Preprint UGVA-DPT 05-927/96

Quantum Theory without Quantification*

C. Piron

Département de Physique Théorique, 24 quai Ernest-Ansermet, CH-1211 Genève 4

Abstract. After having explained Samuel Clarke's conception of the new philosophy of physical reality, we will treat the electron field in this context as a field modifying the void. From this we will be able to derive the so-called quantum rules just from Noether's theorem on conserved currents. Thus quantum theory appears as a kind of nonlocal field theory, in fact a new theory.

1 Introduction

In spite of much remarkable progress, the fashionable physicist's window on world reality has not changed during the past century. It remains always the same, the philosophical view of Descartes and Leibnitz: the reality of the universe is nothing else than a "nothing", the void – just a recipient filled with little particles and other kinds of ether [Le Sage 1818]. In spite of the official claim that obviously fields and quantum particles have replaced such primitive and outmoded concepts, in fact most physicists continue to think in the same terms. They imagine gravitation and electromagnetism as deformations of some substantial ether with a lot of vibrations and corresponding waves. Quantum phenomena reduce to a manifestation of stochastic motions and path averages of little particles [Piron 1995].

To get out of this rut and refuse to allow paradoxes such as infinities and inconsistencies which bog down theoretical physics, we must absolutely forsake this inadequate

* For the Hepp-Hunziker volume of Helvetica Physica Acta

received view and accept as fundamental the views of Newton and Clarke. First of all, we must admit the separate existence of space and time. As Clarke says [1866], the void space is not an attribute without subject but a space without bodies. In other words the void exists in of itself: after all it has properties, it has place, it has three dimensions, and it is Euclidean (or almost Euclidean). At first sight, it may seem that here we run into the difficulty of how to verify such properties, since if we introduce some apparatus then we no longer have the void. Such apparent difficulties have been solved by Dirk Aerts by his formulation of the notion of element of reality and of how the notion of definite experimental project gives a criterion to check the existence of such an element [Aerts 1982]. By experimental project, Aerts means an experiment that one can very well eventually perform on the system, and where the positive result has been defined in advance. Following Einstein and Aerts, we will say that the system has an element of reality, or an actual property, if in the event of the corresponding experiment the positive result would be certainly obtained. As we can see, an element of reality is an actual property of the system itself which exists even in the absence of any apparatus.

The void in a vessel has a volume of one litre if one could exactly fill it with one litre of water if one decided to check its volume. Of course the water-filled vessel is not a vessel filled with the void. When we claim that the void here is Euclidean, we claim the existence of an element of reality, since if one were to construct a triangle here with three solid rods then certainly the sum of the angles would be found to be 180° . The void space itself has such a Euclidean property in the absence of any rod. As a third example, we claim that "the void space at this moment has here an electric field" means that if one decided to place here an electric charge, the charge would be certainly be accelerated. Such a field is an element of reality of the void space in the absence of the test charge: when one makes the experiment to verify the existence of the field, of course one completely destroys the situation and in fact that field in the presence of the charge is even not defined.

Changing one's mind and accepting the existence of void space for itself is not enough however, one must also accept time and the flow of time also as having a separate existence in themselves. By nature, space and time are completely different, each possible place in space is actual in this moment, but for time only the present is actual – the future can be partially chosen and will become actual, whereas the past which has been cannot be changed in any manner whatsoever. The arrow of time exists. This can be checked with the following simple experiment. Let a ball bounce up and down and predict in the middle whether it will arrive at the bottom or the top (as you can see this is a symmetric situation). One can affirm arrival at the top or the bottom after the fact but not before, since in the latter case you could always stop the motion of the ball with your hand. This translates causality and exhibits the arrow of time.

To recapitulate, to describe the world as it really is we must introduce, from the very beginning, both the void space and the time, and so consider particles or other objects as manifestations of space. In this spirit, a particle is considered not as a manifestation of some substance existing alone and by itself, but as some manifest property of the void space intrinsically connected with its surroundings. In this context, the description of a particle in the classical approximation cannot be given just by the position of some hypothetical object, but must be at least completed by specifying the state of the surroundings, that is, the momentum. This is the physical explanation of the 7-dimensional space in classical mechanics. A charge (say an electron) is here in the void space surrounded by its field (the electric displacement $\vec{D}(x)$ in the electron case). Such a field reacts with the void generating another field, a field of force (the electric field $\vec{E}(x)$) which can act on other particles. As we see, even in the vacuum such an action is never direct: it is a major physical error to identify $\vec{E}(x)$ and $\vec{D}(x)$ even in the vacuum.

