QUANTUM DYNAMICS OF TRANSITION PROCESSES IN THE INTERACTING SYSTEMS

Denis V. Juriev

Abstract. The dynamical inverse problem of representation theory (the Wigner problem) [1] is solved for a certain class of interactions of Hamiltonian systems. The constructed quantum dynamics describes transition processes and stationary regimes on an equal footing. Among other numerous applications the results may be useful for the description of the simultaneous quantum dynamics of the open and closed string fields in the unified string field theory with their noncanonical coupling.

The aim of this short note is to describe the quantum picture for transition processes in a certain class of pairs of interacting systems.

Theorem 1. *Let* (V_1, V_2) *be an arbitrary isotopic pair [2],* $\mathrm{Der}(V_1, V_2)$ *be the Lie algebra of its derivations, and* L *be an arbitrary polynomial mapping from* $V_1 \oplus V_2$ *into* $\mathrm{Der}(V_1, V_2)$ *(a nonlinear sectional operator – cf.[3]), then the equations*

$$\begin{cases} \dot{X}_t = L(X_t, A_t)^{(1)} X_t \\ \dot{A}_t = L(X_t, A_t)^{(2)} A_t \end{cases}$$

where $X_t \in V_1$, $A_t \in V_2$, $(L(X_t, A_t)^{(1)}, L(X_t, A_t)^{(2)}) \in \mathrm{Der}(V_1, V_2)$, *preserves the isocommutation relations.*

Corollary 1. *Let* (V_1, V_2) *be an arbitrary isotopic pair,* T *be its representation in the space* U *[2], i.e. a homomorphism of the isotopic pair* (V_1, V_2) *into the operator isotopic pair* $(\mathrm{End}(U), \mathrm{End}(U))$. *Let* L *be an arbitrary polynomial mapping from* $\mathrm{End}(U) \oplus \mathrm{End}(U)$ *into* $\mathrm{Der}(\mathrm{End}(U), \mathrm{End}(U))$, *then the equations*

$$\begin{cases} \dot{T}_t(A) = L(T_t(X), T_t(A))^{(1)} T_t(A) \\ \dot{T}_t(X) = L(T_t(X), T_t(A))^{(2)} T_t(X) \end{cases}$$

where $X \in V_1$, $A \in V_2$, *define a family* T_t $(T_0 = T)$ *of representations of the isotopic pair* (V_1, V_2) *in the space* U. *It means that the operators* $\hat{X}_t = T_t(X)$ *and* $\hat{A}_t = T_t(A)$ *obey the initial isocommutation relations.*

Remark 1. An operator isotopic pair in Corollary 1 may be changed to any isotopic pair (W_1, W_2). The representation T should be changed to a homomorphism π from (V_1, V_2) to (W_1, W_2). The most natural candidates for (W_1, W_2) are geometric isotopic pairs or asymmetric isotopic pairs $(S^2(U), \Lambda^2(U^*))$.

Theorem 2. *An arbitrary derivation of the operator isotopic pair* $(\mathrm{End}(U), \mathrm{End}(U))$ *has the form*

$$\begin{aligned} LX =& [H, X] + S \circ A \\ LA =& [H, A] - S \circ X \end{aligned}$$

where $H, S \in \mathrm{End}(H)$, $[X, Y] = XY - YX$, $X \circ Y = \frac{1}{2}(XY + YX)$. *So the Lie algebra* $\mathrm{Der}(\mathrm{End}(U), \mathrm{End}(U))$ *is isomorphic to* $\mathfrak{gl}(n) \oplus \mathfrak{gl}(n)$ $(n = \dim U)$.

