LMU–TPW 96–18

Complex Structures in Quantum Field Theory

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Invited contribution to the II. International Workshop on Classical and Quantum Integrable Systems, Dubna (Russia), 8–12 July 1996.

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1. Introduction

Covariance under symmetry transformations is a powerful constraint and provides a very useful tool for the investigation of physical systems. A particular example is provided by the infinite–dimensional symmetry of two–dimensional field theories under the Virasoro group [1]. This symmetry is partially inherent to all generally covariant two–dimensional models, since any Riemannian two–manifold admits a conformal gauge, whence two copies of a subalgebra of the Virasoro algebra generate locally the residual symmetry group remaining from the diffeomorphism group after gauge fixing. It is a remarkable fact, however, that many physically interesting models in two dimensions carry representations of the full Virasoro algebra.

The statement that any Riemannian two–manifold admits a conformal gauge is equivalent to the statement that the manifold carries a complex structure, and it is compatibility of quantum field theory on two–manifolds with the underlying complex structure which permits us to draw far reaching conclusions both on operator product expansions and on the field content of these models [2].

Complex structures show up in other places in quantum field theory: In twistor theory a complexification of space–time is employed to set up the twistor correspondence between complexified Minkowski space and twistor space [3]. In string theory it is known that preservation of supersymmetry under compactification from ten to four dimensions as a particular implication requires existence of a complex structure on the internal six–manifold [4]. Furthermore, holomorphy constraints on the field content of superpotentials play an important role in the investigation of low energy effective actions in supersymmetric theories [5], while holomorphy of Kähler potentials is a powerful tool in the investigation of extended supersymmetry [6].

In this talk I will not review these important subjects but concentrate on a few topics: After a brief repetition of a few facts in two–dimensional conformal field theory in section 2, section 3 is devoted to Ward identities and reproducing kernels and section 4 comments on a higher–dimensional Witt algebra. Section 5 comprises a digression into the physics of chiral symmetry breaking, while in section 6 I discuss a map between primary fields of half–integer conformal weight on spheres in momentum space and Weyl spinors in

3+1 dimensions and an application of this map to correlation functions on the cone $p^2 = 0$.

2. Covariant Field Theory in Two Dimensions

There are a few facts in two–dimensional field theory which I would like to recall, since they proved useful in constructing the isomorphy between Weyl spinors and half–differentials in Minkowski space.

A remarkable fact of two–dimensional field theory is the isomorphy between tensors and spinors on a two–manifold with the set of primary fields Φ, which in a conformal gauge transform according to[1]

$$\Phi(z', \overline{z}') = \Phi(z, \overline{z}) \Big(\frac{\partial z'}{\partial z}\Big)^{-\lambda} \Big(\frac{\partial \overline{z}'}{\partial \overline{z}}\Big)^{-\overline{\lambda}}$$

More precisely, the set of tensors and spinors is isomorphic to the set of primary fields with spins $\sigma = \lambda - \overline{\lambda}$ satisfying $2\sigma \in \mathbb{Z}$ [7, 8]. Moreover, Čech cohomology tells us that models with a particular primary field content can be formulated on any Riemannian two–manifold if and only if all spins in the model satisfy $2\sigma \in \mathbb{Z}$.

Of special interest are chiral primary fields of weights $(\lambda, 0)$, since all anomalous terms in their operator product expansions with conserved currents arise from extensions of the Lie algebra generated by the Noether currents and from extensions of the modules $\mathcal{M}_{1-\lambda}$:

$$L_m \circ a_n = [(\lambda - 1)m - n]a_{m+n}$$

and have to be determined from the first cohomology groups of these modules[2]. One finds [9]

$$\dim H^1(\mathcal{A}_1, \mathcal{M}_{1-\lambda}) = \delta_{\lambda,2} + \delta_{\lambda,1} + 2\delta_{\lambda,0}$$

The corresponding short distance expansions of operator products of chiral primary fields with the holomorphic components of the stress tensor T or a Noether current J follow from the Cauchy theorem and are well known[3]:

$$T(z)\phi(\zeta) = \frac{c_\lambda}{(z-\zeta)^{\lambda+2}} \delta_{\lambda^3 - 3\lambda^2 + 2\lambda, 0} + \frac{\lambda}{(z-\zeta)^2}\phi(z) + \frac{1}{z-\zeta}\partial_z \phi(z) \qquad (1)$$

[1] The general factorized transformation behaviour of primary fields without conformal gauge fixing is described in Ref. [7].

