LORENTZ SYMMETRY VIOLATION AND VERY HIGH-ENERGY CROSS SECTIONS

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Abstract

We discuss the implications of a recently proposed pattern of Lorentz symmetry violation on very high-energy cross sections. As a consequence of the breaking of local Lorentz invariance by the introduction of a fundamental length, a , the kinematics is modified and the properties of final states are fundamentally different in collider-like (two incoming particles with equal, opposite momenta with respect to the vacuum rest frame) and fixed-target (one of the incoming particles at rest with respect to the vacuum rest frame) situations. In the first case, the properties of the allowed final states are similar to relativistic kinematics, as long as the relevant wave vectors are much smaller than the critical wave vector scale a^{-1} . But, if one of the incoming particles is close to rest in the vacuum rest frame, energy conservation reduces the final-state phase space at very high energy and can lead to a sharp fall of cross sections starting at incoming-particle wave vectors well below the inverse of the fundamental length. Then, the Froissart bound may cease to be relevant, as total cross sections seem to become much smaller than it would be allowed by local, Lorentz-invariant, field theory. Important experimental implications of the new scenario are found for cosmic-ray astrophysics and for very high-energy cosmic rays reaching the earth.

1 Introduction

In two previous papers (Gonzalez-Mestres, 1997a and 1997b), we suggested that, as a consequence of nonlocal dynamics at Planck scale or at some other fundamental length scale, Lorentz symmetry violation can result in a modification of the equation relating energy and momentum which would write in the vacuum rest frame:

$$E \quad = \quad (2\pi)^{-1} \ h \ c \ a^{-1} \ e \ (k \ a) \tag{1}$$

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where E is the energy of the particle, h the Planck constant, c the speed of light, a a fundamental length scale (that we can naturally identify with the Planck length, but other choices of the fundamental distance scale are possible), k the wave vector modulus and $[e\ (k\ a)]^2$ is a convex function of $(k\ a)^2$ obtained from nonlocal vacuum dynamics.

Rather generally, we find that, at wave vector scales below the inverse of the fundamental length scale, Lorentz symmetry violation in relativistic kinematics can be parameterized writing:

$$e\ (k\ a) \ \simeq \ [(k\ a)^2 \ - \ \alpha\ (k\ a)^4 \ + \ (2\pi\ a)^2\ h^{-2}\ m^2\ c^2]^{1/2} \tag{2}$$

where α is a positive constant between 10^{-1} and 10^{-2} . At high energy, we can write:

$$e\ (k\ a) \ \simeq \ k\ a\ [1 \ - \ \alpha\ (k\ a)^2/2] \ + \ 2\ \pi^2\ h^{-2}\ k^{-1}\ a\ m^2\ c^2 \tag{3}$$

and, in any case, we expect observable kinematical effects when the term $\alpha(ka)^3/2$ becomes as large as the term $2\ \pi^2\ h^{-2}\ k^{-1}\ a\ m^2\ c^2$. Assuming that, apart form the value of the mass, expression (2) is universal for all existing particles whose critical speed in vacuum is equal to the speed of light in the Lorentz-invariant limit, we found three important efects:

a) The Greisen-Zatsepin-Kuzmin (GZK) cutoff on very high-energy cosmic protons and nuclei (Greisen, 1966; Zatsepin and Kuzmin, 1966) does no longer apply.

b) Unstable particles with at least two massive particles in the final state of all their decay channels become stable at very high energy.

c) In any case, unstable particles live longer than naively expected with exact Lorentz invariance and, at high enough energy, the effect becomes much stronger than previously estimated for nonlocal models (Anchordoqui, Dova, Gómez Dumm and Lacentre, 1997) ignoring the small violation of relativistic kinematics.

Furthermore, velocity reaches its maximum at $k \ \approx \ (4\pi^2\ \alpha^{-1}/3)^{1/4}\ (m\ c\ h^{-1}\ a^{-1})^{1/2}$. Above this value, increase of momentum amounts to deceleration. In our ansatz, observable effects of local Lorentz invariance breaking arise, at leading level, well below the critical wavelength scale a^{-1} due to the fact that, contrary to previous models (f.i. Rédei, 1967), we directly apply non-locality to particle propagators and not only to the interaction hamiltonian. In contrast with previous patterns (f.i. Blokhintsev, 1966), $s-t-u$ kinematics ceases to make sense and the motion of the global system with respect to the vacuum rest frame plays a crucial role. The physics of elastic two-body scattering will depend on five kinematical variables. Noncausal dispersion relations (Blokhintsev and Kolerov, 1964) should be reconsidered, taking into account the departure from relativistic kinematics.

