Stratified Low Temperature Phases of Stratified Spin Models : A General Pirogov – Sinai Approach Petr Holický and Miloš Zahradník Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 00 Prague, Czech Republic holickykarlin.mff.cuni.cz, mzahradkarlin.mff.cuni.cz Partially supported by: Commission of the European Union under contracts CHRX-CT93-0411 and CIPA-CT92-4016, Czech Republic grants 202/96/0731 and 96/272. August 30, 1997 Low temperature Gibbs states, stratified phases (with many interfaces) for stratified and nonstratified Hamiltonians, "local" ground states, interfaces, contours, Dobrushin's walls, Pirogov – Sinai theory, Peierls condition, contour functional, "metastable" submodels, (inductively organized) cluster expansions, ground states of one dimensional models, phase diagrams 82A25 We adapt and improve the existing Pirogov – Sinai technology to obtain a general and unifying approach to the study of low temperature "stratified" phases for classical spin models whose Hamiltonian may not even be translation invariant but is "stratified", i.e. invariant with respect to all "horizontal" shifts (not changing the last coordinate). Examples are "stratified" versions of classical models like the Ising model with "vertically dependent" external field; models in halfspaces or layers and also those translation invariant models where Dobrushin's phases with rigid interfaces (one or more) appear. Our method brings some clarification and sharpening even when applied to the ordinary situations of the Pirogov – Sinai theory.

Our main result transcripts the question of characterizing the "stratified" Gibbs states of the given model to the question of finding the *ground states* of some auxiliary *one dimensional* model with infinite range but quickly decaying interactions.

I. Introduction, Notes on the Development of the Problem and Some Examples

The rigorous study of Gibbs states with translation noninvariant structure exhibiting a "rigid interface between the translation invariant phases above and below the interface", goes back to the pioneering Dobrushin's paper 1. Several authors continued this study; we note e.gårticle HKZ where an attempt to combine basic Dobrushin's ideas with the power of Pirogov – Sinai theory was made.

The leading idea in these investigations was to transcript the problem of description of the structure of the rigid interface to a suitable *lower dimensional* problem. In more concrete terms, using the expansion of the partition sums above and below the interface, the behaviour of the "walls" of the interface between the + and − phases in the three dimensional Ising model can be viewed as a behaviour of contours of some auxiliary two dimensional perturbed Ising model.

In HZ, we applied a similar approach based on the reduction to a lower dimensional problem to the study of wetting phenomena and entropic repulsion in the Ising model in halfspace. During our attempt to pursue the method to other interesting situations like the study of "wetting layers" emerging in some phases of the Blume – Capel model and also in the order – disorder – (other) order phases appering in the Potts model below the critical temperature (the article HMZ is under preparation), we found that the additional technical problems are forcing us to look for a more appropriate method. Finally we were lead to a conclusion that the "dimensional reduction method" based on this particular kind of a partial exansion of the considered model should be abandoned. Instead, we found a modification of the Pirogov – Sinai theory which applies *directly* to these "stratified" situations. We hope that the fact that our new version of the Pirogov – Sinai theory gives

even some new insight and a simplification into the traditional "translation invariant" Pirogov – Sinai theory confirms that the method developed by us is adequate.

Methodologically, our approach is based on the version Z of the Pirogov – Sinai theory but the concept of a "stable" ("small") contour and of a "metastable ensemble" is now investigated in a greater detail. Moreover, the concept of a contour ensemble now *disappeared* from our version of Pirogov – Sinai theory completely! The concept of a suitably defined "contour functional" $F(\Gamma)$ (as compared to the contour energy $E(\Gamma)$) remains in our approach as an extremely important *testing quantity* – allowing one to decide whether the contour is "well behaved enough"– but instead of the construction of auxiliary contour models, the central point of our approach is now an idea of a successive partial *expansion* of the model. This is based on an important new technical step which is called *recoloring* of a contour here. Recoloring of a contour Γ in a partially expanded model means that a new, "more expanded" model with the *same* partition functions is constructed where Γ does not appear anymore.

We will see that the "metastable" submodels of the given model (constructed for any stratified boundary condition) can be *expanded completely* and moreover for the "stable" boundary conditions, the corresponding metastable model will be identical with the original "physical" model. The organization of our expansions will make unnecessary previous estimates like "Main Lemma" used in Z. Instead, we have now a more powerful method based on our Theorems 5 and 6.

To summarize, we converted the Pirogov – Sinai theory just to a carefully organized *method* of (successive) *expansion of suitable partition functions*. The use of expansion techniques is absolutely crucial in our situation and the construction of the expansions is a more delicate task than in the translation invariant situations studied before. Namely, contours of the models studied so far were "crusted" in the sense that the events outside and inside the contour were independent. This is *not* valid here in our new situation, where contours can be often interpreted only as "walls" (the terminology of D) of the interface(s) and then the events happening "inside" resp "outside" of the wall are *not* independent ones.

This problem was solved in HKZ by taking expansions "above" and "below" the interface and by replacing the walls by more complex "aggregates" of walls and clusters The whole situation was then projected to $Z^{\nu-1}$ which is the main idea of the paper D.ut for complicated phases with *several* interfaces such an approach is too complicated.

Now we treat both the "crusted" contours and the "noncrusted" ones (walls) in the *same way*. However, the fact that some contours are not "crusted" implies that their contour functional $F(\Gamma)$ (see below for an extensive discussion of this "testing quantity") must be now defined much more carefully Retaining its meaning, vaguely speaking, of the "work needed to install the given contour". We construct succesively, by induction, the expansion of the *whole metastable model*, leaving out the previous idea (of HKZ) of the expansion in *two* different steps (first the expansion of the ensemble of contours and then of the ensemble of the walls resp. of the "aggregates", see HKZ). Let us now mention some typical examples which can be treated by our method – which often gives results more

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e given contour". .