The fundamental fact that some objects called particles are entities (that is cannot be subdivided) and are not localised at a point but occupy place, in other words are not local (a very reasonable hypothesis in our window), is the fundamental fact which explains and justifies the rules of quantum physics. In the following we will present a model of the electron field as an example to illustrate this new philosophy. We will take from the very beginning the point of view explained just above with the goal of justifying the quantum rules just from such a notion of field.

2 Construction of coordinates for space and time

Before introducing the notion of particle, we must give a mathematical model for the void space and the time themselves. We have already treated such a problem in Helvetica Physica Acta [Barut *et al.* 1993]. Here we will not repeat such a construction, but will explain in more detail its physical origin. We build spatial coordinates from a chosen inertial reference frame, which is nothing else than a solid object sufficiently large to not be perturbed by small fluctuations of its surroundings: for example, one could take the "earth" freely falling onto the sun (just perturbed a little by the moon), but a hypothetical earth which does not rotate. Having in this way constructed $\mathbf{R}^3$, a model for space, we add a fourth coordinate, the time t , given by a clock at rest in the chosen reference frame, thereby obtaining $\mathbf{R}^4$, a model for space and time. According to Einstein's principle of relativity, any such $\mathbf{R}^4$ is as good as any other $\mathbf{R}'^4$ obtained using another inertial reference frame. The coordinates in $\mathbf{R}^4$ and $\mathbf{R}'^4$ are related by the usual Lorentz transformation, however to interpret such a transformation we must come back from $\mathbf{R}^4$ and $\mathbf{R}'^4$ to the sole physical space and the physical time with its sole actual instant [Piron 1990].

3 The electron as a field

As we have said in the introduction, fields and particles manifest properties of the existing void during the flow of time. Having built a model $\mathbf{R}^4$ for space and time, physical fields and particles will be described by mathematical fields on $\mathbf{R}^4$, for instance spinor fields $\phi(x)$ or covector fields $A_\mu(x)$. In our example, the electron field will be of such kind, in fact a four component complex spinor field $\Psi(x)$ (and $\Psi^\dagger(x)$). Such a field is of course in interaction with other fields such as the electromagnetic and gravitational fields among others which, for simplicity, will be considered as given independently of the electron field Ψ itself. For this very reason we will call them exterior fields. As we have developed elsewhere [Piron and Moore 1995], an adequate formalism here is the Cartan formalism, where one introduces a 4-form on a mathematical space Σ , here $\mathbf{R}^{12}$, which must reflect all possible states of our electron field model.

More precisely, the space Σ is built with the four coordinates x^μ of a space and time model, together with the four complex components (eight real numbers) of the column matrix Ψ which describes the electron field. The equations for the electron field Ψ (sometimes called improperly equations of "motion" or equations of "propagation") are determined by the Cartan principle, which affirms that $s^*(i_X\, d\omega) = 0$ for such a field $s : \mathbf{R}^4 \to \Sigma$, where X runs over all tangent vector fields on Σ. As we will see, the model is then completely determined by the chosen 4-form ω:

$$\omega = -eA_\mu(x)\Psi^\dagger\alpha^\mu\Psi\,\eta - \mathrm{i}\hbar\Psi^\dagger\alpha^\mu\, d\Psi \wedge \eta_\mu + m\Psi^\dagger\beta\Psi\,\eta$$

where $A_\mu(x)$ is the (given) electromagnetic field, and for notational convenience we have introduced the usual odd forms

$$\begin{aligned}
\eta &= \tfrac{1}{24}\,\varepsilon_{\mu\nu\rho\lambda}\, dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\lambda\,,\\
\eta_\mu &= \tfrac{1}{6}\,\varepsilon_{\mu\nu\rho\lambda}\, dx^\nu \wedge dx^\rho \wedge dx^\lambda\,,\\
\eta_{\mu\nu} &= \tfrac{1}{2}\,\varepsilon_{\mu\nu\rho\lambda}\, dx^\rho \wedge dx^\lambda\,.
\end{aligned}$$

