To be published in

Mathematical Reviews

Reviewer: Luca Salasnich

Address:

Dipartimento di Matematica Pura ed Applicata

Università di Padova

Via Belzoni 7

I-35131 Padova, Italy

Author: C. Jung and T.H. Seligman

Short title: Integrability of the S–Matrix

Journal: Phys. Rep. **285**, 77–141 (1997)

Control number: 1 452 542

Primary classification: 81Q50

Secondary classification(s): 70K50

This interesting paper deals with the connection between the integrability of the scattering matrix S and the integrability of the Hamiltonian H for classical and quantum Hamiltonian systems.

The authors study the dynamics of a system described by the Hamiltonian H and for which the asymptotic dynamics is given by the free Hamiltonian H_0. From these Hamiltonians they derive the quantum evolution operators $U(t)$ and $U_0(t)$, and also the corresponding classical flow maps, which are denoted by $\Phi(t)$ and $\Phi_0(t)$, respectively. Since the quantum S–operator is

defined as

$$S = \lim_{T\to\infty} U_0(T)U(2T)U_0(-T)$$

[see, for instance, J.R. Taylor, *Scattering Theory*, Wiley, New York, 1972], they define a corresponding classical map

$$\tilde{M} = \lim_{T\to\infty} \Phi_0(T)\Phi(2T)\Phi_0(-T) \ .$$

Then the authors introduce the classical map M, which is the map $\tilde{M}$ reduced to the channel space and for fixed energy. For a system with n degrees of freedom, where M has a $(2n-2)$–dimensional domain, they call M completely integrable if there are $n-1$ independent functions in involution such that their common level sets are invariant under M. It is shown that integrability of M always leads to integrability of H, but that integrability of H only leads to integrability of M under an additional condition, namely the asymptotic forward and backward limits must coincide.

In the second part of the paper the authors transfer their classical results to quantum dynamics. For a system with n degrees of freedom they call S completely integrable if there are n independent operators that commute among themselves and with S. Because in quantum mechanics the question of independence of a set of commuting operators is more subtle than in classical mechanics, they consider only operators which have a classical counterpart. In this way it is shown that integrability of S always implies integrability of H, but integrability of H implies integrability of S under the same additional condition obtained in classical mechanics.

Finally the authors give some numerical example of breaking of integrability for the S–matrix. They show that the nearest neighbour spacing

distribution $P(s)$ of the eigenphases of the S–matrix follows the Poisson distribution

$$P(s) = exp(-s) \ ,$$

if the matrix is integrable, and that $P(s)$ is very closed to the Wigner surmise

$$P(s) = (\pi s/2) \exp\left(-\pi s^2/2\right) \ ,$$

if the classical map M is dominated by chaos.