MACROSCOPIC QUANTUM THEORETIC APPROACH TO SUPERCONDUCTIVE ELECTRODYNAMICS*

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Dedication. It is a pleasure to contribute to this volume, dedicated to Maria Marinaro. I shall use this opportunity to present a rather personal approach to superconductivity theory, which I hope will appeal to Maria's eclectic tastes.

Abstract. I present a general, quantum statistical derivation of superconductive electrodynamics from the assumptions of off-diagonal long range order (ODLRO) and gauge covariance of the second kind, without reference to the microscopic mechanism responsible for the ordering. On this basis, I prove that the macroscopic wave function, specified by the ODLRO condition, enjoys the London rigidity property [Lo]; and, from this result, I derive the Meissner and Josephson effects, and the quantisation of trapped magnetic flux. I also outline a framework for the treatment of the open problem of the metastability of supercurrents.

1. Introduction. The object of this article is to present an approach to superconductive electrodynamics, leading to a derivation of the electromagnetic properties of superconductors from their order structure. This approach is based on a quantum treatment of macroscopic variables, and so is at the opposite pole from the standard microscopic, many-body theory.

In order to explain the need for such an approach, let me first recall that, even

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in the case of metallic superconductivity, the widely accepted Bardeen-Cooper-Schrieffer [BCS] theory does not provide a satisfactory electrodynamics, because it fails to meet the basic requirement of gauge covariance of the *second kind* [Sc1, Fr1]. Here, the essential point is that, although one starts with a fully gauge invariant model, given by Fröhlich's electron-phonon system [Fr2], the BCS ansatz is based on a truncated, gauge-dependent version of this model, that retains only those interactions that give rise to Cooper pairing. Attempts [An, Ri] to overcome this difficulty by taking account of the remaining interactions have led to derivations of the Meissner effect that are only *approximately* gauge invariant. Since exact gauge invariance is required for the very definition of local electric currents, this is no solution to the problem. As regards ceramic, i.e. high T_c, superconductors, the microscopic theory is still less developed and has not led to an electrodynamics.

Thus, there is a need for a need for a quantum-based gauge invariant electrodynamics of superconductors. Since, at the observational level, this electrodynamics has such sharply defined *qualitative* characteristics, the task of a corresponding quantum theory is surely to exhibit them in a precise form. It is clear that the traditional techniques of many-body theory [Pi, Th] are unsuited to this purpose, since they are designed for essentially approximative calculations rather than precise classifications. Moreover, the shortcomings of these techniques are quite radical, since, except in the very special case of exactly solvable models, they are based on approximations, which are renderd uncontrollable by the extreme microscopic instability at the root of statistical mechanics (cf. [VK].)

I shall now present a different approach to this problem [Se1, 2], that is based on the characterisation, proposed by Yang [Ya], of the order structure of superconductors. This is completely gauge covariant and circumvents the radical problems of many-body theory. To explain what is involved here, let us first note that the BCS characterisation of the metallic superconductive phase by electron pairing, first proposed by Schafroth [Sc2], has been amply substantiated by experiments on the Josephson effect [Jo] and the quantisation of trapped magnetic flux in multiply-connected superconductors [DF]. Further, it was pointed out by Yang [Ya] that this characterisation is captured by the hypothesis of *off-diagonal long range order* (ODLRO), first introduced by O. Penrose [Pe, PO] for the theory of superfluid Helium. Moreover, the ODLRO hypothesis is fulfilled not only by the BCS ansatz, but by the bipolaron model [AR, AM] of high T_c superconductors, as well as Feynman's [Fe] theory of superfluid Helium [PO]. These observations suggest an approach to superconductive electrodynamics based on the assumption of ODLRO.

This is precisely the approach I pursue. My essential objective is to relate the electromagnetic properties to the order structure of these systems in purely macroscopic quantum terms. Here, I shall formulate the theory within the standard framework of condensed matter physics, by contrast with the mathematically more abstract treatment of [Se2].

I shall organise the treatment as follows. In §2, I shall formulate the generic quantum model of a system of interacting particles of one or more species, satisfying the requirement of gauge covariance of the second kind. Here, I shall specify the condition of ODLRO.

