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VACUUM STRUCTURE, LORENTZ SYMMETRY AND SUPERLUMINAL PARTICLES (I)

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Abstract

If textbook Lorentz invariance is actually a property of the equations describing a sector of the excitations of vacuum above some critical distance scale, several sectors of matter with different critical speeds in vacuum can coexist and an absolute rest frame (the vacuum rest frame) may exist without contradicting the apparent Lorentz invariance felt by "ordinary" particles (particles with critical speed in vacuum equal to c , the speed of light). Sectorial Lorentz invariance, reflected by the fact that all particles of a given dynamical sector have the same critical speed in vacuum, will then be an expression of a fundamental sectorial symmetry (e.g. preonic grand unification or extended supersymmetry) protecting a parameter of the equations of motion. Furthermore, the sectorial Lorentz symmetry may be only a low-energy limit, in the same way as the relation ω (frequency) = c_s (speed of sound) k (wave vector) holds for low-energy phonons in a crystal. We study the consequences of such a scenario, using an ansatz inspired by the Bravais lattice as a model for some vacuum properties. It then turns out that: a) the Greisen-Zatsepin-Kuzmin cutoff on high-energy cosmic protons and no longer apply; b) high-momentum unstable particles have longer lifetimes than expected with exact Lorentz invariance, and may even become stable at the highest observed cosmic ray energies or slightly above. Some cosmological implications of superluminal particles are also discussed.

1 . VACUUM EXCITATIONS AND "ELEMENTARY" PARTICLES

Sparnay and Lamoreaux [1] have experimentally confirmed the Casimir effect [2 - 4] based on the quantum-field-theoretical interpretation of elementary particles, which states that quantum fields are harmonic oscillators in vacuum and have therefore a nonzero ground energy. Thus, the so-called "elementary particles" are actually quantum oscillators of vacuum degrees of freedom and conventional quantum field theory is based on the harmonic approximation to these oscillators. Quantum mechanics seems to arise at a deeper level than the description of the so-called "free particles". This raises a fundamental question: can matter be understood just from a phenomenological study of the excitations of vacuum? Obvioulsy, the matter forming a ionic crystal cannot be described just in terms of the crystal phonon spectrum. On the other hand, our present knowledge of vacuum excitations (quarks, leptons, gluons, electroweak gauge bosons, string models...) is likely to contain, if correctly interpreted, important information on vacuum dynamics itself. A most remarkable fact is that all the above-mentioned excitations of vacuum seem to have the same critical speed in this medium. The dynamical origin of such a symmetry is far from trivial if we adopt the philosophy that the apparent structure of space and time, as seen by matter, is actually determined by the properties of matter itself (e.g. [5 - 11]). This would not really be an unorthodox approach, as standard (inflationary) cosmology generates space from the expansion (creation) of matter.

1a. Vacuum and particles: an analogy with the Bravais lattice

To understand, in the absence of absolute prescriptions from an intrinsic space-time geometry, the meaning of this apparent universality of the critical speed in vacuum, we can attempt a simple analogy with solid state physics. Assume a classical system similar to the monoatomic one-dimensional Bravais lattice [12] with a very large number of coupled oscillators regularly spaced by a , π times the inverse of the critical wave vector k_0 (k_0 and $-k_0$ leading to the same wave function). On each site n , a complex parameter $\phi(n)$ satisfies an equation implying nearest-neighbour coupling, i.e. :

$$d^2/dt^2 \ \ [\phi \ (n)] \ = \ K \ [2 \ \phi \ (n) \ - \ \phi \ (n-1) \ - \ \phi \ (n+1)] \ - \ \omega_{rest}^2 \ \phi \qquad (1)$$

which admits, for wave vector k ($-k_0 \ \leq \ k \ \leq \ k_0$), solutions of the type:

$$\phi_k(n \ , \ t) \ = \ \phi_k \ (0) \ \ exp \ [i \ (k \ n \ a \ - \ \omega \ t)] \qquad (2)$$

with

$$\omega^2 \ (k) \ = \ 2 \ K \ [1 \ - \ cos \ (k \ a)] \ + \ \omega_{rest}^2 \ = \ 4 \ K \ sin^2 \ (ka/2) \ + \ \omega_{rest}^2 \qquad (3)$$

which, at low k and taking positive energy solutions, can be written as:

$$\omega \ (k) \ \simeq \ [K \ (a \ k)^2 \ + \ \omega_{rest}^2]^{1/2} \qquad (4)$$

as in standard special relativity. We expect $\omega_{rest} \ = \ 0$ as long as global fluctuations have no energy. Each plane wave $\phi_k(n \ , \ t)$ can be viewed as a complex harmonic oscillator. The speed $d\omega/dk$ is basically determined by the spring constant K and the critical wavelength $k_0 = \pi \ a^{-1}$. With the above approximations, and interpreting the lattice as the physical vacuum, a classical massive field is obtained which can be quantized to give a charged massive particle. By this simple procedure, a particle has been generated whose critical speed in vacuum is explicitly related to dynamical parameters of this vacuum (K , a). If, as it seems to occur in our world, many particles have the same critical speed in vacuum, they must correspond to oscillators with the same values of these dynamical parameters. The observed symmetries must correspond to symmetries of the inner vacuum dynamics (preonic symmetry, supersymmetry...) and may survive beyond the scales where the particles cease to exist. Instead of a complex parameter, we can take in the previous example a unitary operator $u\ (n\ ,\ n-1)$ associated to links between sites. If $u\ (n\ ,\ n-1)$ belongs to the unitary group $U\ (N)$, λ is a hermitic generator of $U\ (N)$ and α a real parameter, the plane wave:

$$u_k\ (n\ ,\ n-1\ ;\ t) \quad = \quad exp\ [i\ (k\ n\ a\ -\ \omega\ t)]\ exp\ (i\ \alpha\ \lambda) \tag{5}$$

satisfies the equation:

$$d^2/dt^2\ \ [u\ (n\ ,\ n-1)] \quad = \quad K\ [2\ u\ (n\ ,\ n-1)\ -\ u\ (n-1\ ,\ n-2)\ -\ u\ (n+1\ ,\ n)] \tag{6}$$

for:

$$\omega^2\ (k) \quad = \quad 2\ K\ [1\ -\ cos\ (k\ a)] \quad = \quad 4\ K\ sin^2\ (ka/2) \tag{7}$$