The α^μ and β are five $4{\times}4$ matrices which express the dynamical covariance of the theory. In the Lorentz case, according to Dirac we choose

$$\alpha^0 = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}, \qquad \alpha^i = c\begin{bmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{bmatrix}, \qquad \beta = c^2\begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix},$$

but in the Galilean approximation (the usual Schrödinger case) according to Levy-Leblond we will choose

$$\alpha^0 = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad \alpha^i = \begin{bmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{bmatrix}, \qquad \beta = 2\begin{bmatrix} 0 & 0 \\ 0 & -I \end{bmatrix}.$$

As we see, the 4-form ω contains three terms, each one with its own constant prefactor (the electric charge e, the Planck constant $\hbar$, the electron mass m) which is clearly a little redundant. Again, we insist that all fields are fields of properties of the existing void.

Let us first derive the equation for the field Ψ. We have

$$d\omega = -eA_\mu(x)\, d\big(\Psi^\dagger\alpha^\mu\Psi\big) \wedge \eta - \mathrm{i}\hbar\, d\Psi^\dagger \wedge \alpha^\mu\, d\Psi \wedge \eta_\mu + m\, d(\Psi^\dagger\beta\Psi) \wedge \eta\,.$$

Choosing the tangent vector field $X = \hat{e}_{\Psi^\dagger}$ we obtain

$$\begin{aligned}
s^*\big(i_{\hat{e}_{\Psi^\dagger}} d\omega\big) &= s^*\big(- eA_\mu(x)\alpha^\mu\Psi\,\eta - \mathrm{i}\hbar\alpha^\mu\, d\Psi \wedge \eta_\mu + m\beta\Psi\,\eta\big)\\
&= \big(- eA_\mu(x)\alpha^\mu\Psi(x) - \mathrm{i}\hbar\alpha^\mu\partial_\mu\Psi(x) + m\beta\Psi(x)\big)s^*(\eta)\\
&= 0\,.
\end{aligned}$$

Since $s^*(\eta) \neq 0$ we then have that

$$\Big[\alpha^\mu\big(- \mathrm{i}\hbar\partial_\mu - eA_\mu(x)\big) + m\beta\Big]\Psi(x) = 0\,.$$

In the Lorentz case this is exactly the Dirac equation. In the Galilean approximation, we find upon rewriting $\Psi = \begin{bmatrix} \phi \\ \chi \end{bmatrix}$

$$\begin{aligned} \mathrm{i}\hbar\partial_0\phi(x) &= \sigma^j\big(- \mathrm{i}\hbar\partial_j - eA_j(x)\big)\chi(x) - eA_0(x)\phi(x)\,, \\ 0 &= \sigma^j\big(- \mathrm{i}\hbar\partial_j - eA_j(x)\big)\phi(x) - 2m\chi(x)\,. \end{aligned}$$

Substituting for $\chi(x)$ we then have

$$\mathrm{i}\hbar\partial_0\phi(x) = \Big[\tfrac{1}{2m} g^{jk}\big(- \mathrm{i}\hbar\partial_j - eA_j(x)\big)\big(- \mathrm{i}\hbar\partial_k - eA_k(x)\big) + \tfrac{\hbar e}{2m}\sigma_i B^i(x) - eA_0(x)\Big]\phi(x)\,,$$

which is exactly the two component (Pauli-)Schrödinger equation with the good factor $\frac{\hbar e}{2m}$ coupling the spin σ^i to the magnetic field $B^i(x) = \partial_j A_k(x) - \partial_k A_j(x)$.

4 Noether's theorem and the quantum interpretation

One success of the Cartan formalism is the derivation of the Noether theorem it affords. Consider a one parameter group acting on the state space Σ . Suppose that this group is generated by a vector field on Σ :

$$g_\lambda^* : r \mapsto r + \lambda Y\,,$$

where here r is a point in Σ , not to be confused with the x in $\mathbf{R}^4$, and g_λ is the action of the germ of the group. Now suppose that the 4-form ω is invariant under the action of g_λ so that $g_\lambda^*\omega = \omega$. Then as is well known

$$L_Y\omega = i_Y d\omega + d i_Y\omega = 0\,,$$

where L_Y is the Lie derivative by Y .

