On the possible failure of the Gibbs property for measures on lattice systems

Aernout C. D. van Enter
Institute for Theoretical Physics
Rijksuniversiteit Groningen
Nijenborgh 4
9747 AG Groningen
THE NETHERLANDS
AENTER@TH.RUG.NL

April 25, 1997

Abstract

We review results which have been obtained in the last few years on the topic whether various measures of physical interest, defined on lattice systems, have the Gibbs property or not. Most of the measures we consider occur either within renormalization-group theory or within models of non-equilibrium statistical mechanics. In renormalization-group theory examples we discuss in particular the uniqueness regime and the critical regime. Moreover we consider the question how severely the Gibbs property is violated in various examples.

To the memory of R.L. Dobrushin

1 Introduction

In [69, 70] it was shown how various real-space renormalization-group (RG) maps acting on Gibbs measures produce non-Gibbsian measures. In physicists' language, this means that a "renormalized Hamiltonian" can not be defined. The examples presented there were valid at low temperatures or close to a first-order phase transition. The underlying mechanism — pointed out first by Griffiths, Pearce and Israel [22, 23, 33] — is the fact that for the constraints imposed by particular choices of block-spin configurations, the resulting system exhibits a first-order phase transition. For this to happen, it was expected that the original system should be itself at or in the vicinity of a first-order phase transition, cf. the title of [69]. However, more recently counter-examples to this belief have been found, in that renormalized measures for several transformations turn out to be non-Gibbsian for arbitrarily large values of the magnetic field (at low temperatures) [70, 68] or at arbitrarily high temperatures [66].

On the other hand, there exist complementary results, either proving or providing heuristic but plausible arguments, that renormalized Gibbsian measures will be Gibbsian again at very high fields or very high temperatures for "many" transformations, [22, 23, 33, 7, 35], and generally in the uniqueness region after a sufficiently often iterated decimation (possibly combined with another transformation) [52, 53].

What happens in general in intervals around and including critical points was until recently essentially unknown. On the one hand there were various partial results which suggested that transforming a critical Gibbs measure should result in a measure which was Gibbsian again [36, 3, 73, 9], see also [10]. This would be according to the folklore of renormalization-group theory, which requires that a local (= short-range) Hamiltonian is mapped on another local Hamiltonian in the same universality class. However, until very recently, there was essentially no rigorous result available which applied in the critical domain. Here we will discuss how now we know of some examples in the critical regime which give rise to either Gibbsian (see [37, 30]) or non-Gibbsian (see [66] and below) measures.

Further investigations up to 1992 about the (non-)Gibbsianness of various measures of interest in statistical mechanics were rather extensively reviewed in [70], see also [71]. Here we will give a short overview of the more recent results obtained since then, see e.g. [45, 18, 64, 61, 60, 49, 74, 11, 6, 43, 72, 50, 44, 51, 10]. These include new examples as well as a more detailed description on the possible ways of violating the Gibbs property, in particular how severe this violation can be.

2 Gibbs measures and quasilocality

In this section we will describe some definitions and facts we will need about the theory of Gibbs measures. For a more extensive treatment we refer to [20] or [70].

We will consider spin systems on a lattice Z^d, where in most cases we will take a single-spin space Ω_0 which is finite. The configuration space of the whole system is $\Omega = \Omega_0^{Z^d}$. Configurations will be denoted by σ or ω, and their coordinates at lattice site i are denoted by $\omega(i)$ or $\sigma(i)$. A (regular) interaction Φ is a collection of functions Φ_X on $\Omega_0^X, X \in Z^d$ which is translation invariant and satisfy:

$$\Sigma_{0 \in X} \ |\Phi_X|_\infty \leq \infty \tag{2.1}$$

Formally Hamiltonians are given by

$$H^\Phi = \Sigma_{X \in Z^d} \ \Phi_X \tag{2.2}$$

Under the above regularity condition these type of expressions make mathematical sense if the sum is taken over all subsets having non-empty intersections with a finite volume Λ. For regular interactions one can define Gibbs measures as probability measures on Ω having conditional probabilities which are described in terms of appropriate Boltzmann-Gibbs factors:

$$\frac{\mu(\sigma_\Lambda^1|\omega_{\Lambda^c})}{\mu(\sigma_\Lambda^2|\omega_{\Lambda^c})} = exp - (\Sigma_X \ [\Phi_X(\sigma_\Lambda^1 \omega_{\Lambda^c}) \ - \ \Phi_X(\sigma_\Lambda^2 \omega_{\Lambda^c})]) \tag{2.3}$$

for each volume Λ, μ-almost every boundary condition ω_{Λ^c} outside Λ and each pair of configurations σ_Λ^1 and σ_Λ^2 in Λ. As long as Ω_0 is compact, there always exists at least one Gibbs measure for every regular interaction; the existence of more than one Gibbs measure is one definition of the occurrence of a first-order phase transition of some sort. Every Gibbs measure has the property that (for one of its versions) its conditional probabilities are continuous functions of the boundary condition ω_{Λ^c}. It is a non-trivial fact that this continuity, which goes by the name "quasilocality" or "almost Markovianness", in fact characterizes the Gibbs measures [65, 38], once one knows that all the conditional probabilities are bounded away from zero (that is, the measure is *nonnull* or has the *finite energy* property).