The hermitic vector field, which satisfies (6) and (7) similar to u , is actually:

$$A\ (n\ ,\ n-1) \quad = \quad (2i)^{-1}\ [u\ (n\ ,\ n-1)\ -\ u\ (n-1\ ,\ n)] \tag{8}$$

where $u\ (n-1\ ,\ n) \quad = \quad u^\dagger\ (n\ ,\ n-1)$. A is hermitic and contains both positive and negative frequencies:

$$2\ A_k\ (n\ ,\ n-1) \quad = \quad a_k^\dagger\ exp\ [-i\ (k\ n\ a\ -\ \omega\ t)]\ +\ a_k\ exp\ [i\ (k\ n\ a\ -\ \omega\ t)] \tag{9}$$

where $a_k \quad = \quad -i\ exp\ (i\ \alpha\ \lambda)$. In this way, nine massless bosons can be generated, as λ varies, possibly equivalent to a set of $U(N)$ gauge bosons. Matter fields will exhibit (up to a mass) the same relation between energy and momentum as gauge fields if they oscillate with the same parameters K and a as the internal symmetry links, which can be a natural assumption if in both cases we are dealing with nearest-neighbour interactions of the same family of degrees of freedom. In this scenario, the violation of Lorentz symmetry introduced at high energy by equations (3) and (7) will not be accompanied by a breaking of the universality of the relation between energy and momentum in the zero-mass limit. Although masses and other related phenomena will introduce low-energy corrections to this universality, the high-momentum dynamics can keep it unbroken. An ansatz for a general formula describing kinematics in the vacuum rest frame can be as before:

$$\omega^2\ (k) \quad = \quad 2\ K\ [1\ -\ cos\ (k\ a)]\ +\ (2\pi)^2\ h^{-2}\ E_{rest}^2 \tag{10}$$

where h is the Planck constant and E_{rest} the rest energy of the particle. The "speed of light", as measured at low energy in the limit $k\ \rightarrow\ 0$, is $c\ =\ K^{1/2}\ a$. The inertial mass of the particle in the same limit is, as usual, $m\ =\ E_{rest}\ c^{-2}$.

In such an analogy, no basic principle would prevent the vacuum from having other sets of degrees of freedom oscillating with different values of a and K . These degrees of freedom may generate superluminal sectors of matter, as considered in [5 - 11] . If K_i and a_i are the values of K and a for the i-th superluminal sector, we may try for these superluminal particles in the vacuum rest frame the ansatz:

$$\omega^2 \ (k) \ = \ 2 \ K_i \ [1 \ - \ cos \ (k \ a_i)] \ + \ (2\pi)^2 \ h^{-2} \ E^2_{rest} \tag{11}$$

where, from [5 - 10] , the mass of the particle is related to its rest energy by the relation $E_{rest} \ = \ m \ c_i^2$, and $c_i \ = \ K_i^{1/2} \ a_i \ = \ \pi \ K_i \ k_i^{-1}$, where k_i is the critical wavelength. Interaction between different dynamical sectors would then lead to energy and momentum transfer between different kinds of excitation modes. As stressed in [5 - 11] , the superluminal particles we consider would not be tachyons and would have quite different physical properties from those predicted for tachyonic objects [13] . Contrary to tachyons, the new superluminal particles necessarily violate the relativity principle [14] and would produce "Cherenkov" radiation (spontaneous emission of particles with lower critical speed) in vacuum (see [5 - 11] , and also [15]). They can play an important role in high-energy phenomena, as well as in cosmology, and yield detectable signatures at accelerators (e.g. LHC) or in cosmic ray experiments (e.g. AMANDA [16]).

Obviously, kinematics from (10) and (11) is just a rough example parameterizing possible trends of scenarios where Lorentz invariance is only a low-energy limit and "ordinary" particles (those with critical speed in vacuum equal to c , the speed of light) cease to exist at distance scales below a . However, it may allow for useful discussions of Lorentz symmetry violation and of other related phenomena. Consequences in cosmology can be important, especially for the Big Bang scenario [17 , 18] . Quantum field theory [19 - 21] should take these penomena into account, especially when discussing cutoffs and renormalization, but its validity is not in principle altered by their existence. Contrary to tachyons [22 , 23] , the proposed superluminal particles would not violate causality [10] . From formula (10) , the velocity of an "ordinary" particle would be (E = energy, p = momentum):

$$v \ = \ dE/dp \ = \ d\omega/dk \ = \ K^{1/2} \ c \ \omega^{-1} \ sin \ (k \ a) \tag{12}$$

which tends to zero as k approaches $\pm\pi a^{-1}$. Thus, a particle with the highest permitted energy and wave vector would be at rest (zero speed) with respect to the vacuum rest frame, even if it has a high momentum. In this limit, the frequency tends to $\omega^{max} \ = \ [4 \ K \ + \ (2\pi)^2 \ h^{-2} \ E^2_{rest}]^{1/2}$. Similar expressions can be obtained for superluminal particles. In the early Universe, the characteristic phase transition temperature sclaes [2 , 9] would be:

$$T_0 \ \approx \ k_B^{-1} \ h \ K^{1/2} \tag{13}$$

for the ordinary sector, k_B being the Boltzmann constant, and:

$$T_i \ \approx \ k_B^{-1} \ h \ K_i^{1/2} \tag{14}$$

for each superluminal sector. The cosmological phase transition temperatures depend, for each sector, on the sectorial spring constant and not on the critical wavelength scale. An interesting scenario would be, for all i : $a_i = a$; $K_i \gg K$. Then, the "Planck scale" may still make sense as a distance scale, but the "Planck temperature" (equivalent to a Debye temperature) would be different for different sectors. Above T_0 , only superluminal particles would in principle exist and "Cherenkov" radiation in vacuum may have been inhibited by the very short time scale. In this case, the Universe would have quickly cooled down just after having reached each critical temperature (T_0 and the T_i's). Another possible scenario would be: $a_i \gg a$; $c_i \gg c$ which requires very large values of K_i but is not prevented by any basic principle. In such case, a question arises: since superluminal particles remain "elementary" at scales where the internal structure of ordinary particles (quarks, leptons, gauge bosons...) shows up, are the former part of the constituents of the vacuum structure generating the latter? If the answer is positive for one of the superluminal sectors, transparency of vacuum with respect to superluminal particles of this sector would not be a trivial problem. Superluminal particles raise several fundamental questions concerning their origin and properties as possible components of vacuum dynamics.

1b. Further analogies

The analogy with solid state physics raises in itself two interesting questions:

- Do *umklapp* processes [12] take place at very high energy? In such processes, the total momentum would be conserved only up to an integer multiple of $h\ a^{-1}\ \vec{\mathbf{u}}$ for ordinary particles, and of $h\ a_i^{-1}\ \vec{\mathbf{u}}$ for superluminal particles, where $\vec{\mathbf{u}}$ is a unitary vector. This (or a more involved scenario) cannot be excluded, as there is no guarantee that translation invariance remains valid below the a and a_i distance scales. Dynamics would then depend crucially on the topology of momentum space. This topological space can be the product of three circles like in a crystal lattice. But it can also be, for instance, a sphere with antipodes identified on its surface (i.e. $k_0\ \vec{\mathbf{u}} = -\ k_0\ \vec{\mathbf{u}}$ for any unitary vector $\vec{\mathbf{u}}$). Then, rotation invariance would be preserved and expression (10) can naturally apply to three space dimensions. We can also consider a momentum topology where all points on the surface $\mid \vec{\mathbf{u}} \mid = k_0$ of the momentum sphere would be identified to a single point (making the momentum space topologically similar to the $SU(2)$ group). *Umklapp* processes would modify the thermal conductivity of the very early Universe, and make more difficult heat exchanges between different regions of space at high temperature.