In §3, I shall derive the Meissner effect from the assumptions of ODLRO, gauge covariance and translational invariance. The key to this is the incompatibility of ODLRO with a non-zero uniform magnetic induction (Prop. 3.1). This corresponds to a London rigidity [Lo], at the *macroscopic quantum level.*

In §4, I shall extend the above treatment to derive both the quantisation of trapped magnetic flux, in multiply-connected superconductors, and the Josephson effect, from the assumptions of ODLRO and gauge covariance.

In §5, I shall formulate, in outline, a framework for a treatment of the metastability of persistent currents, which, remarkably, remains an open problem.

In §6, I shall briefly summarise the conclusions to be drawn from this work.

2. The Model. We take the quantum model, Σ, to be an infinitely extended system of particles in a Euclidean space X : lattice systems may be formulated analogously. It will be assumed that Σ consists of a system, Σ_{el}, of electrons, and possibly of another component, Σ_i, consisting of ions or phonons. Points in X will generally be denoted by x, but sometimes by y, a or b. It will be assumed that the model enjoys the properties of gauge covariance of the second kind, and that its interactions are translationally invariant. These assumptions are satisfied by Fröhlich's [Fr2] electron-phonon model and Hubbard's [Hu] strong repulsion model, on which the theories of metallic and ceramic superconductivity, respectively, are usually based. Further, at a more fundamental level, they are also satisfied by the electron-ion model with Coulomb interactions.

The electronic part, Σ_{el}, of Σ is formulated in terms of a quantised field

$\psi = (\psi_\uparrow, \psi_\downarrow)$, satisfying the canonical anticommutation relations

$$[\psi_\alpha(x), \psi_\beta(y)^\star]_+ = \delta_{\alpha,\beta}\delta(x-y);\ [\psi_\alpha(x), \psi_\beta(y)]_+ = 0 \tag{2.1}$$

The observables of Σ_{el} are generated by the polynomials in ψ and $\psi^\star$ that are invariant under gauge transformations of the first kind, i.e., $\psi \to \psi e^{i\alpha}$, with α constant. Thus, they are generated algebraically by the monomials

$$\psi^\star(x_1)\ldots\psi^\star(x_n)\psi(x_{n+1})\ldots\psi(x_{2n}).$$

A dynamical characterisation of equilibrium states at inverse temperature β is then given by the Kubo-Martin-Schwinger (KMS) condition, which constitutes a generalisation to infinite systems of the standard Gibbsian one [HHW; Se3,4], and is given by

$$\langle Q_1(t)Q_2\rangle = \langle Q_2 Q_1(t + i\hbar\beta)\rangle \tag{2.2}$$

for all observables $Q_1,\ Q_2$, where $Q(t)$ is the Heisenberg operator representing the evolute of Q at time t.

We shall be concerned with the properties of Σ in the presence of a classical magnetic induction $B = curl A$, and, as stated above, we assume that its dynamics is covariant w.r.t. gauge transformations of the second kind, i.e.,

$$A(x) \to A(x) + \nabla\phi(x);\ \psi(x) \to \psi(x)\exp(ie\phi(x)/\hbar c) \tag{2.3}$$

where ϕ is an arbitrary function of position and $-e$ is the electronic charge. Further, the assumption of translationally invariant interactions implies the covariance of the dynamics w.r.t. space translations

$$A(x) \to A(x+a);\ \psi(x) \to \psi(x+a), \tag{2.4}$$

with a an arbitrary spatial displacement, together with a corresponding transformation for Σ_i. Specialising now to the case where the magnetic induction B is uniform, and so may be represented by the vector potential $A(x) = \frac{1}{2}(B \times x)$, and choosing $\phi(x) = -\frac{1}{2}(B \times x).a$, we have the relation $A(x) + \nabla\phi(x) \equiv A(x-a)$. Hence, by (2.3) and (2.4), the dynamics is covariant w.r.t

$$\psi(x) \to \psi(x+a)\exp\left(\frac{-ie(B \times x).a}{2\hbar c}\right);\ A(x) \to A(x) \tag{2.5}$$

together with the corresponding transformation for Σ_i. Thus, the space translation for the electronic part of Σ, in the presence of a uniform magnetic induction are given by (2.5). We term these the *regauged space translations.* It will be seen that the sinusoidal factor plays a crucial role in our derivation of the Meissner effect in §3, and, subsequently, of other electromagnetic properties of superconductors.