The Noether theorem affirms that the 3-form $i_Y\omega$ is conserved on the solution s :

$$ds^*(i_Y\omega) = 0\,.$$

The proof is straightforward, since

$$ds^*(i_Y\omega) = s^*(d i_Y\omega) = s^*(-i_Y d\omega) = 0\,.$$

The first equality is trivial, the second translates the invariance of ω , and the third is just the field equation.

To interpret such a result mathematically, we write the 3-form $s^*(i_Y\omega)$ as

$$s^*(i_Y\omega) = J^\mu(x)\,\eta_\mu\,.$$

It is then easy to recognise that the current $J^\mu(x)$ is conserved, since the Noether theorem gives

$$\partial_\mu J^\mu(x) = 0\,.$$

To remove one difficulty of the conventional (outmoded) field theory, we want to emphasise that we have derived the conserved currents from the Noether theorem without requiring some physical symmetry of the field equations, using merely some formal invariance of our 4-form. This is particularly transparent in the next example, where only a gauge invariance of the *first* kind is invoked. Indeed, it is easy to check that our ω is invariant under the transformations

$$\Psi^\dagger \mapsto \Psi^\dagger \mathrm{e}^{-\mathrm{i}\lambda} \qquad \text{and} \qquad \Psi \mapsto \mathrm{e}^{\mathrm{i}\lambda}\Psi\,,$$

which are generated by the tangent vector field

$$Y = -\mathrm{i}\Psi^\dagger \hat{e}_{\Psi^\dagger} + \mathrm{i}\hat{e}_\Psi \Psi\,.$$

The Noether theorem then gives the following conserved current

$$s^*(i_Y\omega) = \hbar\,\Psi^\dagger(x)\alpha^\mu\Psi(x)\,\eta_\mu\,.$$

In the Lorentz case, this is nothing else than the conservation of the scalar product

$$\int_{\mathbf{R}^3} \Psi^\dagger(x)\alpha^0\Psi(x)\,\eta_0 = \int_{\mathbf{R}^3} \Psi^\dagger(x)\Psi(x)\,\mathrm{d}V\,,$$

which is exactly the scalar product used by Dirac to solve the one body hydrogen atom problem. We must remark that with the usual argument this supposes that the integral exists and that $\psi(x)$ tends to 0 at infinity. On the other hand, since the norm is invariant the linear field equation can be proved to induce a unitary transformation via the so called Wigner theorem.

In the case of the Galilean approximation, the corresponding conserved scalar product is

$$\int_{\mathbf{R}^3} \Psi^\dagger(x)\alpha^0\Psi(x)\,\eta_0 = \int_{\mathbf{R}^3} \phi^\dagger(x)\phi(x)\,\mathrm{d}V\,.$$

Here only the two first components play a role, exactly justifying the "prescription of quantum mechanics" and the use of the "two component Schrödinger wave equation for spin $\frac{1}{2}$".

As the reader can remark, to obtain quantum theory we have utilised neither "quantum prescriptions" nor the "correspondance principle", but just Noether's theorem applied to field in vacuum space (sic). We can also justify in the same way other "quantum rules", for instance the momentum operator. In the free case ($A_\mu(x) = 0$), which we will consider for the rest of this section, our 4-form ω is invariant by the action of the passive space translations

$$x^j \mapsto x^j + \lambda h^j\,,$$

where the h^j are three numbers, the generators of the translation in the inertial reference frame defining $\mathbf{R}^3$. If the translation is just in the direction $\hat{e}_j$, one of the unit vectors of $\mathbf{R}^3$, we can simply write

$$x^j \mapsto x^j + \lambda\,,$$

and the generator in the state Σ is just the vector field $\hat{e}_j$. In such a case, the conserved current given by Noether's theorem is

$$\begin{aligned} s^*(i_{\hat{e}_j}\omega) &= s^*\big(\mathrm{i}\hbar\Psi^\dagger\alpha^\mu d\Psi\wedge\eta_{\mu j} + m\Psi^\dagger\beta\Psi\,\eta_j\big) \\ &= \mathrm{i}\hbar\Psi^\dagger(x)\alpha^\mu\partial_j\Psi(x)\,\eta_\mu - \mathrm{i}\hbar\Psi^\dagger(x)\alpha^\mu\partial_\mu\Psi(x)\,\eta_j + m\Psi^\dagger(x)\beta\Psi(x)\,\eta_j\,, \end{aligned}$$

which, taking into account the electron field equation, gives simply

$$s^*(i_{\hat{e}_j}\omega) = \mathrm{i}\hbar\Psi^\dagger(x)\alpha^\mu\partial_j\Psi(x)\,\eta_\mu\,.$$