- Is the "acoustic branch" [12] the only dynamical branch of ordinary or superluminal particles? The question is not merely academic, as the contrary may imply the existence of "optical" particles with: a) negative inertial mass at zero momentum, but positive rest energy; b) minimum energy at maximum momentum. Such particles would be very heavy and, at high energy, undergo a repulsive acceleration in the presence of a static, attractive, gravitational feld. They may have crucially influenced the expansion of the Universe. "Optical" particles can be generated replacing in the previous analogy the monoatomic lattice by a diatomic one. We would then be led in the vacuum rest frame, in a similar way to [12] in one space dimension and assuming rotation invariance when generalizing the ansatz to the three-dimensional case, to the frequency spectrum:

$$\omega^2 \ = \ K_0 \ \pm \ K_1\ (k) \qquad (15)$$

with:

$$K_0 \ = \ K \ + \ G \ + \ \lambda_1 \ + \ \lambda_2 \tag{16}$$

$$K_1 \ = \ [K^2 \ + \ G^2 \ + \ 2 \ K \ G \ cos \ (ka) \ + \ (\lambda_1 \ - \ \lambda_2)^2]^{1/2} \tag{17}$$

where $2\lambda_1$, $2\lambda_2$ are monoatomic elastic constants similar to ω^2_{rest} in the previous example and K , G govern nearest-ion interactions [12] . All these constants are positive in realistic examples based on harmonic oscillators. The solution with the − sign ("acoustic" particles) can be dealt with in a similar way to the monoatomic case, and leads to a massless particle if $\lambda_1 \ = \ \lambda_2 \ = \ 0$. Writing $\delta \ = \ \lambda_1 \ - \ \lambda_2$, the solution with a + sign (the "optical branch") has rest (zero momentum) frequency:

$$E_{rest} \ (optical) \ = \ (2\pi)^{-1} \ h \ [K_0 \ + \ (K^2 \ + \ G^2 \ + \ 2 \ K \ G \ + \ \delta^2)^{1/2}]^{1/2} \tag{18}$$

and negative values of $d^2\omega/dk^2$. In the limit where k approaches $\pm\pi \ a^{-1}$, the energy approaches the positive value:

$$E_{min} \ (optical) \ = \ (2\pi)^{-1} \ h \ [K_0 \ + \ (K^2 \ + \ G^2 \ - \ 2 \ K \ G \ + \ \delta^2)^{1/2}]^{1/2} \tag{19}$$

and the particle would also be at rest (zero speed) in this limit, where it has its minimum possible energy and its maximum permitted momentum. Since this is the less energetic state, it is likely that "optical" particles tend to be in such a state in the present epoch. There, for $\vec{\mathbf{k}} \ = \ k_0 \ \vec{\mathbf{u}}$ where $\vec{\mathbf{u}}$ is an arbitrary unitary vector, they would have a large, positive inertial mass in the direction of their momentum and infinite inertial mass in the two other directions. More precisely, we can write for an "optical" particle around maximum wavelength $\vec{\mathbf{k}} \ = \ k_0 \ \vec{\mathbf{u}}$ submitted to a static external force the hamiltonian:

$$H \ = \ (2\pi)^{-1} \ h \ [K_0 \ + \ K_1 \ (k)]^{1/2} \ + \ V \ (\vec{\mathbf{r}}) \tag{20}$$

where $\vec{\mathbf{r}}$ is the position vector and, with the relation $\vec{\mathbf{k}} \ = \ 2\pi \ h^{-1} \ \vec{\mathbf{p}}$ where $\vec{\mathbf{p}}$ is the momentum, the classical Hamilton equations:

$$\vec{\mathbf{v}} \ = \ 2\pi \ h^{-1} \ \vec{\nabla}_{\mathbf{k}} \ H \ = \ - \ (a \ k^{-1}/2) \ [K_0 \ + \ K_1 \ (k)]^{-1/2} \ sin \ (ka) \ \vec{\mathbf{k}} \ dK_1/d[cos(ka)] \tag{21}$$

$$d \ \vec{\mathbf{k}}/dt \ = \ - \ 2\pi \ h^{-1} \ \vec{\nabla} \ V \ (\vec{\mathbf{r}}) \tag{22}$$

where $\vec{\nabla}_{\mathbf{k}}$ stands for gradient in wave vector space. For a unitary vector $\vec{\mathbf{u}}$:

$$d \ \vec{\mathbf{v}}/dt \ |_{\vec{\mathbf{k}} \ = \ k_0\vec{\mathbf{u}}} \ = \ - \ [2 \ E_{min} \ (optical)]^{-1} \ K \ G \ [K \ (k_0)]^{-1} \ a^2 \ [\vec{\mathbf{u}}.\vec{\nabla} \ V \ (\vec{\mathbf{r}})] \ \vec{\mathbf{u}} \tag{23}$$

Therefore, an "optical" particle at minimum energy can be accelerated only by a force parallel to its momentum. A condensate of "optical" particles in vacuum may then spontaneously break rotation invariance, as "optical" particle-antiparticle pairs would be at rest with maximum momenta pointing, locally, in arbitrary directions. The observed isotropy of cosmic microwave background radiation as well as our daily experience and the long range of gravitational forces seem to indicate that such an effect, as felt by "acoustic" particles, must be very small when observed at large distance scales. At shorter distance scales (e.g. those reached by accelerator experiments), it may be worth to perform precision tests of rotation invariance. The threshold for "optical" particle production would be given by the energy at maximum wave vector (we expect masses $\approx 10^{19}$ GeV c^{-2} for $a \approx 10^{-33}$ cm), and not by the $k = 0$ rest energy as for "acoustic" particles. Particles of the "acoustic branch" would be faster than those of the "optical branch", but their maximum energy would be lower than the minimum energy of "optical" particles. "Optical" particles, or instead more complicated objects, may indeed appear at very high energy as a consequence of nontrivial vacuum structure. As they seem to be in "one-to-one" correspondence with "acoustic" particles, we consider them as being part of the same dynamical sector. The question arises whether such "optical" objects, whatever they are, can play a role in the renormalization of quantum field theories. In any case, it seems difficult to imagine how "bare" particles could reasonably be defined without taking into account physics at the natural cut-off scales where "point-like" interactions do no longer make sense.

In principle, there is no basic reason for "optical" particles to couple to the "acoustic" graviton in the same way as "acoustic" particles. For instance, the low-momentum Lorentz symmetry becomes euclidean and with a different value of the critical speed. "Optical" particles correspond to fundamentally different vacuum excitations and would not necessarily decay into "acoustic" ones. If such decays do not occur, some "optical" particles may be stable. Since they are in one-to-one correspondence with "acoustic" particles, "optical" particles can possibly couple to "acoustic" internal-symmetry gauge bosons. If ordinary particles are excitations of vacuum degrees of freedom associated to a condensate of particles of the i-th superluminal sector and involving a long range superluminal force, ordinary "optical" particles at small wave vector may mix with the superluminal field, in the same way as optical phonons mix with the electromagnetic field in a ionic crystal [12]. This would give rise to superluminal "polaritons" propagating in vacuum.