[2] This yields the second cohomology group of the Witt algebra $\mathcal{A}_1$ for $\lambda = 2$. See [7] for a definition of the complex involved in the construction of $H^1(\mathcal{A}_1, \mathcal{M}_{1-\lambda})$.

[3] The cohomology of modules of the Virasoro algebra tells us that there may appear one further anomaly in (1) for $\lambda = 0$. However, this anomaly corresponds to a background charge and breaks translational invariance of the operator product.

$$J(z)\phi(\zeta) = \frac{d_2}{(z-\zeta)^3}\delta_{\lambda,2} + \frac{d_1}{(z-\zeta)^2}\delta_{\lambda,1} + \frac{1}{z-\zeta}\delta\phi(z) \tag{2}$$

where the Noether current is supposed to correspond to a symmetry $\phi \to \phi + \delta\phi$.

3. Ward Identities and Reproducing Kernels

The derivation of operator products from conformal Ward identities in two–dimensional field theory relies heavily on the Cauchy kernel. Therefore, it should be possible to turn the argument around and infer from Ward identities in a symmetric theory the existence of reproducing kernels on the set of classical trajectories of the theory.

Ward identities between correlation functions in quantum field theory can be derived from the generating functional

$$Z[J] = \int D\phi \exp\Big(iS[\phi] + i\int d^4x J(x)\phi(x)\Big) \tag{3}$$

since invariance of the correlation functions under symmetry transformations of the fields $\phi_a(x) \to \phi_a(x) + \delta\phi_a(x)$ implies

$$\langle i\delta\phi_a(x)\rangle = \langle \delta S[\phi]\phi_a(x) + \phi_a(x)\int d^4x' J^b(x')\delta\phi_b(x')\rangle \tag{4}$$

This equation holds for any correlation functions involving also products of fields. However, for the derivation of the short distance parts of the operator product it is enough to consider only the expectation values of single operators $\langle\phi(x)\rangle$. Writing down the Ward identity for correlations of operator products tells us that the Noether currents satisfy a Leibnitz property, i.e. the operator product is associative in the sense of Lie products.

In the sequel we allow for transformations which are restricted to domains $\mathcal{D}$, but do not need to be continuous on boundaries $\partial\mathcal{D}$. If $\delta\phi$ perturbes a solution of the equations of motion

$$-J(x) = \frac{\delta S[\phi]}{\delta\phi(x)}$$

by an internal symmetry, Eq. (4) implies

$$\langle i\delta\phi_a(x)\rangle = \langle \oint_{\partial\mathcal{D}} d^3\sigma'_\mu\, \phi_a(x)\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_b(x'))}\delta\phi_b(x')\rangle = -\langle \oint_{\partial\mathcal{D}} d^3\sigma'_\mu\, \phi_a(x) j^\mu(x')\rangle \tag{5}$$

where $d^3\sigma'$ denotes the surface element on $\partial\mathcal{D}$, and $j(x)$ is the corresponding Noether current. Therefore,

$$K_a{}^{b\mu}(x,x') = -i\phi_a(x)\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_b(x'))}$$

provide the components of a reproducing kernel on the set of classical trajectories of $S[\phi]$. This clearly suggests far reaching generalizations of the notions of analyticity and reproducing kernels we are aware of, in the sense that the space of classical solutions of a quantum field theory with local symmetries allows for a notion of reproducing kernels and a generalized Cauchy theorem.

On the other hand, in cases where notions of analyticity are well under mathematical control, we may exploit established knowledge on reproducing kernels to infer results on operator product expansions from (5), as was done for euclidean quantum field theory on two–manifolds. For another example, consider a spinor field ψ satisfying the Dirac equation $(i\gamma^\mu\partial_\mu - m)\psi(x) = 0$ (which could be considered as a a generalized monogenicity condition from a mathematical point of view). In this case we would infer the operator product

$$\psi(x)j^\mu(x') = S(x-x')\gamma^\mu\delta\psi(x')$$

with $S(x-x')$ denoting a Dirac propagator.