To show that a particular measure is non-Gibbsian it is thus sufficient to find *one* point in configuration space (*one* boundary condition) and *one* conditional probability which has an essential, that is non-removable, discontinuity at that point. The main step in such a proof is to show that when this special configuration is viewed as a constraint, the constrained system has a phase transition.

A second characterization of Gibbs measures uses the variational principle expressing that in equilibrium a system minimizes its free energy. A probabilistic formulation of this fact naturally occurs in terms of the theory of large deviations. A (third level) large deviation rate function is up to a constant and a sign equal to a free energy density. To be precise, let μ be a translation invariant Gibbs measure, and let ν be an arbitrary translation invariant measure. Then the relative entropy density $i(\nu|\mu)$ can be defined as the limit:

$$i(\nu|\mu) = lim_{\Lambda \to Z^d} \frac{1}{|\Lambda|} I_{\Lambda}(\nu|\mu) \tag{2.4}$$

where

$$I_{\Lambda}(\nu|\mu) = \int log(\frac{d\nu_{\Lambda}}{d\mu_{\Lambda}}) d\nu_{\Lambda} \tag{2.5}$$

It has the property that $i(\nu|\mu) = 0$ if and only if the measure ν is a Gibbs measure for the same interaction as the base measure μ. We can use this result in applications if we know for example that a known measure ν cannot be a Gibbs measure for the same interaction as some measure μ we want to investigate. For example, if ν is a point measure, or if it is the case that ν is a product measure and μ is not, we can conclude from the statement: $i(\nu|\mu) = 0$, that μ lacks the Gibbs property.

For another method of proving that a measure is non-Gibbsian because of having the "wrong" type of (in this case too small) large deviation probabilities, see [62].

3 Examples of renormalized measures

We consider the standard nearest neighbor Ising model with (formal) Hamiltonian

$$H = \Sigma_{<i,j>} \; - \sigma(i)\sigma(j) \tag{3.1}$$

at inverse temperature β. The dimension d has to be at least 2.

We will consider various renormalization or block-spin transformations which act on the Ising Gibbs measures. We divide the lattice into a collection of non-overlapping blocks. A renormalization group transformation will be a probability kernel

$$T(\omega'|\omega) = \Pi_{blocks} T(\omega'(j)|\omega(i);\; i \;\in block_j) \tag{3.2}$$

This means that the distribution of the block-spin associated to a block depends only on the spins in this block, in other words the transformation is local in real space. The case of a deterministic transformation is included, by having a T which is either zero or one.

Renormalization group methods are widely in use to study phase transitions and in particular critical phenomena of various sorts (see for example [75, 47, 13]). Some good more recent references in which the theory is explained, mostly at a physical level of rigour, but with careful statements about what actually has and has not been proven are [21, 2].

1) The first class of examples we will consider are (linear) block-average transformations. In [70] it was shown that at low temperatures for a choice of cubic blocks of even length the renormalized measures will be non-Gibbsian. The constraint which acts as a point of discontinuity is taking all the averages in the blocks to be zero. Then in each block one forces an interface, which tends to be in the middle of the block. Minimizing the (free) energy density forces these interfaces to form a collection of parallel lines (c.q. planes or hyperplanes). As these interfaces can be in d directions, and because even if we are given the interfaces, there are still two possible choices where to put the pluses and minuses, this means that the constrained system has $2d$ low-temperature Gibbs measures. Because the constraint: "zero magnetization in each block" removes the field-dependence, the argument which provides the non-quasilocality is valid in arbitrarily strong external magnetic fields. This result in [70] applies at very low temperatures. In [3] it was numerically investigated in $d = 2$ up to which temperatures one should expect the lack of quasilocality to persist. The result was that for 2 by 2 blocks the critical point is outside the region of pathologies, although the maximal temperature for which pathologies occur is only a few percent below the critical temperature. Here we show on the other hand the new result that if one takes anisotropic blocks of size 1 by 2, the critical point is included in the region of pathologies. To be definite, let us take blocks of length 2 in the vertical direction and unit length in the horizontal direction. We take the (unnormalized) block-average:

$$\sigma'_{i_1,i_2} = \sigma_{i_1,2i_2} + \sigma_{i_1,2i_2+1} \tag{3.3}$$

The block-spin configuration which will be the point of discontinuity, is again the configuration which is zero in each block, $\sigma'_{i_1,i_2} = 0$ for all i_1, i_2. The constrained system thus allows for 2 configurations compatible with the constraint in each block, namely plus up and minus below (configuration A), and minus up and plus below (configuration B). This means that the constrained block-spin system is equivalent to a standard nearest neighbour Ising model, with an antiferromagnetic bond in the vertical direction and a ferromagnetic bond of twice the original strength in the horizontal direction. Such a model trivially has its critical point at a higher temperature than the original model. The arguments how to translate the phase transition for the constrained system into a violation of quasilocality for the renormalized system are the same as in [70], sections 4.1 to 4.3, in particular compare the proofs of Theorems 4.6 and 4.8. Thus we conclude:

Theorem: Let $\mu_{\beta,h}$ be a Gibbs measure for the two-dimensional Ising model. In an infinite half-plane in the (β, h)-plane, which contains the critical point, the measures obtained by taking an block-average transformation on 1 by 2 blocks are non-Gibbsian.

2) Majority rule and Kadanoff transformations.

In the case of majority rule transformations [59] applied to odd blocks, in a strong plus field, the proof of the non-Gibbsianness uses as a constraint the configuration in which the majority is minus, that means it opposes the field in each block. Thus at very low temperatures the majority is minimal, which implies that the total spin is close to zero, and the constrained system has a phase transition of similar type as the block-average equal to zero constrained system of the first example had [68]. On the othere hand there are strong indications [36, 10] that there is a region around the critical point in $d = 2$ which is free from pathologies under majority rule transformations.

The Kadanoff transformation is a soft version (a proper example of a stochastic transformation) of the majority rule:

$$T(\sigma'(j)|\sigma(i);\ i \in block_j)) = C\ exp\ [p\sigma'(j)\Sigma_{i \in block_j}\ \sigma(i)] \tag{3.4}$$

In [70] it was shown that at low temperatures, and for small external fields, the transformed measures become non-Gibbsian. In [30] for the Ising model on a triangular lattice it is shown that there is an interval around the critical point for which the transformed measures are Gibbsian, hence free of pathologies.

3) a highly non-linear transformation.

We consider the following transformation :

Divide the lattice Z^d into blocks B_j^L of linear size L and define block-spins $\sigma'(j)$ by

$$\sigma'(j) = \Sigma_{i \in B_j^L}\ \sigma(i) \tag{3.5}$$

except in the case where all spins in the block are minus, in which case

$$\sigma'(j) = +L^d \tag{3.6}$$

Now we have the following theorem:

Theorem: Let μ_β be the Gibbs measure of the nearest neighbor Ising model in dimension $d \geq 2$, at inverse temperature $\beta > 0$. Then there exists a constant L_0 such that for all $L > L_0$, $\mu'_L = \mu \circ T_L$ is non-Gibbsian.

Proof: The proof follows again the scheme of [70], section 4.2. The special block-spin configuration which will be the point of (essential) discontinuity for a suitably chosen conditional probability will be the choice $\sigma'(j)$=L^d in each block. This means that within each block all spins are unanimous, either in the plus or in the minus-direction. Thus imposing the special block-spin configuration as a constraint means that the sum of all spins in each block can take only two values (either unanimously positive or unanimously negative). Hence the thus constrained system is again a nearest neighbour Ising model, but now at an inverse temperature $\beta' = L^{d-1} \times \beta$. For L large enough, β' becomes so large that the constrained system is in the phase transition (low temperature) regime (step 1 of the proof). The block-spin boundary conditions selecting the plus and minus measures are $\sigma'(j) = +(L^d - 2)$ or $-(L^d - 2)$ in each boundary block. This means that in every block there is only one spin opposing all the others. When $L > 2$, a standard Peierls argument proves the phase transition (step 2 of the proof). As the expectation of $\sigma'(0)$ (unfixed according to step 3 of the proof) is different in the plus and minus phases, we see that the scheme of proof of [70], section 4.2, can again be straightforwardly applied.

4)Decimations and projections.

We will call a "decimation" taking the marginal of a measure restricted to the spins on a sublattice of the same dimension as the original system, and a "projection" taking the marginal to a lower-dimensional sublattice. In some sense these transformations are the simplest, and the most is known about them. In the region of strong uniqueness (complete analyticity for regular volumes), repeating the transformation maps a Gibbs measure again onto a Gibbs measure, even though after one or a finite number of transformations the transformed measure can be non-Gibbsian, and the same is true if one composes a sequence of decimations with a finite number of applications of any other transformation [70, 52, 53, 54, 55]. Recently Haller and Kennedy showed that there is an interval around the critical temperature where the once decimated measures on the sublattice with even coordinates are Gibbsian [30].