If "optical" particles have significant couplings to "acoustic" gravitation and internal-symmetry gauge bosons, several interesting phenomena can be expected. Interactions between "optical" and "acoustic" particles would present rather unconventional features. The low-energy hamiltonian for a system formed by an "acoustic" particle of mass m_a and an "optical" particle of effective inertial mass M_O will be in the vacuum rest frame:

$$H \simeq p_a^2 \ (2 \ m_a)^{-1} \ + \ (2M_O)^{-1} \ (2\pi \ a)^{-2} \ h^2 \ sin^2(ka) \ + \ V \ (\vec{\mathbf{r}}) \tag{24}$$

where $\vec{\mathbf{p}}_a$ is the momentum of the "acoustic" particle, m_a its mass, M_O the effective inertial mass of the "optical" particle, $\vec{\mathbf{k}}$ its wave vector, $\vec{\mathbf{r}} = \vec{\mathbf{r}}_a - \vec{\mathbf{r}}_O$, $\vec{\mathbf{r}}_a$ and $\vec{\mathbf{r}}_O$ the position vectors of the "acoustic" and the "optical" particle, and V $(\vec{\mathbf{r}})$ the potential energy. If the "optical" particle is close to $k = k_0$, we can write:

$$sin^2 \ (k \ a) \ = \ sin^2 \ (k \ a \ - \ k_0 \ a) \ \approx \ (k \ a \ - \ k_0 \ a)^2 \tag{25}$$

$$H \simeq p_a^2 \ (2 \ m_a)^{-1} \ + \ (2M_O)^{-1} \ (p_O \ - \ h \ a^{-1}/2)^2 \ + \ V \ (\vec{\mathbf{r}}) \tag{26}$$

where $\vec{\mathbf{p}}_O$ is the momentum of the "optical" particle, and Hamilton equations lead to:

$$dp_O/dt \ = \ - \ dp_a/dt \ = \ \vec{\nabla} \ V \ (\vec{\mathbf{r}}) \tag{27}$$

$$\vec{\mathbf{v}}_a \ = \ m_a^{-1} \ \vec{\mathbf{p}}_a \tag{28}$$

$$\vec{\mathbf{v}}_O \;=\; M_O^{-1}\,(p_O \;-\; h\,a^{-1}/2)\,p_O^{-1}\,\vec{\mathbf{p}}_O \tag{29}$$

where $\vec{\mathbf{v}}_a$ and $\vec{\mathbf{p}}_O$ stand for the velocity of the "acoustic" and "optical" particle. The last equation can be turned into:

$$\vec{\mathbf{p}}_O \;=\; \vec{\mathbf{v}}_O\,v_O^{-1}\,(M_O\,v_O) \;+\; h\,a^{-1}/2) \tag{30}$$

so that the conserved total momentum $\vec{\mathbf{P}}$ is given by:

$$\vec{\mathbf{P}} \;\simeq\; m_a\,\vec{\mathbf{v}}_a \;+\; M_O\,\vec{\mathbf{v}} \;+\; h\,a^{-1}\,v_O^{-1}\,\vec{\mathbf{v}}_O/2 \tag{31}$$

It directly follows from this expression that, if the momentum transfer amounts only to a rotation of $\vec{\mathbf{p}}_O$, it will be mainly spent in the rotation of the zero-speed component of the momentum of the "optical" particle. Only a fraction $\simeq\ 2\,M_O\,v_O\,h^{-1}\,a$ would in such case lead to a modification the effective momentum $M_O\,v_O$ of the "optical" particle. In the limit where the "optical" particle is initially at rest, there will be no acceleration for a transverse infinitesimal momentum transfer. Thus, naive conservation laws will not apply to the "effective" total momentum $m_a\,\vec{\mathbf{v}}_a \;+\; M_O\,\vec{\mathbf{v}}_O$.

"Optical" particles may be unconventional dark matter candidates playing an important role in the formation of structure in the early Universe. As for "acoustic" particles, annihilation may have been prevented by matter-antimatter asymmetry. If such particles exist, are stable and have not annihilated, they will be present in the matter-dominated Universe. If their density is of the order of standard cosmological matter densities, and if they are practically at rest with respect to the vacuum rest frame (assumed to be close to that defined by cosmic microwave background), they will move at a speed $\approx 2.10^{-3}\,c$ with respect to the Local Group. We can then expect on earth fluxes $\approx 10^{-6}\,m^{-2}\,year^{-1}$ which would not be unaccessible to future cosmic ray detectors. An interesting question is whether they can be found in "acoustic" matter. If they have a significant cosmological density, couple to "acoustic" gravitation and have accreted with "acoustic" matter, they should be present in terrestrial materials. However, it should be noticed that the gravitational force they would feel on the surface of earth is $\approx 10^{12}\,eV\,m^{-1}\,G_{O,a}\,G_N^{-1}$ for a mass $M_O\ \approx 10^{-5}\,gm$ (the mass scale suggested by the above considerations), $G_{O,a}$ being the gravitational coupling constant between "acoustic" and "optical" particles, and G_N Newton's constant. If $G_{O,a}\,G_N^{-1}$ is not much below 1 , this force may have been strong enough to favour the diffusion of "optical" particles trough terrestrial matter towards the center of the earth. Dedicated experiments could, for instance, search in fine powders for very heavy particles with unconventional responses to external forces.

1c. The low-energy limit

The above analogies remind simple, well-known examples of how dynamics at a small scale of distance (the interatomic distance) with energy scale $\approx\ 1\,keV\ (T\ \approx\ 10^7\,K)$ can generate, and naturally protect, objects (the phonons) existing as massless excitations down to microkelvin temperatures ($\approx\ 10^{-10}\,eV$). A new dynamical interpretation of massless particles and gauge interactions may potentially emerge from such analogies, with inner vacuum dynamics replacing the standard geometric formulation of the gauge principle and chiral symmetries. For instance, rather than associating gauge fields to a mathematical way to compare local definitions of space-time or internal symmetry frames, we can admit that there is a well-defined physical way to make such comparisons but that the local frames and parameters fluctuate. Because of nearest-neighbour interactions, fluctuating dynamical links between sites are generated giving rise to the gauge bosons. In the above simple-minded model of links, the masslessness of the gauge boson would be equivalent to assuming that, whatever its value, no energy is spent in setting the matrix $u\ (n\ ,\ n-1)$ to be different from the identity provided its value is the same for all links. Gauge fields could possibly be interpreted as polarization states of vacuum with nearest-neighbour interaction between polarizations (similar to spin systems). The concept of "vacuum polarization" already appeared long ago in quantum electrodynamics without any explicit use of a precise vacuum structure [19 , 21] . With more than one space dimension, since the local polarization variable would naturally be a vector, we can associate it with links between sites but also with the sites themselves. In this last case, the above $u\ (n\ ,\ n-1)$ would rather become a space-vector $U(N)$ matrix $\vec{\mathbf{P}}\ (n)$ describing the polarization state of each site on the lattice. The use of links or of site variables would depend on the details of the dynamics and, possibly, on the scale at which the lattice is built. More fundamental is the absence of local elastic constants, such as ω_{rest} in (1) , in order to preserve the masslessness of the gauge bosons.