4. A Witt Algebra in Higher Dimensions

In this section I would like to make a cautionary remark about extensions of two–dimensional conformal field theory to higher dimensions. An immediate way to generalize two–dimensional conformal field theory to higher dimensions relies on higher–dimensional complex manifolds, which locally carry a higher–dimensional analog of the Witt algebra. This can be inferred from a local basis of meromorphic vector fields on $\mathbb{C}^n$

$$L_{j\{m\}} = -\prod_{i=1}^{n} z_i{}^{m_i}\cdot z_j\frac{\partial}{\partial z_j}$$

where $\{m\} = \{m_1,\ldots m_n\}$ is a vector of integers. This yields an algebra $\mathcal{A}_n$:

$$[L_{i\{k\}}, L_{j\{m\}}] = k_j L_{i\{k+m\}} - m_i L_{j\{k+m\}}$$

To determine the anomalous extensions of the product of two stress tensors, we have to classify the central extensions

$$[L_{i\{k\}}, L_{j\{m\}}] = k_j L_{i\{k+m\}} - m_i L_{j\{k+m\}} + C_{ij\{k\}\{m\}}$$

satisfying a cocycle condition due to preservation of the Jacobi identity:

$$k_bC_{ac\{k+l\}\{m\}} - k_cC_{ab\{k+m\}\{l\}} - l_cC_{ab\{k\}\{l+m\}} - l_aC_{bc\{k+l\}\{m\}} \tag{6}$$
$$-m_aC_{bc\{l\}\{k+m\}} + m_bC_{ac\{k\}\{l+m\}} = 0$$

The possibility to shift the generators by central elements implies the trivial solutions or coboundaries

$$C^{\text{trivial}}_{ab\{k\}\{m\}} = m_af_{b\{k+m\}} - k_bf_{a\{k+m\}} \tag{7}$$

Unfortunately, in solving this cohomology problem we find the somewhat disappointing result

$$\dim H^2(\mathcal{A}_n) = \delta_{n,1}$$

Therefore, when we really want to imitate the successes of two–dimensional conformal field theory in higher dimensions, we seem to end up with three canonical choices: We may rely on the finite–dimensional conformal group, or we may rely on the twistor correspondence employing a complexification of Minkowski space, or we may employ some other variant of quaternionic analyticity. However, we may also opt for another possibility: Making use of low–dimensional complex structures in real Minkowski space. Before discussing a specific example, where this option can be employed, I will digress from mathematics to physics for a while and discuss some aspects of chiral symmetry breaking in gauge theories.

5. Remarks on Chiral Symmetry Breaking

Chiral symmetry breaking is a characteristic feature of low energy QCD which remains puzzling from a theoretical point of view. Spontaneous breaking of chiral $SU(N_f)$ symmetry is expected to arise as a consequence of confinement or as an instanton effect [10, 11], but it is not clear which mechanism drives chiral symmetry breaking. Chiral symmetry breaking effects of confining forces have been discussed in [12, 13], and it has been pointed out that in dual QCD monopole condensation not only yields confinement but also a chiral condensate through a gap equation [14]. In QCD we would like to understand how monopoles break chiral symmetry in spite of their chiral coupling, or whether instantons or other non–perturbative effects break chiral symmetry before confinement. This problem is also relevant for the nature of the phase transition, since the absence of an order parameter for confinement in the presence of light flavors excludes a second order phase transition, if chiral symmetry breaking is causally connected to confinement. Pisarski and Wilczek pointed out that an ϵ–expansion for the corresponding σ–model indicates a first order transition for more than two light flavors, but that a second order transition in the universality class of the $O(4)$ vector model is likely to appear in case of two light flavors [15, 16].

