the dynamical basis is defined through the prolongation of the η_i via the canonical basis. Writing the section $\mathbf{f}$ in terms of the two bases

$$\begin{aligned} \hat{\mathbf{f}} &= \mathbf{f}e_0 + (D^+\mathbf{f})e_- + (D^-\mathbf{f})e_+ + \frac{1}{2}\{D^+, D^-\}\mathbf{f}e_1 && (8.19) \\ &= \mathbf{f}_0\hat{\eta}_0 + \mathbf{f}_+\hat{\eta}_- + \mathbf{f}_-\hat{\eta}_+ + \mathbf{f}_1\hat{\eta}_1 \end{aligned}$$

and then reexpressing $\mathbf{f}$, $D^{\pm}\mathbf{f}$, $\frac{1}{2}\{D^+, D^-\}\mathbf{f}$ in terms of the $\mathbf{f}_0$, $\mathbf{f}_{\pm}$, $\mathbf{f}_1$, we find the following

$$\mathbf{f} = \eta_0\mathcal{X} \tag{8.20}$$

$$
\begin{aligned}
D^{\pm}\mathbf{f} &= D^{\pm}\eta_0\mathcal{X} + \frac{\eta_0}{D^{\mp}\tilde{\theta}^{\pm}}\left[D^{\pm}\left(\tilde{\theta}^{-}D^{\mp}\tilde{\theta}^{\pm}\right)\mathcal{Y}^{+}\right. && (8.21)\\
&\quad \left.+D^{\pm}\left(\tilde{\theta}^{+}D^{\mp}\tilde{\theta}^{\pm}\right)\mathcal{Y}^{-}\right] \\
\frac{1}{2}\{D^{+},D^{-}\}\mathbf{f} &= \partial_z\eta_0\mathcal{X} + \eta_0\left(\left[D^{+},D^{-}\right](\tilde{\theta}^{+}\tilde{\theta}^{-}) - D^{+}\tilde{\theta}^{-}D^{-}\tilde{\theta}^{+}\right)\mathcal{Z} \\
&\quad -\eta\partial_z\theta^{-}\mathcal{Y}^{+} - \eta\partial_z\theta^{+}\mathcal{Y}^{-} && (8.22)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{X} &= \mathbf{f}_0 + \mathbf{f}_{+}\tilde{\theta}^{-} + \mathbf{f}_{-}\tilde{\theta}^{+} + \mathbf{f}_1\tilde{z} && (8.23)\\
\mathcal{Y}^{\pm} &= \mathbf{f}_{\pm} + \mathbf{f}_1\tilde{\theta} && (8.24)\\
\mathcal{Z} &= \mathbf{f}_1. && (8.25)
\end{aligned}
$$

Equations 8.23-8.25 describe a flag in the $N = 2$ super 1-jet space

$$
\mathcal{F}_{(1|0)} \quad \subset \quad \mathcal{F}_{(1|2)} \quad \subset \quad \mathcal{F}_{(2|2)} \qquad (8.26)
$$

where

$$
\begin{aligned}
\mathcal{F}_{(1|0)} &= \{\mathcal{X}\} \\
\mathcal{F}_{(1|2)} &= \{\mathcal{X},\mathcal{Y}^{+},\mathcal{Y}^{-}\} \\
\mathcal{F}_{(2|2)} &= \{\mathcal{X},\mathcal{Y}^{+},\mathcal{Y}^{-},\mathcal{Z}\}.
\end{aligned}
$$

Superconformal transformations generate deformations of these flags

$$
\begin{pmatrix} 1 & -\tilde{\theta}^{-} & -\tilde{\theta}^{+} & \tilde{z} \\ 0 & 1 & 0 & \tilde{\theta}^{+} \\ 0 & 0 & 1 & \tilde{\theta}^{-} \\ 0 & 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} \mathbf{f}_0 \\ \mathbf{f}_{+} \\ \mathbf{f}_{-} \\ \mathbf{f}_1 \end{pmatrix}
=
\begin{pmatrix} \mathbf{f}_0 + \mathbf{f}_{+}\tilde{\theta}^{-} + \mathbf{f}_{-}\tilde{\theta}^{+} + \tilde{z}\mathbf{f}_1 \\ \mathbf{f}_{+} + \tilde{\theta}^{+}\mathbf{f}_1 \\ \mathbf{f}_{-} + \tilde{\theta}^{-}\mathbf{f}_1 \\ \mathbf{f}_1 \end{pmatrix}. \qquad (8.27)
$$

8.3 General case

We conjecture that for the case of arbitrary N, the superconformal transformations will induce deformations of a super flag in the N-extended 1-jet of the associated super Riemann surface. The flag should be

$$
\mathcal{F}_{(1|0)} \quad \subset \quad \mathcal{F}_{(1|N)} \quad \subset \quad \mathcal{F}_{(2|N)} \qquad (8.28)
$$

where the subspaces of the flag are

$$
\begin{aligned}
\mathcal{F}_{(1|0)} &= \{\mathcal{X}\} \\
\mathcal{F}_{(1|N)} &= \left\{\mathcal{X},\mathcal{Y}^{1},\ldots,\mathcal{Y}^{N}\right\} \\
\mathcal{F}_{(2|2)} &= \left\{\mathcal{X},\mathcal{Y}^{1},\ldots,\mathcal{Y}^{N},\mathcal{Z}\right\}
\end{aligned}
$$