In this note, we would like to discuss another important consequence of the new kinematics, i.e. the appearence of strong limitations in the allowed phase space for final states of two-body collisions, especially when the target is moving slowly with respect to the vacuum rest frame. As in previous papers (Gonzalez-Mestres, 1997a and 1997b), we assume that c and α are universal constants for all particles under consideration. If this were not the case, our analysis would require modifications but other new physical phenomena would equally emerge. Such an alternative will be discussed in a forthcoming paper.

2 The new kinematics

No special constraint seems to arise from (2) if, in the vacuum rest frame, two particles with equal, opposite momenta of modulus p with $\alpha\ (k\ a)^2 \ll 1$ collide to produce a multiparticle final state. When the term $\alpha\ (k\ a)^2\ p\ c/2$ becomes $\approx m^2\ c^2\ p^{-1}/2$ or larger, the new kinematics favours large momenta and allows for new final-state phase space, as compared to relativistic kinematics. But, as a consequence of Lorentz symmetry violation (the required transformation would have relative speed $v \simeq c$), the situation becomes fundamentally different at very high energy if one of the incoming particles is close to rest with respect to the "absolute" frame where formulae (1) - (3) apply.

Assume a very high-energy particle (particle 1) with momentum $\vec{\mathbf{p}}$, impinging on a particle at rest (particle 2) in the vacuum rest frame. We take both particles to have mass m , and $p \gg mc$. In relativistic kinematics, we would have elastic final states where particle 1 has, with respect to the direction of $\vec{\mathbf{p}}$, longitudinal momentum $p_{1,L} \gg mc$ and particle 2 has longitudinal momentum $p_{2,L} \gg mc$ with $p_{1,L} + p_{2,L} = p$. A total transverse energy $E_T \simeq mc^2$ would still be left for the outgoing particles. However, the situation is drastically modified if the kinematics is given by expressions (1) - (3) and if $\alpha\ (k\ a)^2\ p$ (k being the modulus of the wave vector of the incoming particle) becomes of the same order as $m\ c$ or larger. As the energy increases, stronger and stronger limitations of the available final-state phase space appear: with the approximation (3), the final-state configuration $p_{1,L} = p - p_{2,L} = (1 - \lambda)\ p$ becomes kinematically forbidden for $\alpha\ (k\ a)^2\ p > 2\ m\ c\ \lambda^{-1}(1 - \lambda)^{-1}/3$. Thus, for momenta above $\approx (m\ c\ a^{-2}\ h^2)^{1/3}$, "hard" interactions become severely limited by kinematical constraints.

Similarly, with the same initial state, a multiperipheral final state configuration with N particles ($N > 2$) of mass m and longitudinal momenta $g^{i-1}\ p'_L$ ($i = 1, ..., N$, $g > 1$), where $p'_L = p\ (g - 1)\ (g^N - 1)^{-1}$, $g^N \gg 1$ and $p'_L \gg m\ c$, would have in standard relativity an allowed total transverse energy $E_T\ (N\ ,\ g) \simeq m\ c^2\ [1 - m\ c\ (2\ p'_L)^{-1}\ (1 - g^{-1})^{-1}]$ which is positive definite. Again, using the new kinematics and the approximation (3), we find that such a longitudinal final state configuration is forbidden for values of the incoming momentum such that $\alpha\ (k\ a)^2\ p\ c > 2\ (3\ g)^{-1}\ (1 + g + g^2)\ E_T\ (N\ ,\ g)$.