In the next section I report on recent results for the the fermion correlation $\langle q(p_1)\overline{q}(p_2)\rangle$ on the orbit $p^2 = 0$, in an attempt to shed new light on the problem from an unexpected angle [17]. Dynamical breaking of chiral flavor symmetry in gauge theories is a puzzle, because it is very different from spontaneous magnetization: In a ferromagnet the interaction tends to align the dipoles, while thermal fluctuations restore disorder if the system has enough energy. Gluon exchange, on the other hand, does not necessarily align left– and right–handed fermions. Stated differently, the massless Dyson–Schwinger equation for the trace part of the fermion propagator always admits a trivial solution, if a quark condensate is not inserted *ab initio*[4]. Therefore, chiral symmetry breaking has to be implemented in unusual ways, if we want to recover it from gauge dynamics: By requiring a condensate as part of initial conditions, in double scaling limits, through chiral symmetry breaking boundary conditions, in chiral symmetry breaking regularizations, etc. While this does not invalidate standard approaches to the problem, it serves to remind the reader that some poorly understood mechanism leaves its footprint on the long distance properties of the QCD vacuum, and motivates the group theoretical construction of Lorentz covariant correlation functions given below.

The main ingredient of the work reported below is a mapping between massless spinors in 3+1 dimensions and primary fields, which relates the order parameter to automorphic functions under the Lorentz group. From a mathematical point of view, the novel feature of the automorphic functions under investigation is that they provide correlations between primary fields on spheres of different radii, thus providing true representations of the Lorentz group and extending the determination of correlation functions in 2D conformal field theory. The nontrivial behavior of radii under the boost sector of the Lorentz group allows for chiral symmetry preserving terms in the correlation functions which could not appear in a two–dimensional framework, while the chiral symmetry breaking terms in turn appear closely related to 2D fermionic correlation functions.

There exists wide agreement that chiral symmetry breaking in QCD arises both dynamically, as a genuine QCD phenomenon, and through electroweak symmetry breaking, which in a standard scenario accounts for the quark current masses[5]. Dynamical chiral symmetry breaking is then expected to ac-

[4]In the latter case the corresponding Dyson–Schwinger equation yields a gap equation for the condensate. The puzzle then concerns the identification of the "phonons" in QCD.

[5]The electroweak sector also contributes to breaking of chiral $SU(N_f)$ through the axial anomaly since the charge operator Q^2 is not flavor symmetric.

count partially for the difference between current and constituent masses [18]. From this point of view, the large discrepancy between current and constituent masses, and the fact that there is not even an approximate parity degeneracy in the hadron spectrum provides strong evidence for dynamical breaking of chiral symmetry.

Another argument in favor of dynamical breaking of chiral symmetry comes from the Gell-Mann–Oakes–Renner relation:

$$2m_q\langle\overline{q}q\rangle = -f_\pi^2 m_\pi^2 \tag{8}$$

where m_q stands for a mean value of current quark masses. This relation is expected to hold in the sense of a leading approximation in m_q, and works phenomenologically the better the smaller the value of m_q is [19]. While this does not strictly imply $\lim_{m_q\to 0}\langle\overline{q}q\rangle \neq 0$, it implies at least that the condensate vanishes weaker than first order in m_q.

The necessity of including non–vanishing condensates in QCD sum rules provides further strong indication for spontaneous breaking of chiral symmetry[6], while yet another hint for chiral symmetry breaking is provided by 't Hooft's result that decoupling of heavy fermions does not comply with local chiral flavor symmetry [20]. Last not least, chiral symmetry breaking can also be addressed in lattice simulations of QCD. In this framework non–vanishing condensates have been reported e.g. in [21, 22].

This review comprises a short summary of some compelling arguments in favor of dynamical chiral symmetry breaking. There is strong evidence that chiral symmetry breaking in QCD is not solely of electroweak origin.

6. Half–Differentials and Fermion Propagators

Chiral spinors in 3+1 dimensions can be described as primary fields of conformal weight $\frac{1}{2}$ on spheres in momentum space [17]. To exploit this observation, we work in the Weyl representation of Dirac matrices, and parametrize the unit sphere in momentum space in terms of stereographic coordinates:

$$z = \frac{p_1 + ip_2}{|\mathbf{p}| - p_3} \qquad\qquad \tilde{z} = -\frac{p_1 - ip_2}{|\mathbf{p}| + p_3} \tag{9}$$

Proper orthochronous Lorentz transformations act on these coordinates according to

$$z' = z(\mathbf{p}') = \overline{U}\circ z(\mathbf{p}) = \frac{\bar{a}z + \bar{b}}{\bar{c}z + \bar{d}} \tag{10}$$

[6]This becomes particularly evident in heavy–light systems, where the condensate of the light quark is expected to contribute to the meson propagator even in the limit of vanishing current mass.