Apart from gravitational lenses and similar experiments able to feel the effect of dark matter, all the above phenomena would remain invisible to low-energy experiments. For instance, in the ordinary sector with $h\ k\ \approx\ 1\ keV/c$ and $a\ \approx\ 10^{-33}\ cm$, one has $(k\ a)^2\ \approx\ 10^{-52}$, which is clearly much too small to produce any detectable effect. High-energy experiments would be the only way to possibly reach sensitivity to this kind of Lorentz symmetry violation. Thus, the impressive bounds derived from low-energy experimental tests of Lorentz invariance (e.g. [24 , 25]) are not incompatible with the proposed scenario. A basic question can be raised: can we still justify the existence of gravity as a gauge interaction, if Lorentz invariance is just a low-energy limit? In the previous analogy, an answer could be that local fluctuations of the vacuum rest frame and of local dynamical parameters exist in any case. For instance, if $g_{\mu\nu}$ is the effective metric tensor in the low-energy Lorentz-invariant limit of the equations of motion, a fluctuation of the coefficients of the time derivative and of the nearest-neighbour interaction in (1) amounts to a fluctuation of the metric. More generally, we can consider (1) as a particular configuration of the equation:

$$A\ d^2/dt^2\ \ [\phi\ (n)]\ +\ H\ d/dt\ [\ \phi\ (n+1)\ -\ \phi\ (n-1)]\ -$$

$$-\ K_{fl}\ [2\ \phi\ (n)\ -\ \phi\ (n-1)\ -\ \phi\ (n+1)]\ -\ \omega_{rest}^2\ \phi\ =\ 0 \tag{32}$$

In the continuum limit, the coefficients $A\ =\ g_{00}$, $H\ =\ g_{01}\ =\ g_{10}$ and $-K_{fl}\ =\ g_{11}$ can be regarded as the matrix elements of a space-time bilinear metric with equilibrium values: $A\ =\ 1$, $H\ =\ 0$ and $K_{fl}\ =\ K$. Then, a small local fluctuation:

$$A\ =\ 1\ +\ \gamma \tag{33}$$

$$K_{fl}\ =\ K\ (1\ -\ \gamma) \tag{34}$$

with $\gamma \ll 1$ would be equivalent to a small, static gravitational field created by a far away source. The "graviton" would remain massless if the local fluctuations of A , K_{fl} and H , $\delta A\ (n)$, $\delta K_{fl}\ (n)$ and $\delta H\ (n)$, satisfy equations like (1) without the rest-frequency term (as is the case for the vibrations of a crystal, where only nearest-neighbour interactions play a role). Then, gravitation would still exist even if Lorentz symmetry is no longer an exact linear symmetry. The graviton would lead to a long-range force, as long as global fluctuations of A , K_{fl} and H would cost no vibrational energy. No obvious incompatibility seems to arise between this scenario and the violation of Lorentz symmetry at very high energy, which is related to the finite value of a and, as we shall see in the next chapter, can produce detectable effects even at very small values of $k\ a$.

2. LORENTZ SYMMETRY VIOLATION IN HIGH-ENERGY PHYSICS

The study of high-energy phenomena must incorporate the fact that, if Lorentz invariance is broken, relativistics kinematics can no longer be applied. As an illustrative example, we shall use the kinematics derived from the analogy with the one-dimensional monoatomic Bravais lattice. But, obviously, our basic arguments have a more general validity. Several consequences can follow from this modification of the usual framework:

2a. The GZK cutoff does no longer apply

Assume that, in the vacuum rest frame, the kinematics of "ordinary" particles is indeed given by (10) with universal values of K and a , and $K^{1/2}\ a\ =\ c$. As an example, we take $a\ \simeq\ 10^{-33}\ cm$ and $K\ \simeq 10^{87}\ s^{-2}$. At the distance scales associated to the highest cosmic ray energies, i.e. energy $E\ \approx\ 10^{20}\ eV$ and $(k\ a)^2\ \approx\ 10^{-17}$, we can expand E as follows:

$$E\ \simeq\ (2\pi)^{-1}\ h\ [K\ (k^2a^2\ -\ k^4\ a^4/12)\ +\ (2\pi)^2\ h^{-2}\ E_{rest}^2]^{1/2} \tag{35}$$

and, at high energy, we can write:

$$E\ \simeq\ (2\pi)^{-1}\ h\ c\ k\ [1\ +\ 2\pi^2\ (h\ c\ k)^{-2}\ E_{rest}^2\ -\ k^2\ a^2/24] \tag{36}$$

Corrections from Lorentz symmetry breaking are thus of order 10^{-18} , i.e. $\approx 100\ eV$. This is to be compared with the term from rest energy in (36) which, for a proton and at the same energy, is $\approx 10^{-2}\ eV$. Thus, Lorentz symmetry violation can play an important role in kinematics at these energies. A proton with $E\ >\ 10^{20}\ eV$ interacting with a cosmic microwave background photon would be sensitive to these corrections. For instance, after having absorbed a $10^{-3}eV$ photon moving in the opposite direction, the proton gets an extra $10^{-3}eV$ energy, whereas its momentum is lowered by $10^{-3}eV/c$. In the conventional scenario with exact Lorentz invariance, this is enough to allow the excited proton to decay into a proton or a neutron plus a pion, losing an important part of its energy. However, it can be checked that in our scenario with Lorentz invariance violation such a reaction is strictly forbidden. Writing for the maximum allowed energy transfer the equation:

$$2\pi\ (h\ c)^{-1}\ [E\ (k\ +\ 2\pi.10^{-3}\ eV/hc)\ +\ 10^{-3}\ eV]\ -\ k\ =\ \delta_p\ +\ \delta_\pi \tag{37}$$

with

$$\delta_p\ =\ (k\ -\ k_\pi)\ [2\pi^2\ h\ ^{-2}(k\ -\ k_\pi)^{-2}\ m_p^2c^2\ -\ (k\ -\ k_\pi)^2\ a^2/24] \tag{38}$$

$$\delta_\pi \quad = \quad k_\pi \ [2\pi^2 \ (h \ k_\pi)^{-2} \ m_\pi^2 \ c^2 \ - \ k_\pi^2 \ a^2/24] \tag{39}$$

where m_p and m_π are respectively the proton and the pion mass, k_π the momentum of the produced pion, and the left-hand-side of the equation can be approximated by:

$$2\pi \ (h \ c)^{-1} \ [E \ (k \ + \ 2\pi.10^{-3} \ eV/hc) \ + \ 10^{-3} \ eV] \ \ \approx \ \ E \ (k) \ + \ 2.10^{-3} \ eV \tag{40}$$

it turns out that corrections due to Lorentz symmetry violation completely preclude the reaction. Elastic $p \ + \ \gamma$ scattering is permitted, but allows the proton to release only a small amount of its energy, as can be seen writing:

$$2\pi \ (h \ c)^{-1} \ [E \ (k \ + \ 2\pi.10^{-3} \ eV/hc) \ + \ 10^{-3} \ eV] \ - \ k \ \ = \ \ \delta_p \ + \ \delta_\gamma \tag{41}$$

with δ_p defined as before replacing k_π by the outgoing photon wave vector k_γ and:

$$\delta_\gamma \quad = \quad - \ k_\gamma^3 \ a^2/24 \tag{42}$$

whose solution is in the range $k_\gamma \ \approx \ 10^{-5} \ k$. Thus, the outgoing photon energy for an incoming $10^{20} \ eV$ proton cannot exceede $\Delta E^{max} \ \approx \ 10^{-5} \ E \ = \ 10^{15} \ eV$ instead of the value $\Delta E^{max} \ \approx \ 10^{19} \ eV$ obtained with exact Lorentz invariance. Similar or more stringent bounds exist for channels involving lepton production. Furthermore, obvious phase space limitations will also lower the event rate, as compared to standard calculations using exact Lorentz invariance which predict photoproduction of real pions at such cosmic proton energies. The effect seems strong enough to invalidate the Greisen-Zatsepin-Kuzmin (GZK) cutoff [26] and explain the existence of the highest-energy cosmic rays [27] . It will become more important at higher energies, as we get closer to the a^{-1} wavelength scale. Similar arguments apply to heavy nuclei, again invalidating the GZK cutoff. Since, in both cases, the cosmic ray energy was expected to degrade over distances $\approx 10^{24} \ m$ according to conventional estimates, the correction by several orders of magnitude we just introduced applies to distance scales much larger than the estimated size of the presently observable Universe. It is not possible to extend the argument to photons of $E \ \approx 10^{14} \ eV$, because in this case one would have $(k \ a)^2 \ \approx \ 10^{-29}$ leading to too small corrections.