For Potts models applying a decimation slightly above the transition temperature results in a non-Gibbsian measure [68], while at sufficiently high temperatures the transformed measure is Gibbsian [43, 44].

As for projections, it is known that on the phase transition line of the 2-dimensional Ising model the projection to Z of any Gibbs measure is non-Gibbsian [62]. In the whole uniqueness regime, except possibly at the critical point, this projection results in a Gibbsian measure [48, 43, 44]. In three dimensions the projected measures to two-dimensional planes are again non-Gibbsian on the transition line [18, 49], however, now, due to the presumably existing surface (layering) transition between different Basuev states, one expects that the projected measures also in a small field will be non-Gibbsian [43, 44].

The composition of a projection and a decimation in the phase transition region gives rise to a new phenomenon, namely the possibility of a state-dependent result. The transformed plus and minus measures are Gibbs measures for different interactions, while the transformed mixed measures are non-Gibbsian [45].

4 Further results on non-Gibbsian measures

Further investigations about non-Gibbsian measures have been done about the random-cluster models of Fortuin and Kasteleyn [60, 24, 4, 29], about some image analysis model [50], and about some non-equilibrium models [64, 74, 57, 53].

1) Random-cluster models

About the random-cluster model, it was shown in [70], following [1], that all measures which are consistent with the specification are not quasilocal when $q > 1$. We did not know at the time whether the wired and free boundary condition limit states are in fact consistent with the random-cluster specification. Due to the lack of quasilocality of the specification this is not immediate from standard arguments, but this assumption was shown to hold in [60, 4] for the free b.c. state and in [24] for both free (disordering) and wired (ordering) boundary conditions. In the proofs essential use is made of the attractivity (or FKG-)property. By considering the Fortuin-Kasteleyn-Swendsen-Wang representation [15] one has, similarly to the renormalization-group examples, again that the marginal of a Gibbsian — quasilocal — measure turns out to be non-quasilocal.

2) Contracting the single-spin space.

Various results about measures obtained by contracting single-spin spaces have been obtained, following the first results on "projected Gaussians" of [39]. In [50] the so-called fuzzy Potts model was considered, in which the q Potts states are grouped together in $q' = q/n$ groups. This together grouping can give rise to non-quasilocal conditional probabilities. This model is of interest for image analysis. It is remarkable that a similar procedure applied to the maximal entropy measures for subshifts of finite type (SOFTs), which one can view as ground states, sometimes results in Gibbsian measures [25, 26, 27, 28].

3)Non-equilibrium models.

It is known that the invariant measures of the voter model and the Martinelli-Scoppola cluster dynamics are non-Gibbsian [70, 57, 40]. More recently it was shown that the invariant measures for the two-species TASEP (totally asymmetric exclusion process) is non-Gibbsian because its conditional probabilities are neither quasilocal [64] nor bounded away from zero [74]. In this model there are particles of type 1, particles type 2 and holes. Both type of particles can jump to a hole on the right, and a type 1 particle can exchange its position with a type 2 particle neighbouring it on its right. The result applies to measures which have a positive density of both types of particles and also a positive density of holes.

5 Robustly non-Gibbsian measures?

The non-Gibbsian character of the various measures considered comes often as an unwelcome surprise. A description in terms of effective or coarse-grained or renormalized potentials is often convenient, and seems for some applications even essential. The fact that such a description is not available thus is a severe drawback. There have been attempts to tame the non-Gibbsian pathologies, and here we want to give a short description of how far one gets with some of those attempts.

1) The fact that the constraints which act as points of discontinuity often are

configurations which are very untypical for the measure under consideration, suggests a notion of a *weakly* non-Gibbsian measure as a measure which has conditional probabilities which are continuous on a set of full measure. This notion was first suggested by Dobrushin and Shlosman, and first appeared in print in [46]. It has been developed and was applied to various models in [18], and later in [50] and [60]. Examples of measures which are at the worst weakly non-Gibbsian measures in this sense are decimated or projected Gibbs measures in an external field, random-cluster measures on regular lattices, and low temperature fuzzy Potts measures. In the random-cluster measures one can actually identify explicitly all bond configurations which give rise to the non-quasilocality. They are precisely those configurations in which (possibly after a local change) more than one infinite cluster coexist.

On a tree, because of the possible coexistence of infinitely many infinite clusters with positive probability, the random-cluster measure can violate the weak non-Gibbsianness condition and be strongly non-Gibbsian [29].