if $E = |\mathbf{p}|$, and

$$z' = U^{-1T} \circ z(\mathbf{p}) = \frac{dz - c}{a - bz} \tag{11}$$

if $E = -|\mathbf{p}|$.
U denotes the positive chirality spin $\frac{1}{2}$ representation of the Lorentz group:

$$U(\omega) = \exp(\frac{1}{2}\omega^{\mu\nu}\sigma_{\mu\nu}) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{C})$$

We identify local functions written in co-ordinates $(z, \bar{z}, |\mathbf{p}|)$ and $(\tilde{z}, \bar{\tilde{z}}, |\mathbf{p}|)$ via

$$\psi(\tilde{z}, \bar{\tilde{z}}, |\mathbf{p}|) = -z\psi(z, \bar{z}, |\mathbf{p}|) \tag{12}$$

$$\phi(\tilde{z}, \bar{\tilde{z}}, |\mathbf{p}|) = \bar{z}\phi(z, \bar{z}, |\mathbf{p}|) \tag{13}$$

and these overlap conditions can be rephrased as Weyl equations:

$$(|\mathbf{p}| + \mathbf{p}\cdot\sigma)\begin{pmatrix} \psi(z, \bar{z}, |\mathbf{p}|) \\ \psi(\tilde{z}, \bar{\tilde{z}}, |\mathbf{p}|) \end{pmatrix} = 0$$

$$(|\mathbf{p}| - \mathbf{p}\cdot\sigma)\begin{pmatrix} \phi(\tilde{z}, \bar{\tilde{z}}, |\mathbf{p}|) \\ \phi(z, \bar{z}, |\mathbf{p}|) \end{pmatrix} = 0$$

Under (10) ϕ and ψ transform according to

$$\phi'(z', \bar{z}', |\mathbf{p}'|) = (c\bar{z} + d)\phi(z, \bar{z}, |\mathbf{p}|) \tag{14}$$

$$\psi'(z', \bar{z}', |\mathbf{p}'|) = (\bar{c}z + \bar{d})\psi(z, \bar{z}, |\mathbf{p}|) \tag{15}$$

if $E = |\mathbf{p}|$. Due to (12,13) this is equivalent to

$$\begin{pmatrix} \phi'(\tilde{z}', \bar{\tilde{z}}', |\mathbf{p}'|) \\ \phi'(z', \bar{z}', |\mathbf{p}'|) \end{pmatrix} = U \cdot \begin{pmatrix} \phi(\tilde{z}, \bar{\tilde{z}}, |\mathbf{p}|) \\ \phi(z, \bar{z}, |\mathbf{p}|) \end{pmatrix}$$

$$\begin{pmatrix} \psi'(z', \bar{z}', |\mathbf{p}'|) \\ \psi'(\tilde{z}', \bar{\tilde{z}}', |\mathbf{p}'|) \end{pmatrix} = U^{-1\dagger} \cdot \begin{pmatrix} \psi(z, \bar{z}, |\mathbf{p}|) \\ \psi(\tilde{z}, \bar{\tilde{z}}, |\mathbf{p}|) \end{pmatrix}$$

The case $E = -|\mathbf{p}|$ corresponds to $U \leftrightarrow U^{-1\dagger}$ in the equations above, but we will stick to positive energy in the sequel. Correlations for $E = -|\mathbf{p}|$ can easily be recovered from the covariance cosiderations for positive energy through a reflection $\mathbf{p} \to -\mathbf{p}$.

The implication of the Weyl equation in the derivation of the map between massless spinors and half–differentials means, that we rely on an interaction picture when we employ the results above to gain information on Lorentz covariant correlation functions. To take advantage of this construction, we write a spinor on the half–cone $E = |\mathbf{p}|$:

$$\Psi(p) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} \bar{z} \\ 1 \end{pmatrix} \phi(z, \bar{z}, |\mathbf{p}|) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ -z \end{pmatrix} \psi(z, \bar{z}, |\mathbf{p}|) \tag{16}$$

with a corresponding representation of the correlation function of massless fermions in the Dirac picture