Thus, compared to the model considered by Coleman and Glashow [15] , the present scenario (where K and a have an exactly universal value for all "ordinary" particles) produces the reverse effect. Not only the existence of very high-energy cosmic rays is not an evidence against Lorentz symmetry violation, but the experimental failure of the GZK cutoff for protons and nuclei would be an evidence for a deviation from relativistic kinematics. Obvioulsy, a better understanding of the dynamics at Planck scale is needed.

2b. Unstable high-momentum particles live longer than naively expected

In standard relativity, we can compute the lifetime of any unstable particle in its rest frame and, with the help of a Lorentz transformation, obtain the Lorentz-contracted lifetime for a particle moving at finite speed. This is no longer possible with the kinematics defined by (10). If a particle of mass m and momentum k decays into two particles (particles 1 and 2) with masses m_1 and m_2 , $m \ > \ m_1 \ + \ m_2$, we can write at high energy, in the energetically most favourable configuration (no transverse energy):

$$E \ (k \ , \ m) \quad = \quad E \ (k' \ , \ m_1) \ + \ E \ (k \ - \ k' \ , \ m_2) \tag{43}$$

which, with a simple change of variables using (10) can be turned into:

$$[sin^2\ (\theta/2)\ +\ \alpha]^{1/2}\ =\ [sin^2\ (\theta_1/2)\ +\ \alpha_1]^{1/2}\ +\ [sin^2\ (\theta_2/2)\ +\ \alpha_2]^{1/2} \tag{44}$$

where $\theta\ =\ k\ a$, $\alpha\ =\ (\pi\ m\ c\ a)^2\ h^{-2}$ and similarly for θ_1 , θ_2 , α_1 and α_2 . Writing for simplicity $\alpha_1\ =\ \alpha_2$ and setting the configuration $\theta_1\ =\ \theta_2$, (always allowed with exact Lorentz invariance in which case it requires nonzero transverse energy), the equation becomes:

$$sin^2\ (\theta/2)\ +\ \alpha\ =\ 4\ [sin^2\ (\theta_1/2)\ +\ \alpha_1] \tag{45}$$

and, for $\theta_1\ =\ \theta_2/2$:

$$1\ -\ cos\ (k\ a/2)\ =\ (\alpha\ -\ 4\ \alpha_1)^{1/2} \tag{46}$$

and, for α , α_1 and $k\ a \ll\ 1$, can be approximated by:

$$k\ \simeq\ 2^{1/2}\ (\pi\ c)^{1/2}\ (a\ h)^{-1/2}\ (m^2\ -\ 4\ m_1^2)^{1/4} \tag{47}$$

so that the configuration is forbidden above this value of k . This implies an important reduction of phase space for the decay of very high-energy particles: their lifetimes get longer than it was expected with exact Lorentz invariance. More complicate expressions with similar meaning can be derived for $m_1\ \neq\ m_2$. If m_1 and m_2 are both nonzero, a stronger constraint can be obtained which forbids the decay at very high energy. To derive it, we can write: $\theta\ =\ \theta_1\ +\ \theta_2$, and expand equation (44). We are thus led to:

$$\alpha\ -\ \alpha_1\ -\ \alpha_2\ =\ 2\ [sin^2\ (\theta_1/2)\ sin^2\ (\theta_2/2)\ +\ D\ (\theta_1, \theta_2, \alpha_1, \alpha_2)] \tag{48}$$

where:

$$D\ (\theta_1, \theta_2, \alpha_1, \alpha_2)\ =\ [sin^2\ (\theta_1/2)\ +\ \alpha_1]^{1/2}\ [sin^2\ (\theta_2/2)\ +\ \alpha_2]^{1/2}\ -$$

$$-\ sin\ (\theta_1/2)\ sin\ (\theta_2/2)\ cos\ (\theta_1/2)\ cos\ (\theta_2/2) \tag{49}$$

If m_1 or m_2 vanishes, it will always be possible to keep the left-hand side of (48) larger than the right-hand side taking, for instance, $m_1\ =\ 0$ and $sin\ (\theta_1/2)$ small enough (although most of the usual phase space would then be lost). However, this possibility does no longer exist if none of the two masses vanish. In general, any decay with at least two massive particles being part of the final state is forbidden at very high energy. We can check the existence of such bounds minimizing the right-hand side in (48) . If m_2 is the smallest mass, a typical bound will forbid the decay for $E\ >\ E^{st}$ where:

$$E^{st}\ \approx\ c^{3/2}\ h^{1/2}\ (a\ m_2)^{-1/2}\ (m^2\ -\ m_1^2\ -\ m_2^2)^{1/2} \tag{50}$$

Thus, as a result of Lorentz symmetry violation, unstable particles and nuclei may become stable when accelerated to very high momenta, provided all decay channels contain at least two massive particles. The energy scale above which the decay is forbidden varies like the inverse square root of the mass of the lightest particle produced by the decay.

The neutron would become stable for $E\ \gtrsim\ 10^{20}\ eV$. At the same energies or slightly above, *some unstable nuclei would also become stable.* Similarly, *some hadronic resonances* (e.g. the Δ^{++} , whose decay product must contain at least a proton and a positron) *would become stable* at $E \gtrsim 10^{21}\ eV$. Most of these objects will decay before they can be accelerated to such energies, but they may result of a collision at very high energy or of the decay of a superluminal particle. The study of very high-energy cosmic rays can thus reveal as stable particles objects which would be unstable if produced at accelerators.