In a related approach Dobrushin [11, 6, 51] showed that for a projected pure phase on the coexistence line of the 2-dimensional Ising model it is possible to find an almost everywhere defined interaction. This approach, which is via low-temperature expansions, provides a way of obtaining good control for the non-Gibbsian projection. For some renormalization examples an investigation via low temperature expansions into the possibility of recognizing non-Gibbsianness was started in [61].

Another simple counter-example of a strongly non-quasilocal measure, where each configuration can act as a point of discontinuity, is a mixture of two Gibbs measures for different interactions [72].

2) Stability under decimation (and other transformations).

In [52, 53] it was shown how decimating once renormalized non-Gibbsian measures results in Gibbs measures again after a sufficiently large number of iterations. These often decimated measures are in the high-temperature regime, in which the usually applied renormalization-group maps are well-defined (this does not hold true for all maps though, in view of the highly non-linear example given before).

On the other hand, in examples where the non-Gibbsian property is associated with large deviation properties which are not compatible with a Gibbsian character (this holds for example for projected Gaussians, invariant measures of the voter and the Martinelli-Scoppola model) the non-Gibbsian property survives all sort of transformations [53, 72]. The argument is that when some obviously non-Gibbsian measure has a rate function zero with respect to the measure under consideration, this property is generally preserved ([70] Section 3.2 and 3.3).

The family of projected Gaussians include scaling limits for majority-rule transformations in high dimensions [14]. Transforming those scaling limits corresponds to making a move from a fixed point in what is usually called a “redundant” direction (cf. Wegner’s contribution to [13]) in some space of Hamiltonians, although here of course the whole point is that such Hamiltonians do not exist.

3) The two criteria mentioned above are distinct. Maes and Vande Velde [51]

have a simple one-dimensional example of a one-dependent measure which has a set of discontinuity points of full measure, but due to the one-dependence the measure becomes after decimation independent and therefore trivially Gibbsian.

6 Final Comments, Conclusions and Some Open Problems

1.The highly non-linear block-spin map defined above is admittedly artificial from a physical point of view. However, the heuristic considerations of [3, 9], which were designed for the block-average transformation formally equally well apply to this map, but lead to an incorrect conclusion. This illustrates the danger of relying on formal as opposed to fully rigorous arguments. Also, the fact that the question whether the critical point is free of pathologies for block-average transformations has different answers depending on the block shape, illustrates that a general argument justifying the use of RG-algoritms may not exist. In view of the many numerically sucessful and intuitively appealing applications of the theory this situation is a rather unsatisfactory.

2.If we consider the sequence $\mu \circ T_L$ for the highly non-linear example mentioned above, starting from a high temperature state, as a corollary to the (local) central limit theorem [32, 8, 7, 58], it is easy to see that, with a proper rescaling of the block spins, this sequence converges weakly to an independent Gaussian zero-mean measure. The measures in the sequence are almost all (that is, all except for possibly a finite number) non-Gibbsian.

3.If we repeat the 1 by 2 block-average transformation, in sufficiently high dimension (presumably $d \geq 3$), acting on the critical Ising Gibbs measure, a central limit theorem holds [58], and the critical measure shows "high temperature" behaviour as opposed to the occurrence of non-Gibbsianness which is more like a "low-temperature" behaviour.

4.One approach which might look more hopeful is to consider renormalization transformations which are non-local in some sense. One could either think of momentum-space transformations, or try to renormalize non-local objects like contours. It seems that the momentum-space maps suffer from the same type of problems as real-space transformations [31, 67], but an implementation of renormalization-group ideas on contour models looks more promising, at least for the description of first-order phase transitions [19, 5].

5.In non-equilibrium statistical mechanics there are many open questions about the occurrence of non-Gibbsian measures. The term non-Gibbsian or non-reversible is often used for invariant measures in systems in which there is no detailed balance [41, 17, 16]. It is an open question when such measures are non-Gibbsian in the sense we have described here. It has been conjectured that such measures for which there is no detailed balance are quite generally non-Gibbsian in systems with a stochastic dynamics, see for example [40] or [17], Appendix 1; on the other hand it has been predicted that non-Gibbsian measures are rather exceptional ([41], Open problem IV.7.5, p.224) at least for non-reversible spin-flip processes under the assumptions of rates which are bounded away from zero. The examples we have are for the moment too few to develop a good intuition on this point.

Thus, although the number of examples of non-Gibbsian measures is on the increase, a good general description or classification is still an outstanding problem. Indeed, non-Gibbsian measures occur in quite different areas of statistical mechanics, and can have quite different properties. By now it seems somewhat surprising that it took so long to realize the fact that the Gibbs property is rather special, in particular in view of Israel's [34] result that in the set of all translation invariant (ergodic, nonnull) measures, Gibbs measures are exceptional.