and the coordinates in the jet space are

$$\mathcal{X} = \mathbf{f}_0 + \mathbf{f}_\alpha \tilde{\theta}^\alpha + \mathbf{f}_z \tilde{z} \tag{8.29}$$

$$\mathcal{Y}^\alpha = \mathbf{f}_\alpha + \mathbf{f}_z \tilde{\theta}^\alpha \tag{8.30}$$

$$\mathcal{Z} = \mathbf{f}_z. \tag{8.31}$$

The superconformal transformations generate deformations of the superflag

$$\begin{pmatrix} 1 & -\tilde{\theta}^1 & -\tilde{\theta}^2 & \cdots & -\tilde{\theta}^N & \tilde{z} \\ 0 & 1 & 0 & \cdots & 0 & \tilde{\theta}^1 \\ 0 & 0 & 1 & \ddots & \vdots & \tilde{\theta}^2 \\ \vdots & \vdots & 0 & \ddots & 0 & \vdots \\ 0 & 0 & 0 & 0 & 1 & \tilde{\theta}^N \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{f}_0 \\ \mathbf{f}_1 \\ \mathbf{f}_2 \\ \vdots \\ \mathbf{f}_N \\ \mathbf{f}_z \end{pmatrix} = \begin{pmatrix} \mathbf{f}_0 + \mathbf{f}_\alpha \tilde{\theta}^\alpha + \tilde{z}\mathbf{f}_z \\ \mathbf{f}_1 + \tilde{\theta}^1 \mathbf{f}_z \\ \mathbf{f}_2 + \tilde{\theta}^2 \mathbf{f}_z \\ \vdots \\ \mathbf{f}_N + \tilde{\theta}^N \mathbf{f}_z \\ \mathbf{f}_z \end{pmatrix}. \tag{8.32}$$

9 Conclusions and Outlook

We have presented a one dimensional constrained pseudoparticle mechanical model which may be written as an $\mathrm{Osp}(N|2M)$ phase space gauge theory. The partial gauge fixing of these models yields theories which may be interpreted as chiral sectors of two-dimensional theories of matter coupled to superconformal $\mathcal{W}$-gravity. Partial gauge fixing of the $\mathrm{Osp}(N|2)$ pseudoparticle mechanical model results in supergravity theories with $\mathrm{SO}(N)$ invariant N-extended superconformal symmetry of Bershadsky and Knizhnik. Written as a phase space gauge theory, the pseudoparticle model explains the success of the two-dimensional zero curvature approach to finding extended conformal algebras. In terms of the particle model, the zero curvature method is essentially equivalent to the fact that the compatibility of the matter equation of motion and matter gauge transformation law yields the gauge field transformation law, even after partial gauge fixing. The finite transformations of the matter and non-gauge fixed gauge fields may be obtained by integrating the $\mathrm{osp}(N|2M)$ transformations after redefinitions of gauge parameters has been performed to put transformations into "standard" form, transforming the matter and gauge fields by successive $\mathrm{Osp}(N|2M)$ transformations, and finally performing the partial gauge fixing at finite level. We have carried this procedure explicitly for the cases of Osp(2|1) and Osp(2|2), thus giving a new derivation of the $N = 1$ and $N = 2$ superschwarzian derivatives. An alternate derivation of the superschwarzian is given by writing the matter equation of motion as a superfield and demanding covariance of the equation under superconformal transformations. The component version of this derivation could be useful in the case of $\mathcal{W}$-algebras if there is some sort of "$\mathcal{W}$-field" structure analogous to superfield structure. With regard to the matter content of these theories, if one wishes to have a conformal theory where the stress tensor transforms as a quasi-primary field, then the conformal weight and spin of the matter fields occurring in these supergravity theories is pinned down due to the rigidity of the compatibility equation. Thus, there appears to be an obstruction in coupling matter with arbitrary conformal weight and spin to the untwisted pseudoparticle model as it has been presented. To arrive at twisted and topological theories, a dynamically significant deformation of the original pseudoparticle model must be made, i.e. some Lagrange multiplier gauge fields must be given dynamics. Unlike the untwisted theory, in the twisted theories one has the freedom to couple matter of arbitrary conformal weight to supergravity, due to the appearance of the arbitrary twisting parameter α_0.

The pseudoparticle model facilitates an the interpretation of the SO(N) invariant N-extended superconformal transformations as deformations of flags in the N-supersymmetrized 1-jet bundles over super Riemann surfaces. The matter conformal weight and charge dictates what power of the canonical super line bundle one should use to begin the construction. The weight and charge are constrained by the requirement that the gauge fields transform as quasi-tensors. The matter equation of motion then defines a dynamical basis in the N-supersymmetrized 1-jet which allows one to write down the flag. Superconformal transformations change the way spaces are embedded in higher dimensional spaces in the flag.

This procedure that we have given for finding the finite transformations should extend to $N \geq 3$ to obtain the finite transformations of the nonlinear SO(N) N-extended superconformal theories. There is no real modification necessary in going to $N \geq 3$, only the addition of more auxiliary fields to close the algebra. More generally, this procedure should allow one to obtain the finite superconformally extended $\mathcal{W}$-transformations. The difficulty in considering Osp($N|2M$) for $M > 1$ is that there is not a well defined notion of "standard" $\mathcal{W}$-transformations. The non-supersymmetric case has been investigated in [21].

Finally, it is natural to suspect a relation of the pseudoparticle model with integrable hierarchies of nonlinear equations such as the generalized Korteweg-de-Vries hierarchies. Another interesting question is whether the quantization of particle models of this type make sense. These questions are currently under investigation.

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