Neutrino masses and oscillations are also important in order to discuss the lifetimes of high-energy particles, as we are often confronted to decay modes involving muon and electron neutrinos. For instance, if one of the light neutrinos (ν_e , ν_μ) has a mass in the $\approx 10\ eV$ range, *the muon would become stable* at energies above $\approx 10^{22}\ eV$. Let E_1 , E_2 , m_1 and m_2 be the energies and masses of neutrinos ν_1 and ν_2 , which mix to give neutrinos ν and ν' with energies E , E' and masses m , m' . We write (10) as: $E = F\ (k\ ,\ m)$ for neutrino ν , and similar expressions for the other neutrinos. A simple mixing scheme would be to add to the hamiltonian non-diagonal elements:

$$< \nu_1 \mid H \mid \nu_2 > \ = \ < \nu_2 \mid H \mid \nu_1 > \ = \ \Delta\ (k) \tag{51}$$

and require that the hamiltonian has eigenvalues E and E' given by the above described expressions. We then get:

$$\Delta^2\ (k) \ = \ F\ (k\ ,\ m_1)\ F\ (k\ ,\ m_2) \ - \ F\ (k\ ,\ m)\ F\ (k\ ,\ m') \tag{52}$$

and a mixing angle ψ given by:

$$tan\ (2\ \psi) \ = \ 2\ \Delta(k)\ [F\ (k\ ,\ m_1) \ - \ F\ (k\ ,\ m_2)]^{-1} \tag{53}$$

Going back to a more fundamental level, if ϕ_1 and ϕ_2 are two different (but related) degrees of freedom satisfying equation (1) and describing ν_1 and ν_2 with the same value of K and different values of ω_{rest}^2 , adding a term proportional to $\mid \phi_1 - \phi_2 \mid^2$ to the lagrangian would indeed result in a pure mass mixing of the type we just presented *with constant* ψ , without changing the effective value of K (therefore leaving the critical speed in vacuum unchanged). However, since Lorentz invariance does no longer hold, the form of the dispersion relation (10) may be modified at high wavelength by anharmonic effects. Mixing with superluminal particles would produce different effects, as it necessarily implies particles associated to oscillations with different values of K and cannot leave sectorial kinematics unchanged. Such a mixing can strongly modify the parameters of our above discussion (see also [11] for an explicit example), but in any case it contributes to invalidate the GZK cutoff and to possibly permit unstable particles to become stable at high momentum.

3. SUPERLUMINAL PARTICLES AND STANDARD COSMOLOGY

It is well known that, in the standard Big Bang model without inflation [17 , 18] , the horizon problem arises basically from the fact that the most distant sources we can observe now with microwave antennae pointing in opposite directions must have been ≈ 100 horizon lengths apart when the cosmic microwave background radiation decoupled (T , temperature, $\approx 3\ .\ 10^3\ K$, t , age of the Universe, $\approx 10^{13}\ s$). Given the observed isotropy of cosmic background radiation, it seems difficult to understand how regions of the Universe that were not in causal contact could have acquired the same temperature up to $\approx 10^{-5}$ fluctuations. Temperature fluctuations are related to density fluctuations which were in principle generated much earlier in the history of the Universe.

Superluminal particles may provide a natural alternative to inflation in order to solve the horizon problem. For instance, assuming that the size of the presently observable Universe is $\approx 10^{26}\ m$ and its age $\approx 10^{17} s$, and a standard evolution for R (the cosmological distance scale measured by the radius of the presently observable Universe), we find a ratio $R\, t^{-1} \approx 10^4\ c$ at cosmic time $t \approx 10^7\ s$ and $k_B\ T \approx 1\ keV$. A superluminal particle with critical speed $c_i \approx 10^{12}\ c$ and rest energy $E_{rest} = mc_i^2 \approx 100\ GeV$ ($m \approx 10^{-13}\ eV\ c^{-2}$) in thermal equilibrium (i.e. with $v \approx 10^8\ c$) can cross the Universe provided it can keep most of its energy for more than $\approx 10^7\ s$. The effect of collisions has been taken into account, assuming that the particle undergoes a few scatterings per second. This scenario requires "Cherenkov" radiation in vacuum to be very weak. At such energies, the emitted ordinary particles woud necessarily be photons and neutrino-antineutrino pairs. Therefore, the superluminal particle should have weak enough electroweak couplings. By traveling at very high speed, such particles can emit pairs of "back-to-back" ordinary photons and neutrinos (see, e.g. [9 , 10]) or locally thermalize slower (therefore, heavier) superluminal particles which in turn would thermalize ordinary matter. In this way, it may be possible to solve the horizon problem and simultaneously find dark matter candidates, if superluminal particles are abundant enough to compensate the expected weak coupling between different sectors. More precise considerations would require building a global cosmological model, of which we give some possible ingredients below.

To attempt a description of the cosmological role of superluminal particles, we assume that a theory of all gravitation-like forces can be built, taking at each point the vacuum rest frame, and generalize Friedmann equations writing for a flat Universe in the present epoch (where pressure can be neglected):

$$R^{-1}\ d^2R/dt^2 \approx -4\pi\ Z_2\ Z_1^{-1}/3 + \Lambda/3 \tag{54}$$

$$(R^{-1}\ dR/dt)^2 \approx 8\pi\ Z_2\ Z_1^{-1}/3 + \Lambda/3 \tag{55}$$

where Λ is the cosmological constant [17 , 18] and, in a simplified scheme:

$$Z_1 = \rho_a + \rho_O + \Sigma_i\ (\rho_{a,i} + \rho_{O,i}) \tag{56}$$

$$Z_2 = G_a\ \rho_a^2 + G_O\ \rho_O^2 + \Sigma_i\ (G_{a,i}\ \rho_{a,i}^2 + G_{O,i}\ \rho_{O,i}^2) \tag{57}$$

where $G_a = G_N$ is Newton's gravitational constant, ρ_a the density of "acoustic" ordinary matter, ρ_O the density of "optical" ordinary matter (taken to be positive), $\rho_{a,i}$ and $\rho_{O,i}$ the densities of "acoustic" and "optical" matter of the i-th superluminal sectors (again, taking the densities of "optical" particles to be positive), and the G's are effective gravitation-like coupling constants. $Z_2\ Z_1^{-1}$ replaces the usual expression $G_N\ \rho$ in standard Friedmann equations. Z_1 is the total density of "particle matter", where the expression "particle matter" designs all possible excitations of vacuum that we can describe as particles.

Expressions (54) to (57) can be derived, for instance, by associating standard Friedmann equations (with only one "gravitational" component) to a lagrangian in terms of R and

dR/dt , and generalizing the expressions for kinetic and potential energies in the limit where gravitational couplings between different components of Z_1 are small. An interaction between the different "gravitational" components of Z_1 is, even in this case, implicitly generated by the constraint that R and dR/dt are space-time variables common to all the kinds of matter we consider. Since, at the same time, cosmology considers space-time as being generated by matter, this is indeed an effective dynamical interaction between matter from different sectors. The role of vacuum is crucial in the generation of a single, absolute space-time with a local absolute rest frame.