Acknowledgments: My contribution reflects mainly joint work with Roberto Fernández and Alan Sokal, and also partly with Roman Kotecký and József Lörinczi. I had very useful conversations with the late R. L. Dobrushin, who first asked me about the possible occurrence of pathologies at high and possibly very high temperatures during a workshop in les Houches, and at a workshop at the Erwin Schrödinger Institute in Vienna and who presented his ideas and results on robustness of the Gibbs property at a workshop in Renkum the Netherlands. I am grateful for the occasions I had to profit from his always stimulating and challenging discussions and questions. His scientific contributions have been of immense value and were matched by a wonderful personality. I thank Geoffrey Grimmett and Enzo Olivieri, who explained to me how imposing constraints can raise, as well as lower, critical temperatures. At the Budapest meeting I had useful discussions with Enzo Olivieri and Tom Kennedy about the behaviour of renormalization group maps around critical points. I also thank Jean Bricmont, Krist Maes and Koen Vande Velde and Marinus Winnink for very helpful discussions and correspondence. This research has been supported by EU contract CHRX-CT93-0411.

References

[1] M. Aizenman, J. T. Chayes, L. Chayes and C. M. Newman. Discontinuity of the magnetization in the one-dimensional $\frac{1}{|x-y|^2}$ Ising and Potts models. *J. Stat. Phys.*, 50:1–40, 1988.

[2] G. Benfatto and G. Gallavotti. *Renormalization group*, Physics Notes, Vol.1. Princeton University Press, Princeton, 1995.

[3] G. Benfatto, E. Marinari and E. Olivieri. Some numerical results on the block spin transformation for the $2d$ Ising model at the critical point. *J. Stat. Phys.*, 78:731–757, 1995.

[4] C. Borgs and J. T. Chayes. The covariance matrix of the Potts model: a random cluster analysis. *J. Stat. Phys.*, 82:1235-1297, 1996.

[5] A. Bovier and C. Külske. A rigorous renormalization group method for interfaces in random media. *Rev. Math. Phys.*, 6:413-496, 1994.

[6] J. Bricmont and A. Kupiainen. Private communication, 1995.

[7] C. Cammarota. The large block spin interaction. *Nuovo Cimento B*, 96:1–16, 1986.

[8] E. A. Carlen and A. Soffer. Entropy production by block spin summation and central limit theorems. *Comm. Math. Phys.*, 140:339–371, 1992.

[9] M. Cassandro and G. Gallavotti. The Lavoisier law and the critical point. *Nuovo Cimento B*, 25:691–705, 1975.

[10] E. N. M. Cirillo and E. Olivieri. Renormalization-group at criticality and complete analyticity of constrained models: a numerical study. Bari-Roma preprint, 1996.

[11] R. L. Dobrushin. Lecture given at the workshop "Probability and Physics", Renkum, Holland, 1995.

[12] R. L. Dobrushin and S. B. Shlosman. Completely analytical interactions: constructive description. *J. Stat. Phys.*, 46:983–1014, 1987.

[13] C. Domb and M. S. Green (Eds.). *Phase transitions and critical phenomena*, Vol.6. Academic Press, New York, 1976.

[14] T. C. Dorlas and A. C. D. van Enter. Non-Gibbsian limit for large-block majority spin transformations. *J. Stat. Phys.*, 55:171-181, 1989.

[15] R. G. Edwards and A. D. Sokal. Generalization of the Fortuin-Kasteleyn-Swendsen-Wang representation and Monte Carlo algorithm. *Phys. Rev. D*, 38:2009–2012, 1988.

[16] M. H. Ernst and H. J. Bussemaker. Algebraic spatial correlations in lattice gas automata violating detailed balance. *J. Stat. Phys.*, 81:515-536, 1995.

[17] G. L. Eyink, J. L. Lebowitz and H. Spohn. Hydrodynamics and fluctuations outside of local equilibrium: driven diffusive systems. preprint, 1995, Austin math. phys. archives, 95-168.

[18] R. Fernández and C.-Ed. Pfister. Non-quasilocality of projections of Gibbs measures. EPFL preprint, 1994, Austin math. phys. archives, 94-113.

[19] K. Gawędzki, R. Kotecký and A. Kupiainen. Coarse graining approach to first order phase transitions. *J. Stat. Phys.* 47:701-724, 1987.

[20] H.-O. Georgii. *Gibbs Measures and Phase Transitions.* Walter de Gruyter (de Gruyter Studies in Mathematics, Vol. 9), Berlin–New York, 1988.

[21] N. Goldenfeld. *Lectures on phase transitions and the renormalization group.* Addison-Wesley, Frontiers in Physics 85, 1992.