With the same hypothesis as for the scalar product, this means that

$$\int\limits_{\mathbf{R}^3}\Psi^\dagger(x)\alpha^0(-\mathrm{i}\hbar\partial_j)\Psi(x)\,\eta_0$$

is conserved. Physically this is the total momentum of the field, and the corresponding density is given by the well known momentum operator

$$p_j = -\mathrm{i}\hbar\partial_j\,.$$

Our 4-form is also invariant invariant by the action of the passive time translations, leading in the same way to conservation of the total energy of the field

$$\int\limits_{\mathbf{R}^3}\Psi^\dagger(x)\alpha^0(\mathrm{i}\hbar\partial_t)\Psi(x)\,\eta_0 = \int\limits_{\mathbf{R}^3}\Psi^\dagger(x)\big[\alpha^i(-\mathrm{i}\hbar\partial_i) + m\beta\big]\Psi(x)\,\eta_0\,.$$

As a final example, consider the passive rotations λ about the $\hat{e}_i$ axis. Our 4-form is invariant by such transformations, which by definition act simultaneously on x , Ψ et $\Psi^\dagger$. The corresponding generator is the vector field

$$Y = x_j\hat{e}_k - x_k\hat{e}_j - \mathrm{i}\hat{e}_\Psi\,\tfrac{1}{2}\,\sigma^i\Psi + \mathrm{i}\Psi^\dagger\,\tfrac{1}{2}\,\sigma^i\hat{e}_{\Psi^\dagger}$$

since ψ and $\psi^\dagger$ are spinors. The Noether theorem gives the conservation of the total angular momentum

$$\int\limits_{\mathbf{R}^3}\big[\Psi^\dagger(x)\alpha^0x_j(-\mathrm{i}\hbar\partial_k)\Psi(x) - \Psi^\dagger(x)\alpha^0x_k(-\mathrm{i}\hbar\partial_j)\Psi(x) + \Psi^\dagger(x)\tfrac{\hbar}{2}\,\sigma^i\Psi(x)\big]\,\eta_0$$

and the corresponding density is the angular momentum operator

$$x_jp_k - x_kp_j + \tfrac{\hbar}{2}\,\sigma^i\,.$$

As we can see, this operator decomposes into an orbital part

$$x_j p_k - x_k p_j$$

and a highly non-local part, the spin

$$\tfrac{\hbar}{2}\,\sigma^i\,.$$

5 Conclusion

If one accepts this new window, the existence of the void space and one of its manifestations, the four component complex spinor field Ψ, one is led to the fundamental rules of "quantum mechanics". These turn out to be nothing else than the rules of a non-local physics. With such a $\Psi(x)$ we have built a Hilbert space whose rays are the states of the field, the electron field. Our position operator x^j and our momentum operator p_j are the usual ones satisfying the Heisenberg commutation relations. It is our deep conviction that the particle aspect of the theory arises only from interactions and selection rules due to the conservation laws, above all the angular momentum, which contains a highly non-local term, the spin.

Finally, I would like to express my thanks to David Moore for his help and encouragement in the difficult process of translating my thoughts into English.

REFERENCES

D. Aerts 1982; "Description of many separated physical entities without the paradoxes encountered in quantum mechanics" *Found. Phys.* **12** 1131-1170.

A. O. Barut, D. J. Moore and C. Piron 1993; "The Cartan formalism in field theory" *Helv. Phys. Acta* **66** 795-809.

S. Clarke 1866 in; "Oeuvres philosophiques de Leibnitz t. II" Quatrième replique de Clarke à Leibnitz, §9, p.643. Librarie Philosophique de Ladrange (Paris).

Le Sage 1818; "Traité de mécanique physique" Prévost (Genève).

C. Piron 1990; "Mécanique quantique bases et applications" Chap. 6. Presses polytechniques et universitaires romandes (Lausanne).

C. Piron and D. J. Moore 1995; "New aspects of field theory" *Turk. J. Phys.* **19** 202-216.