The new formulae reflect the fact that the "graviton" coupled to superluminal matter is in principle not the same which couples to "ordinary" matter (indeed, local fluctuations of the vacuum rest frame and dynamical constants can be sector-dependent), assume for simplicity a similar separation between "optical" and "acoustic" matter inside the same sector and, to a first approximation, neglect effects due to the mixing between the effective gravitons coupled to different components. Modifications to this schematic description can be readily introduced, but would not change our basic reasoning and conclusions concerning the new flexibility of cosmological fits and the allowed values of the cosmological constant. For instance, we can add to Z_2 "non-diagonal" terms due to gravitation-like interactions between different components of Z_1 . Defining, as usual [17 , 18], the critical density ρ_c as the "acoustic" ordinary matter density which, alone, would make the standard Friedmann equations compatible with a flat Universe without cosmological constant and with the measured value of Hubble's constant, i.e. $\rho_c \ = \ 3 \ (8\pi \ G_N)^{-1} \ H_0^2$ where H_0 is the current value of $R^{-1} \ dR/dt$, we can write:

$$\Omega_\Lambda \ = \ (8\pi \ G_N \ \rho_c)^{-1} \ \Lambda \ = \ (3 \ H_0^2)^{-1} \ \Lambda \tag{58}$$

$$\Omega_\alpha \ = \ G_\alpha \ \rho_\alpha^2 \ (G_N \ \rho_c \ Z_1)^{-1} \ = \ (3 \ Z_1 \ H_0^2)^{-1} \ G_\alpha \ \rho_\alpha^2 \tag{59}$$

where $G_\alpha \ \rho_\alpha^2$ is one of the components of Z_2 in (57) , i.e. $\alpha \ = \ a \ , \ O \ , \ (a \ , \ i) \ , \ (O \ , \ i)$. We then get, for a flat Universe, the relation:

$$\Omega \ = \ \Omega_\Lambda \ + \ \Sigma_\alpha \ \Omega_\alpha \ = \ 1 \tag{60}$$

In the recent years, there have been claims [28 , 29] in favour of a comparatively large cosmological constant which could amount, using the above definitions of the Ω-like parameters, to values of Ω_Λ (the contribution of the cosmological constant to the expansion of the Universe) as large as $\approx \ 0.6$. The scenario we propose, with many components of Z_1 and Z_2 , would allow for rather small values of $Z_2 \ Z_1^{-1}$ (typically, if there are many components with similar weights and weakly interacting with eachother), and therefore possibly for Ω_Λ naturally close to 1 in a flat Universe, even if ρ_a/ρ_c is equal to 0.3 or larger. Thus, at the price of considerably weakening the connection between the density of ordinary "acoustic" matter and the parameters governing the expansion of a flat Universe but possibly getting closer to reality, the new formulation would really make easier cosmological fits willing to simultaneously describe matter in the Universe, galaxy formation, the age of the Universe, and the spectrum and isotropy of cosmic microwave background. If the Universe is not flat, the following term accounting for cosmic curvature should be added to the right-hand side of (55):

$$Z_{curv} \ = \ - \ k_c \ R_U^{-2} \ Z_1^{-1} \ \Sigma_\alpha \ \rho_\alpha \ c_\alpha^2 \tag{61}$$

where $k_c = \pm 1$ is the curvature constant, R_U the curvature radius of the Universe (most likely, $R_U \gg R$) and c_α the critical speed of each component of Z_1 . From (54 - 57), the formula for the deceleration parameter q [17 , 18] becomes:

$$q = - (R\, d^2R/dt^2)\, (dR/dt)^{-2} = H^{-2}\, (4\pi\, Z_2\, Z_1^{-1} - \Lambda)/3 \tag{62}$$

where $H = R^{-1}\, dR/dt$ is Hubble's "constant" and, writing:

$$- Z_{curv} = - H^2 + (8\pi\, Z_2\, Z_1^{-1} + \Lambda)/3 = H^2\, (2\, q - 1) + \Lambda \tag{63}$$

we generalize the well-known relation between curvature, deceleration and Hubble's constant [17 , 18] in the presence of a nonvanishing cosmological constant. Defining Ω as before, expression (60) applied to the present Universe becomes now:

$$\Omega = \Omega_\Lambda + \Sigma_\alpha\, \Omega_\alpha = 1 - Z_{curv}\, H_0^{-2} = 1 + k_c\, R_U^{-2}\, H_0^{-2}\, Z_1^{-1}\, \Sigma_\alpha\, \rho_\alpha\, c_\alpha^2 \tag{64}$$

or, eliminating Z_{curv} in terms of the present value of q , q_0 :

$$\Omega_{particles} = \Sigma_\alpha\, \Omega_\alpha = 2\, (q_0 + \Omega_\Lambda) \tag{65}$$

which is similar to standard formulae, but with the definitions (58) and (59). Whether the Universe is flat or curved, the equalities (64) and (65) lead to the standard relations:

$$\Omega_\Lambda = (1 - 2q_0 - Z_{curv}\, H_0^{-2})/3 \tag{66}$$

$$\Omega_{particles} = 2\, (1 + q_0 - Z_{curv}\, H_0^{-2})/3 \tag{67}$$

with $Z_{curv} = 0$ in a flat Universe. The requirement that $\Omega_{particles}$ be positive, combined with experimental bounds on q_0 , puts bounds on a positive value of Z_{curv} (corresponding to $k = -1$). The situation seems less obvious for negative values of Z_{curv} , if superluminal particles exist. In recent fits [28 , 29] , Ω_Λ tends to get close to its maximum value compatible with experimental lower bounds on q_0 , and a negative Z_{curv} would allow for a larger Ω_Λ provided the contribution of superluminal sectors to $\Omega_{particles}$ is large enough. In (54) and (55), the cosmological constant Λ is actually:

$$\Lambda = Z_1^{-1}\, \Sigma_\alpha\, \Lambda_\alpha\, \rho_\alpha \tag{68}$$

where the Λ_α are sectorial cosmological constants, varying much slower than R . The expansion of the Universe seems thus to generate the vacuum matter which produces the sectorial cosmological constants. It is quite naturally that a significant cosmological constant would arise in our approach. If Z_1 and Z_2 describe the role of vacuum excitations, but the expansion of the Universe is still generating the matter which forms the ground state of vacuum (i.e. "vacuum" itself), this evolution is expected to spend a sizeable amount of energy in the creation of new matter: it must be driven by vacuum dynamics and vacuum energy. Then, the presence in (54) and (55) of terms describing inner vacuum dynamics seems compelling, even at a deeper level than for inflationary models. For instance, vacuum may have two sets of degrees of freedom: a) one, presently at low temperature, which produces all the objects that we call "particles"; b) a second one at higher temperature (possibly undergoing a second-order phase transition and weakly coupled to the particles we observe), whose cooling generates vacuum matter and drives vacuum expansion. It is well kown, in condensed matter physics, that two weakly interacting sets of degrees of freedom can remain for a long time at different temperatures (e.g. in adiabatic demagnetization). It is not obvious how well the simplified approach we adopted allows to describe the possibly complex role of vacuum, as a dynamical system with many degrees of freedom, in the present expansion of the Universe. Most likely, in spite of the important successes of present models [30 , 31] , crucial ingredients describing the role of inner vacuum dynamics are still missing in standard cosmology. Current "Pre-Big Bang" cosmology [32] is based on a superstring approach where vacuum dynamics is accounted for by the spectrum and properties of the complete set of vacuum excitations described by the superstrings. Alternatives approaches could be directly based on inner vacuum dynamics, naturally generating superluminal particles and Lorentz symmetry violation.

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[29] M.S. Turner, "The Cosmology of Nothing", in "Vacuum and Vacua: the Physics of Nothing", Ed. A. Zichichi, World Scientific Pub. 1996 , and references therein.
[30] M.S. Turner, "Cosmology: Standard and Inflationary", in "Particle Physics, Astrophysics and Cosmology", XXIIth SLAC Summer Institute, Ed. J. Chan and L. De Porcel (Conf-9408100 ; National Technical Information Service, U.S. Department of Commerce, 1994) , and references therein.
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