[22] R. B. Griffiths and P. A. Pearce. Position-space renormalization-group transformations: Some proofs and some problems. *Phys. Rev. Lett.*, 41:917–920, 1978.

[23] R. B. Griffiths and P. A. Pearce. Mathematical properties of position-space renormalization-group transformations. *J. Stat. Phys.*, 20:499–545, 1979.

[24] G. Grimmett. The stochastic random-cluster process and the uniqueness of random-cluster measures. Cambridge University preprint, to appear in *Ann. Prob.*, 23, 1995, see also The Random-cluster Model. Chapter 3 in: Probability, Statistics and Optimisation, Ed. F. P. Kelley, Wiley and Sons Ltd., 1994.

[25] O. Häggström. On the relation between finite-range potentials and subshifts of finite type. *Prob. Th. Rel. Fields*, 101:469-478, 1995.

[26] O. Häggström. A subshift of finite type that is equivalent to the Ising model. *Erg. Th. Dyn. Sys.*, 15:543-556, 1995.

[27] O. Häggström On phase transitions for subshifts of finite type. *Isr. J. Math.*, to appear, 1996.

[28] O. Häggström. Aspects of spatial random processes. Chalmers University of Göteborg thesis, 1994.

[29] O. Häggström. Almost sure quasilocality fails for the random-cluster model on a tree. Chalmers University of Göteborg preprint, to appear in *J. Stat. Phys.*, 1996.

[30] K. Haller and T. Kennedy. Absence of renormalization group pathologies near the critical temperature–two examples. University of Arizona preprint, Austin Archives 95-505, 1995.

[31] A. Hasenfratz and P. Hasenfratz. Singular renormalization group transformations and first order phase transitions (I). *Nucl. Phys. B*, 295[FS21]:1-20, 1988.

[32] D. Iagolnitzer and B. Souillard. Random fields and limit theorems. In J. Fritz, J. L. Lebowitz, and D. Szász, editors, *Random Fields (Esztergom, 1979), Vol. II*, pages 573–592. North-Holland, Amsterdam, 1981.

[33] R. B. Israel. Banach algebras and Kadanoff transformations. In J. Fritz, J. L. Lebowitz, and D. Szász, editors, *Random Fields (Esztergom, 1979), Vol. II*, pages 593–608. North-Holland, Amsterdam, 1981.

[34] R. B. Israel. Private communication, 1992.

[35] I. A. Kashapov. Justification of the renormalization-group method. *Theor. Math. Phys.*, 42:184–186, 1980.

[36] T. Kennedy. Some rigorous results on majority rule renormalization group transformations near the critical point. *J. Stat. Phys.*, 72:15–37, 1993.

[37] T. Kennedy. Budapest meeting on disordered systems and statistical physics, rigorous results, august 1995.

[38] O. K. Kozlov. Gibbs description of a system of random variables. *Prob. Inf. Transmission*, 10:258–265, 1974.

[39] J. L. Lebowitz and C. Maes. The effect of an external field on an interface, entropic repulsion. *J. Stat. Phys.*, 46:39–49, 1987.

[40] J. L. Lebowitz and R. H. Schonmann. Pseudo-free energies and large deviations for non-Gibbsian FKG measures. *Prob. Th. and Rel. Fields*, 77:49-64, 1988.

[41] T. M. Liggett. Interacting particle systems. Springer, Berlin, 1995.

[42] J. Lörinczi. Some results on the projected two-dimensional Ising model. In M. Fannes, C. Maes, and A. Verbeure, editors, *Proceedings NATO ASI Leuven Workshop "On Three Levels"*, pages 373–380, Plenum Press, 1994.

[43] J. Lörinczi. On limits of the Gibbsian formalism in thermodynamics. University of Groningen thesis, 1995.

[44] J. Lörinczi. Quasilocality of projected Gibbs measures through analyticity techniques. University of Groningen preprint, 1995, to appear in *Helv. Phys. Acta*, 1996.

[45] J. Lörinczi and K. Vande Velde. A note on the projection of Gibbs measures. *J. Stat. Phys.*, 77:881–887, 1994.

[46] J. Lörinczi and M. Winnink. Some remarks on almost Gibbs states. In N. Boccara, E. Goles, S. Martinez, and P. Picco, editors, *Cellular Automata and Cooperative Systems*, pages 423–432, Kluwer, Dordrecht, 1993.

[47] S. K. Ma. *Modern theory of critical phenomena.* Benjamin, Reading, Massachusetts, 1976.

[48] C. Maes and K. Vande Velde. Defining relative energies for the projected Ising measure. *Helv. Phys. Acta*, 65:1055–1068, 1992.