The above, or similar, considerations apply to strong interactions as well as to electromagnetic processes. For the initial state configuration where the target is at rest in the vacuum rest frame, and compared to standard expectations based on relativistic kinematics, a *sharp fall of elastic, multiparticle and total cross sections* can be expected at very high energy. For "soft" strong interactions, the approach were the two-body total cross section is the less sensitive to final-state phase space is, in principle, that based on dual resonance models and considering the imaginary part of the elastic amplitude as being dominated by the shadow of the production of pairs of very heavy resonances of masses M_1 and M_2 of order $\approx (p\ m\ c^3/2)^{1/2}$ in the direct channel (Aurenche and Gonzalez-Mestres, 1978 and 1979). But, even in this scenario, we find important limitations to the allowed values of M_1 and M_2 , and to the two-resonance phase space, when $\alpha\ (k\ a)^2\ p$ becomes $\approx m\ c$ or larger. In all cases, the departure from the standard relativistic situation occurs, if the target is close to rest in the vacuum rest frame, at incoming energies E above $\approx (m\ a^{-2}\ h^2\ c^4)^{1/3}$ which corresponds to a transition energy scale $\approx 10^{22}\ eV$ for $m\ \approx\ 1\ GeV/c^2$ and $a\ \approx\ 10^{-33}\ cm$, and $\approx 10^{21}\ eV$ if the target mass is $\approx 500\ keV/c^2$. Lowering the critical wave vector scale a^{-1} to $\approx\ 10^{26}\ cm^{-1}$ (just above the wave vector scale of the highest-energy cosmic rays), the fall of cross sections would start at $E\ \approx 10^{16}\ -\ 10^{17}\ eV$, which seems excluded by cosmic ray data if the earth is moving slowly with respect to the vacuum rest frame. In astrophysical processes, the new kinematics may inhibit phenomena such as GZK-like cutoffs, photodisintegration of nuclei, decays, radiation emission under external forces, momentum loss (which at very high energy does not imply deceleration) through collisions, production of lower-energy secondaries... *potentially solving the basic problems raised by the highest-energy cosmic rays.* Above $E\ \approx (m\ a^{-2}\ h^2\ c^4)^{1/3}$, nonlocal effects play a crucial role and invalidate considerations based on Lorentz invariance and local field theory used to derive the Froissart bound (Froissart, 1961), which seems not to be violated but ceases to be significant given the expected behaviour of total cross sections which, at very high-energy, seem to fall far below this bound. An updated study of noncausal dispersion relations, incorporating the new kinematics from nonlocal dynamics, can possibly lead new bounds. As previously stressed (Gonzalez-Mestres, 1997a) , this apparent nonlocality may actually reflect the existence of superluminal sectors of matter (Gonzalez-Mestres, 1996) where causality would hold at the superluminal level (Gonzalez-Mestres, 1997c).

Other initial state configurations can be considered. We may have two incoming particles with momenta of moduli p_1^i and p_2^i and opposite directions in the vacuum rest frame, and $p_1^i\ \gg\ p_2^i\ \gg\ mc$. Keeping a constant value of $\lambda\ =\ p_2^i\ (p_1^i)^{-1}$, we find that the fall of final-state phase space occurs for p_1^i above $\approx \lambda^{1/2}\ a^{-1}\ h$. The incoming momenta p_1^i and p_2^i may also be pointing in the same direction. Then, the final-state phase space starts to fall at $p_1^i\ \approx \lambda^{-1/4}\ (m\ c\ h\ a^{-1})^{1/2}$. A more complete discussion, including non-parallel incoming momenta and the case $m\ =\ 0$, will be presented elsewhere.

3 Experimental considerations

Lorentz symmetry violation prevents naive extrapolations from reactions between two particles with equal, opposite momenta in the vacuum rest frame (similar to colliders) to reactions where the target is at rest in this frame (similar to cosmic-ray events). *Assuming the earth to move slowly with respect to the vacuum rest frame* (for instance, if the "absolute" frame is close to that defined by the requirement of cosmic microwave background isotropy), the described kinematics predicts the existence of a maximum energy deposition for high-energy cosmic rays in the atmosphere, in the rock or in a given underground or underwater detector. Well below Planck energy, a very high-energy cosmic ray would not necessarily deposit most of its energy in the atmosphere: its energy deposition decreases for energies above a transition scale, far below the energy scale associated to the fundamental length. The maximum allowed momentum transfer in a single collision occurs at an energy just below $E \approx (m\ a^{-2}\ h^2\ c^4)^{1/3}$. For E above $\approx (m\ a^{-2}\ h^2\ c^4)^{1/3}$, the allowed longitudinal momentum transfer falls, typically, like p^{-2} (obtained differentiating the term $\alpha\ k^2\ a^2\ p\ c/2$). To set upper limits, we can take for m the mass of oxigen or nitrogen in the case of air, oxigen in water, and heavier elements in the rock. At energies around $\approx (m\ a^{-2}\ h^2\ c^4)^{1/3}$, the cosmic ray will in our scenario undergo several scatterings in the atmosphere and still lose there most of its energy, possibly leading to unconventional longitudinal cascade development profiles that could be observed by very large-surface air shower detectors like the AUGER observatory (AUGER Collaboration, 1997). Above $E \approx (m\ a^{-2}\ h^2\ c^4)^{1/3}$, it can indeed cross the atmosphere keeping most of its momentum and energy and deposit its energy in the rock or in water, or possibly reach and underground or underwater detector. Thus, some cosmic ray events of apparent energy far below $10^{20}\ eV$ (perhaps apparently muon or neutrino-like, or exotic-like), as seen by earth-surface (e.g. air shower), underground or underwater detectors, may actually be originated by extremely-high energy cosmic rays well above this energy scale.