We define the *pair field*

$$\Psi(x_1, x_2) = \psi_{\uparrow}(x_1)\psi_{\downarrow}(x_2) \tag{2.6}$$

The property of ODLRO may then be expressed in terms of this field by the condition that

$$\lim_{|y|\to\infty}[\omega(\Psi(x_1, x_2)\Psi^{\star}(x_1' + y, x_2' + y) - \Phi(x_1, x_2)\Phi^{\star}(x_1' + y, x_2' + y)] = 0 \tag{2.8}$$

for all $x_1,\ x_2,\ x_1'$ and x_2' in X, where Φ is a classical field, that does not tend to zero at infinity, i.e., for some $x_1, x_2,\ \Phi(x_1 + y, x_2 + y)$ does not tend to zero as $|y|\to\infty$. Φ is then termed the *macroscopic wave function.*

Note that, although Ψ is not an observable, the quantity in angular brackets in (2.8) is one.

Lemma 2.1. *The ODLRO conditions define the macroscopic wave function up to a constant phase factor, i.e., if Φ_1, Φ_2 both satisfy these conditions, for the same state of Σ, then $\Phi_2 = \Phi_1\exp(i\eta)$, where η is a real-valued constant.*

Proof. Assuming that Φ_1, Φ_2 both satisfy the ODLRO conditions with respect to the same state, it follows from (2.8) that

$$\lim_{|y|\to\infty}[\Phi_1(x_1, x_2)\Phi_1^{\star}(x_1' + y, x_2' + y) - \Phi_2(x_1, x_2)\Phi_2^{\star}(x_1' + y, x_2' + y)] = 0 \tag{2.9}$$

Since this is valid for all $x_1, x_2, x_1', x_2'\in X$, we may replace x_1, x_2 here by arbitrary points x_1'', x_2'', thereby obtaining

$$\lim_{|y|\to\infty}[\Phi_1(x_1'', x_2'')\Phi_1^{\star}(x_1'+y, x_2'+y)) - \Phi_2(x_1'', x_2'')\Phi_2^{\star}(x_1'+y, x_2'+y)] = 0 \tag{2.10}$$

On multiplying (2.9) by $\Phi_2(x_1'', x_2'')$ and (2.10) by $\Phi_2(x_1, x_2)$), and then taking the difference, we see that

$$\lim_{|y|\to\infty}\Phi_1^{\star}(x_1' + y, x_2 + y)[\Phi_1(x_1, x_2)\Phi_2(x_1'', x_2'') - \Phi_1(x_1'', x_2'')\Phi_2(x_1, x_2)] = 0$$

Since, by the above definition of ODLRO, Φ does not tend to zero at infinity, it follows that the quantity in the square brackets of this last equation vanishes. Therefore, since $\Phi_{1,2}$ are non-zero, by the same stipulation,

$$\Phi_2(x_1, x_2) = c\Phi_1(x_1, x_2)\ \forall x_1, x_2\in X$$

where c is a complex-valued constant. Consequently, as Φ_1, Φ_2 both satisfy (2.8), it follows immediately that c is just a constant phase factor $\exp(i\eta)$.

3. The Meissner Effect. The essential distinction between normal diamagnetism and the Meissner effect is that the former can support a uniform, static, non-zero magnetic induction and the latter cannot. Thus, we base our derivation of the Meissner effect on considerations of the response of a state possessing ODLRO to the action of a uniform magnetic field.

Proposition 3.1. *The system cannot support uniform (non-zero) magnetic induction in translationally invariant states possessing the property of ODLRO.*

The following Corollary follows immediately from this Propostion and elementary thermodynamics.

Corollary 3.2. *Assuming that there is no translational symmetry breakdown in the equilibrium state, then either*

(1) ODLRO prevails and $B = 0$, *or*

(2) the system is normally diamagnetic and does not possess ODLRO.

Further, assuming that, in the absence of a magnetic field, the free energy density of the ODLRO phase is lower, by Δ, *than that of the normal one, the former phase will prevail, and thus the system will exhibit the Meissner effect, provided that the applied field,* H, *satisfies the condition that* $|H| < H_c := (8\pi\Delta)^{1/2}$

Comment. This result signifies that the Meissner effect arises from a rigidity of the macroscopic wave-function in the face of an applied magnetic field. This corresponds precisely to London's [Lo] idea of rigidity, transferred to the macroscopic level.