[49] C. Maes and K. Vande Velde. The (non-)Gibbsian nature of states invariant under stochastic transformations. *Physica A*, 206:587–603, 1994.

[50] C. Maes and K. Vande Velde. The fuzzy Potts model. *J. Phys. A, Math. Gen* 28:4261–4271, 1995.

[51] C. Maes and K. Vande Velde. Relative energies for non-Gibbsian states. Leuven preprint, 1996.

[52] F. Martinelli and E. Olivieri. Some remarks on pathologies of renormalization-group transformations. *J. Stat. Phys.*, 72:1169–1177, 1993.

[53] F. Martinelli and E. Olivieri. Instability of renormalization-group pathologies under decimation. *J. Stat. Phys.*, 79:25–42, 1995.

[54] F. Martinelli and E. Olivieri. Approach to equilibrium of Glauber dynamics in the one phase region I. *Comm. Math. Phys.*, 161:447–486, 1994.

[55] F. Martinelli and E. Olivieri. Approach to equilibrium of Glauber dynamics in the one phase region II. *Comm. Math. Phys.*, 161:487–515, 1994.

[56] F. Martinelli, E. Olivieri and R. H. Schonmann. For 2-D lattice spin systems weak mixing implies strong mixing. *Comm. Math. Phys.*, 165:33–48, 1994.

[57] F. Martinelli and E. Scoppola. A simple stochastic cluster dynamics: rigorous results. *J. Phys. A, Math. Gen.*, 24:3135–3157, 1991.

[58] C. M. Newman. Normal fluctuations and FKG inequalities. *Comm. Math. Phys.*, 74:119-128, 1980.

[59] Th. Niemeijer and J. M. J. van Leeuwen. Renormalizaton Theory for Ising-like spin systems. In *Phase Transitions and Critical Phenomena*, Vol.6, pp.425–505, C. Domb and M. S. Green, editors, Academic Press, New York, 1976.

[60] C.-Ed. Pfister and K. Vande Velde. Almost sure quasilocality in the random cluster model. *J. Stat. Phys.*, 79:765–774, 1995.

[61] J. Salas. Low temperature series for renormalized operators: the ferromagnetic square Ising model. *J. Stat. Phys.*, 80:1309–1326, 1995.

[62] R. H. Schonmann. Projections of Gibbs measures may be non-Gibbsian. *Comm. Math. Phys.*, 124:1–7, 1989.

[63] R. H. Schonmann and S. B. Shlosman. Complete analyticity for 2D Ising completed. *Comm. Math. Phys.*, 170:453–482, 1995.

[64] E. R. Speer. The two species totally asymmetric simple exclusion process. *Proceedings NATO ASI Leuven Workshop "On three levels"*, M. Fannes, C. Maes and A. Verbeure, editors, 91–103, Plenum Press, 1994.

[65] W. G. Sullivan. Potentials for almost Markovian random fields. *Comm. Math. Phys.*, 33:61–74, 1973.

[66] A. C. D. van Enter. Ill-defined block-spin transformations at arbitrarily high temperatures. *J. Stat. Phys.*, 83:761–765, to appear, 1996.

[67] A. C.D. van Enter and R. Fernández. In preparation.

[68] A. C. D. van Enter, R. Fernández, and R. Kotecký. Pathological behavior of renormalization group maps at high fields and above the transition temperature. *J. Stat. Phys.*, 79:969–992, 1995.

[69] A. C. D. van Enter, R. Fernández, and A. D. Sokal. Renormalization transformations in the vicinity of first-order phase transitions: What can and cannot go wrong. *Phys. Rev. Lett.*, 66:3253–3256, 1991.

[70] A. C. D. van Enter, R. Fernández, and A. D. Sokal. Regularity properties and pathologies of position-space renormalization-group transformations: Scope and limitations of Gibbsian theory. *J. Stat. Phys.*, 72:879–1167, 1993.

[71] A. C. D. van Enter, R. Fernández, and A. D. Sokal. Renormalization transformations: Source of examples and problems in probability and statistics. *Resenhas do Instituto de Matematica e Estatistica da Universidade de Sao Paulo*, 1:233-262, 1994.

[72] A. C. D. van Enter and J. Lörinczi. Robustness of the non-Gibbsian property: some examples. University of Groningen preprint, 1995, *J. Phys. A, Math. and Gen.*, to appear, 1996.

[73] K. Vande Velde. Private communication, 1994.

[74] K. Vande Velde. On the question of quasilocality in large systems of locally interacting components. K.U. Leuven thesis, 1995.

[75] K. G. Wilson and J. Kogut. The renormalization group and the ϵ-expansion. *Phys. Rep.*, 12C:75-200, 1974.