Corollary 2. *Let* (V_1, V_2) *be an arbitrary isotopic pair, T be its representation in the space U, and* $\hat{H}$, $\hat{S}$ *be polynomial expressions of operators* $\hat{X}$, $\hat{A}$, *then the dynamics*

$$\begin{cases} \frac{\partial}{\partial t}\hat{X}_t =& [\hat{H}(\hat{X}_t, \hat{A}_t), \hat{X}_t] + \hat{S}(\hat{X}_t, \hat{A}_t) \circ \hat{X}_t \\ \frac{\partial}{\partial t}\hat{A}_t =& [\hat{H}(\hat{X}_t, \hat{A}_t), \hat{A}_t] - \hat{S}(\hat{X}_t, \hat{A}_t) \circ \hat{A}_t \end{cases}$$

(the coupled Manakov dynamics, cf.[4]) preserves the initial isocommutation relations between operators $\hat{X}$ *and* $\hat{A}$.

Remark 2. The coupled Manakov dynamics may be written in the alternative form

$$\begin{cases} \frac{\partial}{\partial t}\hat{X}_t =& \hat{M}(\hat{X}_t, \hat{A}_t)\hat{X}_t - \hat{X}_t\hat{N}(\hat{X}_t, \hat{A}_t) \\ \frac{\partial}{\partial t}\hat{A}_t =& \hat{N}(\hat{X}_t, \hat{A}_t)\hat{A}_t - \hat{A}_t\hat{M}(\hat{X}_t, \hat{A}_t) \end{cases}$$

where $\hat{M} = \hat{H} + \hat{S}$ and $\hat{N} = \hat{H} - \hat{S}$.

Remark 3. The equations of Corollary 2 are the quantum counterparts of classical dynamical equations for two systems with potential Hamiltonian, magnetic–type (gyroscopic) and Rayleigh–Lienart type nonHamiltonian interactions. The classical equations describe transition processes as well as stationary regimes in pairs of the interacting systems with auto–oscillations. It is remarkable that both cases are described on an equal footing in the quantum picture.

Remark 4. Quantum dynamical equations of the paper [5] is a particular case of the equations of Corollary 2.

Remark 5. All results allow a supergeneralization (cf.[2]).

Remark 6. The results may be useful for the description of the simultaneous quantum dynamics of closed and open string fields in the unified string field theory with their *noncanonical coupling*. It seems that the algebraic structure of the quantum string field theory [6] supports such description.

References

[1] Wigner E., Do the equations of motion determine the quantum mechanical commutation relations? Phys. Rev. 77 (1950), 711-712;
Juriev D., Dynamical inverse problem of representation theory and the noncommutative geometry. Report RCMPI-95/03 (1995) [e-print version (SISSA Electronic Archive on Funct. Anal.): *funct-an/9507001* (1995)].

[2] Juriev D., Topics in isotopic pairs and their representations. Theor. Math. Phys. 105(1) (1995), 18-28 [e-print version (Texas Univ. Electronic Archive on Math. Phys.): *mp_arc/94-267* (1994)]; Topics in isotopic pairs and their representations. II. A general supercase. Report RCMPI-95/06 (1995) [e-print version (Duke Univ. Electronic Archive on Quant. Alg.): *q-alg/9511012* (1995)].

[3] Trofimov V.V., Fomenko A.T., Algebra and geometry of integrable Hamiltonian dynamical systems. Moscow, 1995.

[4] Manakov S.V., The inverse scattering problem method and two–dimensional evolution equations. Russian Math. Surveys 31(5) (1976), 245-246.

[5] Juriev D., Classical and quantum dynamics of noncanonically coupled oscillators and Lie superalgebras. Russian J. Math. Phys., to appear [e-print version (SISSA Electronic Archive on Funct. Anal.): *funct-an/9409003* (1994)].

[6] Juriev D., Infinite dimensional geometry and quantum field theory of strings. II. Infinite dimensional noncommutative geometry of a self–interacting string field. Russian J. Math. Phys. 4 (1996), to appear [e-print version (LANL Electronic Archive on Theor. High Energy Phys.): *hep-th/9403148* (1994)].

Thalassa Aitheria, Ltd.
Res Center for Math Phys and Informatics,
Miklukho-Maklaya 20-180 Moscow 117437 Russia
E-mail address: denis@juriev.msk.ru