Proof of Proposition 3.1. Let $\Psi_a \equiv \gamma(a)\Psi$ be the transform of Ψ under the regauged space translation (2.5). Then, by (2.5) and (2.8), $\Psi \rightarrow \Psi_a \equiv \gamma(a)\Psi$, where

$$\Psi_a(x_1, x_2) \equiv (\gamma(a)\Psi)(x_1, x_2) = \Psi(x_1 + a, x_2 + a)\exp(\frac{-ie(B\times(x_1 + x_2)).a}{2\hbar c}) \quad (3.1)$$

It follows immediately from this formula that the transformations γ do not form an abelian group and, in fact, that

$$\gamma(-a - b)[\gamma(a)\gamma(b) - \gamma(b)\gamma(a)] = 2i\sin(\frac{e(B\times a).b}{\hbar c}) \quad (3.2)$$

We now note that, by the translational invariance of the equilibrium state,

$$\langle\Psi_a(x_1,x_2)\Psi_a^\star(x_1',x_2')\rangle\equiv\langle\Psi(x_1,x_2)\Psi^\star(x_1',x_2')\rangle \tag{3.3}$$

Further, it follows from (3.1), (3.3) and the ODLRO condition (2.8) that

$$\lim_{y\to\infty}[\langle\Psi(x_1,x_2)\Psi^\star(x_1'+y,x_2'+y)\rangle-\Phi_a(x_1,x_2)\Phi_a^\star(x_1+y,x_2'+y)]=0 \tag{3.4}$$

where

$$\Phi_a\equiv\gamma(a)\Phi \tag{3.5}$$

with γ defined as in (3.1). Since equations (2.8) and (3.4) imply that Φ_a, as well as Φ, serves as a macroscopic wave function for the equilibrium state, it follows from Lemma 2.1 that $\Phi_a=c(a)\Phi$, where $c(a)$ is a complex number of unit modulus. Since this is valid for all spatial displacements a, it follows, using (3.5), that

$$\gamma(a)\Phi=c(a)\Phi;\ \ \gamma(b)\Phi=c(b)\Phi;\ \ \forall a,b\in X$$

Consequently, as $c(a),\ c(b)$ are intercommuting complex numbers,

$$\gamma(-a-b)[\gamma(a)\gamma(b)-\gamma(b)\gamma(a)]\Phi=0$$

and therefore, by (3.2),

$$\sin(\frac{eB.(a\times b)}{\hbar c})\Phi=0\ \forall a,b\in X$$

from which it follows that *either* $B=0$ *or* $\Phi=0$, which is the required result.

4. Results on Flux Quantisation and Josephson Effect. The above derivation of the Meissner effect was achieved by a very general argument, centred on the constraints imposed by translational symmetry and gauge covariance on the macroscopic wave function Φ. The phenomena of magnetic flux quantisation and the Josephson effect have also been derived form similar general arguments [Se2], whose results we shall now summarise.

4.1. Flux Quantisation. Suppose now that a cylindrical region, T, is removed from the body of Σ. Then the matter is confined to the complementary region $X'=X\backslash T$, and T forms a tunnel through it. Then, any magnetic field in T is excluded, by Meissner effect, from Σ, except for a 'surface region', where the induced currents screen out the field. The trapped magnetic flux, $\mathcal{F}$, is then given by

$$\mathcal{F} = \int_{\Pi} B.dS \tag{4.1}$$

where B is the space-dependent magnetic induction, and the integral is taken over a plane, Π, perpendicular to T. We shall assume that

(1) the cross-section of T is circular,* and the field B is directed parallel to the generators of this tunnel; and

(2) the dynamics of Σ is covariant, and its equilibrium states invariant, with respect to both rotations about the central axis of T and time-reversals.

Proposition 4.1. *Under the above assumptions, together with that of ODLRO, the trapped magnetic flux is quantised in units of $hc/2e$, i.e.,*

$$\mathcal{F} = \nu hc/2e \tag{4.2}$$

where ν is an integer. Furthermore, states with different values of this quantum number are globally different from one another, and consequently there is a superselection rule, whereby transitions between them cannot be effected by purely local operations

The proof [Se2] of this result is centred on the single- valuedness of the macroscopic wave function, Φ, and the factor 2 in the denominator of (4.2) arises from the electron pairing represented by this function.