Interesting constraints on the fundamental length a can be derived from this analysis, assuming simultaneoulsy (Gonzalez-Mestres, 1997a and 1997b) that the absence of GZK cutoff is due to the same pattern of Lorentz symmetry violation. The combined absence of GZK cutoff and existence of $\approx 10^{20}\ eV$ energy deposition from cosmic rays in the atmosphere lead to a in the range $10^{-35}\ cm\ <\ a\ <\ 10^{-30}\ cm$ (energy scale between 10^{16} and $10^{21}\ GeV$). The lower bound comes from the requirement that the violation of local Lorentz invariance at the fundamental length scale be able to influence particle interactions at the $10^{19}\ -\ 10^{20}\ eV$ energy scale strongly enough to suppress the GZK cutoff. The upper bound is derived from the existence of events with $\approx 10^{20}\ eV$ energy deposition in the atmosphere (Linsley, 1963; Lawrence, Reid and Watson, 1991; Afanasiev et al., 1995; Bird et al., 1994; Yoshida et al., 1995).

Then, very high-energy accelerator and cosmic-ray experiments would indeed be complementary research lines: the results of both kinds of experiments would not be equivalent up to Lorentz tranformations. If the transition energy scale for cross-sections corresponds to $p_1^i\ c\ \approx 10^{20}\ eV$, a $p\ -\ p$ collider at $\approx 700\ TeV$ per beam could make possible direct tests of Lorentz symmetry violation, comparing collisions at the accelerator with collisions between a $\approx 10^{21}\ eV$ proton of cosmic origin and a proton or nucleus from the atmosphere. Simultaneously, other kinds of tests may be possible through the lifetimes and decay products of very high-energy unstable particles (Gonzalez-Mestres, 1997a and 1997b) in the cosmic-ray events producing the highest-energy secondaries. We would be confronted to a new situation, contrary to previous expectations, if the cosmic rays at the highest possible energies interact more and more weakly with matter because of kinematical constraints. The existence of a maximum energy of events generated in the atmosphere would not correspond to a maximum energy of incoming cosmic rays. Unconventional events originated by such particles may have been erroneously interpreted as being associated to cosmic rays of much lower energy. New analysis seem necessary, as well as new experimental designs using perhaps in coincidence very large-surface detectors devoted to interactions in the atmosphere with very large-volume underground or undewater detectors.

Acknowledgement

It is a pleasure to thank P. Espigat and other colleagues at LPC, Collège de France, for useful discussions.

References

Afanasiev, B.N. et al., Proc. of the 24^{th} International Cosmic Ray Conference, Rome, Italy, Vol. 2 , p. 756 (1995).
Anchordoqui, L., Dova, M.T., Gómez Dumm, D. and Lacentre, P., *Zeitschrift für Physik C* 73 , 465 (1997).
AUGER Collaboration, "The Pierre Auger Observatory Design Report" (1997).
Aurenche, P. and Gonzalez-Mestres, L., *Phys. Rev. D* 18 , 2995 (1978).
Aurenche, P. and Gonzalez-Mestres, L., *Zeitschrift für Physik C* 1 , 307 (1979).
Bird, D.J. et al., *Ap. J.* 424 , 491 (1994).
Blokhintsev, D.I. and Kolerov, G.I., *Nuovo Cimento* 34 , 163 (1964).
Blokhintsev, D.I., *Sov. Phys. Usp.* 9 , 405 (1966).
Froissart, M., *Phys. Rev.* 123, 1053 (1961).
Gonzalez-Mestres, L., "Physical and Cosmological Implications of a Possible Class of Particles Able to Travel Faster than Light", contribution to the 28^{th} International Conference on High-Energy Physics, Warsaw July 1996 . Paper hep-ph/9610474 of LANL (Los Alamos) electronic archive (1996).
Gonzalez-Mestres, L., "Vacuum Structure, Lorentz Symmetry and Superluminal Particles", paper physics/9704017 of LANL electronic archive (1997a).
Gonzalez-Mestres, L., "Absence of Greisen-Zatsepin-Kuzmin Cutoff and Stability of Unstable Particles at Very High Energy, as a Consequence of Lorentz Symmetry Violation", paper physics/9705031 of LANL electronic archive (1997b).
Gonzalez-Mestres, L., "Space, Time and Superluminal Particles", paper physics/9702026 of LANL electronic archive (1997c).
Greisen, K., *Phys. Rev. Lett.* 16 , 748 (1966).
Lawrence, M.A., Reid, R.J.O. and Watson, A.A., *J. Phys. G* , 17 , 773 (1991).
Linsley, J., *Phys. Rev. Lett.* 10 , 146 (1963).
Rédei, L.B., *Phys. Rev.* 162 , 1299 (1967).
Yoshida, S. et al., Proc. of the 24^{th} International Cosmic Ray Conference, Rome, Italy, Vol. 1 , p. 793 (1995).
Zatsepin, G.T. and Kuzmin, V.A., *Pisma Zh. Eksp. Teor. Fiz.* 4 , 114 (1966).