$$\langle \Psi(p) \overline{\Psi}(p') \rangle = \tag{17}$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} \bar{z}z' & \bar{z} \\ z' & 1 \end{pmatrix} \langle \phi(\mathbf{p}) \phi^+(\mathbf{p}') \rangle + \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & -\bar{z}' \\ -z & z\bar{z}' \end{pmatrix} \langle \psi(\mathbf{p}) \psi^+(\mathbf{p}') \rangle$$

$$+ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} \bar{z} & -\bar{z}\bar{z}' \\ 1 & -\bar{z}' \end{pmatrix} \langle \phi(\mathbf{p}) \psi^+(\mathbf{p}') \rangle + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} z' & 1 \\ -zz' & -z \end{pmatrix} \langle \psi(\mathbf{p}) \phi^+(\mathbf{p}') \rangle$$

The 2–point functions on the right hand side transform under a factorized representation of the Lorentz group. This makes this representation very convenient for the investigation of correlations $\langle \Psi(p) \overline{\Psi}(p') \rangle$ which comply with Lorentz covariance. Stated differently, we ask which correlations of spinors of the form $\Psi(p)$ could be constructed from a non–trivial vacuum, or more general, between any Lorentz invariant states.

The investigation in Ref. [17] revealed

$$\langle \psi(\mathbf{p}_1) \psi^+(\mathbf{p}_2) \rangle = \langle \phi(\mathbf{p}_2) \phi^+(\mathbf{p}_1) \rangle = f_1\left(\frac{|\mathbf{p}_1|}{|\mathbf{p}_2|} \right) \frac{1 + z_1 \bar{z}_2}{\sqrt{|\mathbf{p}_1||\mathbf{p}_2|}} \, \delta_{z\bar{z}}(z_1 - z_2) \tag{18}$$

$$\langle \psi(\mathbf{p}_1) \phi^+(\mathbf{p}_2) \rangle = \overline{\langle \phi(\mathbf{p}_2) \psi^+(\mathbf{p}_1) \rangle} = \frac{1}{z_1 - z_2} \, f_2\left(|\mathbf{p}_1||\mathbf{p}_2| \frac{(z_1 - z_2)(\bar{z}_1 - \bar{z}_2)}{(1 + z_1\bar{z}_1)(1 + z_2\bar{z}_2)} \right) \tag{19}$$

where Lorentz covariance does not fix f_1 and f_2[7]. The orbit $p^2 = 0$ thus contributes to a condensate

$$tr \langle \Psi(p) \overline{\Psi}(p') \rangle = -2 f_2 (|\mathbf{p}||\mathbf{p}'| \sin^2(\frac{\theta}{2})) \tag{20}$$

where θ denotes the angle between $\mathbf{p}$ and $\mathbf{p}'$.

If the correlation function in configuration space gives the positive energy contribution to a propagator of initial conditions (modulo $i\gamma_0$), like in the perturbative vacuum, then consistency of the result above is expressed by the fact

[7] The on–shell correlation in the vacuum of the free theory $\langle \psi(\mathbf{p}) \overline{\psi}(\mathbf{p}') \rangle = -2p \cdot \gamma |\mathbf{p}| \delta(\mathbf{p} - \mathbf{p}')$ is recovered from Eqs. (17,18,19) for $f_1(x) = \delta(x - 1)$, $f_2 = 0$.

that the f_2 terms do not anticommute with γ_5, while the f_1 terms anticommute with γ_5 and imply a restriction for external momenta to be parallel. On the other hand, since we pretend to deal with a confining theory (albeit disguised in a non–perturbative vacuum), there is no reason to believe that the propagator can be reconstructed from data on a single orbit of the Lorentz group. The results above presumably make sense only within a confining theory, if observables are expressed in terms of meson or baryon correlations, and the 4–point function is on my agenda since quite a while now.

Acknowledgements: I would like to thank the organizers of the meeting in Dubna for their warm hospitality and for the invitation to give a talk at the workshop. I would also like to thank Yum–Tong Siu and the staff of the Mathematical Sciences Research Institute in Berkeley for hospitality and the opportunity to participate in the program on Several Complex Variables. Part of this work was performed during my stay in Berkeley in spring. Research at MSRI is supported in part by NSF grant DMS–9022140. This work has also been supported in part by DFG grants DI 601/1–1 and DI 601/2–1.

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