4.2. The Josephson Effect. Let Σ_c be a system composed of two superconductors, Σ_1 and Σ_2, separated by an insulating film, $\mathcal{I}$. We assume that

(1) the state of Σ_c possesses ODLRO, and the macroscopic wave functions in Σ_1 and Σ_2 tend to spatially constant values, $\Phi_{1,\infty}$, $\Phi_{2,\infty}$, 'infinitely far' from the insulating film, as in the case of isolated systems with translational symmetry; and

(2) the difference, η, between the phases of $\Phi_{1,\infty}$ and $\Phi_{2,\infty}$ takes a value η_0 at equilibrium, and otherwise the $\eta - \eta_0$ is a driving force, that generates an $\eta-$dependent tunnelling current across the insulating film.

* This assumption is convenient, but inessential, for the derivation of Prop. 4.1 [Se2].

Proposition 4.2. *Suppose that Σ_1 and Σ_2 are kept at electric potentials that differ by ΔV. Then, under the above assumptions, the frequency of the tunnelling current is $2e\Delta V/h$.*

This Proposition is that of the Josephson effect. Its proof [Se2] follows from a simple general argument, based on the single- valuedness of the macroscopic wave-function and of the tunnelling current.

5. Problem of the Metastability of Supercurrents. The phenomenon of superconductivity itself, i.e. of the persistence of electric currents in the absence of any applied field, is intimately connected to the Meissner effect [Lo]. For these supercurrents are the source of a magnetic field, which they themselves screen, by Meissner effect, from the interior of the system. Thus, the phenomenological fact is that the currents and magnetic field stabilise one another. Further, it is clear that the supercurrent-carrying states must be *metastable*, rather than absolutely stable, since both the currents and the magnetic field carry positive energy.

Our objective now is to sketch a framework for a treatment of the quantum mechanical basis of the metastability of supercurrent- carrying states. Since the associated magnetic field, B, is essential to these states, we shall base our considerations on the model $\tilde{\Sigma}$, comprising Σ and the field B. A state of $\tilde{\Sigma}$ then corresponds to a pair (ω, B), where ω is a state of Σ,* and B is the magnetic induction vector field, which we shall treat as classical.

The body of Σ must be multiply-connected, e.g. with the topology of a torus, in order to be able to support supercurrents. For simplicity, we shall confine our present considerations to the situation described in §4.1, where a tunnel T is carved through the body of a superconductor, Σ, which thus occupies the complementary region $X' = X \backslash T$.

We express the constitutive equations relating the position dependent current density, J, and polarisation, i.e. spin density, M, to the magnetic induction, B, in the form

$$J(x) = \mathcal{J}_\nu(B; x) \tag{5.1}$$

* Here, we use the term 'state' to in the generalised sense, where it signifies an expectation functional on the observables. This serves to extend the convential meaning of pure or mixed states to infinite systems [Se4].

and

$$M(x) = \mathcal{M}_\nu(B;x) \tag{5.2}$$

where the subscript ν indicates a dependence on this macroscopic quantum number, which governs the flux quantisation in Prop. 4.1. The magnetic field, H, is given, in appropriate units, by the standard formula

$$H = B - 4\pi M \tag{5.3}$$

Further, assuming that there are no external sources, J and M are related by the Maxwell equation

$$curl H = 4\pi J \tag{5.4}$$

We assume that the equations (5.1)-(5.4) yield a unique solution, B_ν, for B, subject to the standard conditions for $B,\ H,\ J$ on the boundary of T, together with the vanishing of B infinitely far from this cylinder (Meissner effect!). We then denote by ω_ν the equilibrium state of Σ, characterised by the KMS condition (2.2), when the magnetic induction is fixed at B_ν. We shall assume here that $\nu\neq 0$, which implies that the induction and current are non-zero. This current is therefore a supercurrent, provided that the state $\tilde{\omega}_\nu \equiv (\omega_\nu, B_\nu)$ of $\tilde{\Sigma}$ is stabilised.

Our aim now is to investigate the question of the metastability of this state. Here, we take the view [Se3] that metastability corresponds to stability against localised perturbations, as distinct from the global stability of true equilibrium states. Thus, we define the localised perturbations of B_ν to be the set $\mathcal{B}$ of all divergence-free vector fields b, with compact support.* We then define $\omega_\nu^{(b)}$ to be the equilibrium state of Σ obtained by perturbing B_ν by $b(\in\mathcal{B})$, and $\phi_\nu(b),\ \tilde{\phi}_\nu(b)$ to be the corresponding increments in the Gibbs free energies of Σ and $\tilde{\Sigma}$, respectively. Thus,

$$\tilde{\phi}_\nu(b) = \phi_\nu(b) + \frac{1}{4\pi}\int_X (b.B_\nu + \frac{1}{2}B_\nu^2)dx \tag{5.5}$$

the last term being the contribution due to the magnetic field energy.

The key question we shall now address is that of whether $\tilde{\phi}_\nu$ is minimised at $b = 0$. For, *if* it is, then the free energy of $\tilde{\Sigma}$ cannot be lowered by any localised modification, either of the state of Σ or of the magnetic induction. This is evident from the fact that, for given B, the free energy of state of Σ minimises its free

* A function has compact support if it vanishes outside some bounded spatial region.

energy. Thus, since there are no viable processes that raise the free energy of $\tilde{\Sigma}$, it follows that, *if* $\tilde{\phi}_\nu$ is absolutely minimised at $b=0$, then $\tilde{\omega}_\nu$ is metastable, in the sense of being infinitely long-lived in the face of localised perturbations [Se3]. On the other hand, even if $\tilde{\phi}_\nu$ is not minimised there, the state $\tilde{\omega}_\nu$ could still be sufficiently long-lived to be sensibly metastable, provided that a sufficiently large activation energy is required for the system to escape from this state. We shall discuss this after Proposition 5.1.

To study the relevant properties of $\tilde{\phi}_\nu$, we note that, for arbitrary $b(\in\mathcal{B})$ and real-valued λ, the perturbation of B_ν by λb leads to an interaction Hamiltonian

$$-\lambda\int(m(x).b(x)+j_\nu(x).a(x))dx \ + \ O(\lambda^2),$$

where a is a vector potential for b and $m,\ j_\nu$ are the operators representing the polarisation and current density, respectively, for $B=B_\nu$. Hence, it follows from our definitions of $\mathcal{J}_\nu,\ \mathcal{M}_\nu,\ \phi_\nu$ that

$$\frac{\partial}{\partial\lambda}\phi_\nu(\lambda b)_{|\lambda=0}=-\int[\mathcal{M}_\nu(B_\nu,x).b(x)+\mathcal{J}_\nu(B_\nu,x).a(x)]dx$$

and therefore, by (5.5),

$$\frac{\partial}{\partial\lambda}\tilde{\phi}_\nu(\lambda b)_{|\lambda=0}=\int[(4\pi)^{-1}B_\nu(x).b(x)-\mathcal{M}_\nu(B_\nu,x).b(x)-\mathcal{J}_\nu(B_\nu,x).a(x)]dx \quad (5.6)$$

Since $b=curla$ and $\int a.curlc\equiv\int c.curla$, it follows from (5.1)-(5.4) that the r.h.s. of (5.6) vanishes. Hence, we have the following result.

Proposition 5.1. *Under the above assumptions, the incremental free energy function $\tilde{\phi}_\nu$ is stationary at $b=0$, i.e.*

$$\frac{\partial}{\partial\lambda}\tilde{\phi}_\nu(\lambda b)_{|\lambda=0}=0\ \forall b\in\mathcal{B}$$

Discussion. In view of this result, one sees from the considerations of [Se3] that the only two ways in which the metastability of the supercurrent-carrying state $\tilde{\omega}_\nu$ can arise are the following.

(1) The stationary value of $\tilde{\phi}_\nu$, at $b=0$, corresponds to an absolute minimum, in which case $\tilde{\omega}_\nu$ is metastable, with infinite lifetime.

(2) $\tilde{\phi}_\nu$ is not absolutely minimised at $b=0$, but the escape of the system from the state $\tilde{\omega}_\nu$ requires an activation energy much larger than kT, and consequently

this state has a sufficiently long lifetime to be sensibly metastable (cf. [CCO] for a treatment of this kind of metastability in Ising systems.)

Thus, the basic problem concerning the metastability of supercurrents, i.e. of *superconductivity*, is the following.

Problem. *Which of the above alternatives, (1) and (2), prevails as the basis of the persistence of supercurrents?*

Conjecture. We would conjecture that it is the second one, for reasons similar to those of [Se3], that ruled out the alternative corresponding to (1) in the case of superfluidity in the absence of any magnetic field.

6. Concluding Remarks. The theory presented here provides a general connection between the order structure and the electrodynamics of superconductors. The principal open problem, within the terms of this theory, is that of the metastability of supercurrents, posed at the end of the previous Section. In my view, this remains an outstanding problem of condensed matter physics, even in the case of metallic superconductors.

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