Self/Anti-Self Charge Conjugate States for $j = 1/2$ and $j = 1$*

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We briefly review recent achievements in the theory of neutral particles
(the Majorana-McLennan-Case-Ahluwalia construct for self/anti-self charge conjugate states for $j = 1/2$ and $j = 1$ cases). Among new results we present a theoretical construct in which a fermion and an antifermion have the same intrinsic parity; discuss phase transformations and find relations between the Majorana-like field operator ν, given by Ahluwalia, and the Dirac field operator. Also we give explicit forms of the $j = 1$ "spinors" in the Majorana representation.

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The construct for self/anti-self charge conjugate states defined in the momentum representation has been proposed in refs. [1,2]. This is the straightforward development of the Majorana ideas [3]

and the ideas of McLennan [4] and Case [5].

Let us present the previous results:

- Self/anti-self charge conjugate spinors have been defined in the $(1/2,0)\oplus(0,1/2)$ representation in the momentum space [1c]:

$$\lambda^{S,A} = \begin{pmatrix} \pm i\Theta\phi_L^*(p^\mu) \\ \phi_L(p^\mu) \end{pmatrix} \quad , \quad \rho^{S,A} = \begin{pmatrix} \phi_R(p^\mu) \\ \mp i\Theta\phi_R^*(p^\mu) \end{pmatrix} \tag{1}$$

and have been named as the type-II spinors. They are eigenstates of the charge conjugation operator:

$$S^c_{[1/2]} = e^{i\theta_c} \begin{pmatrix} 0 & i\Theta \\ -i\Theta & 0 \end{pmatrix} \mathcal{K} \quad , \quad \Theta \equiv -i\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad ; \tag{2}$$

$$S^c_{[1/2]}\lambda^{S,A}(p^\mu) = \pm\lambda^{S,A}(p^\mu) \quad , \tag{3a}$$

$$S^c_{[1/2]}\rho^{S,A}(p^\mu) = \pm\rho^{S,A}(p^\mu) \quad . \tag{3b}$$

Similar (to a certain extent) states can be constructed in the higher representations of the Lorentz group, e.g., in the $(j,0)\oplus(0,j)$ representation, $j > 1/2$.

- The field operator

$$\nu^{DL}(x^\mu) = \sum_\eta \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p}\Big[\lambda^S_\eta(p^\mu)a_\eta(p^\mu)\exp(-ip\cdot x) + \lambda^A_\eta(p^\mu)b^\dagger_\eta(p^\mu)\exp(+ip\cdot x)\Big] \tag{4}$$

has been proposed for this sort of states [1c].

- λ and ρ spinors are *not* eigenspinors of the $(j,0)\oplus(0,j)$ helicity operator

  $$h=\begin{pmatrix}\mathbf{J}\cdot\hat{\mathbf{n}} & 0\\ 0 & \mathbf{J}\cdot\hat{\mathbf{n}}\end{pmatrix} \qquad (5)$$

  (by the definition, indeed, because $\Theta_{[j]}\mathbf{J}\Theta_{[j]}^{-1}=-\mathbf{J}^*$). The new quantum number (*chiral helicity*) corresponding to the operator $\eta=-\gamma^5 h$ has been introduced.

- λ and ρ spinors are *not* eigenspinors of the parity operator, see formulas (36a,b) in ref. [1c]. "This is not related to the fact that $S^c_{[1/2]}$ and $S^s_{[1/2]}$ do not commute. Since $S^c_{[1/2]}$ is *not* linear, it is possible to have a simultaneous

  set of eigenspinors, but such a set does not have its eigenspinors of type-II", in the opinion of D. V. Ahluwalia, ref. [1c].

- The introduction of the interaction in the usual manner ("covariantization" $\partial_\mu\to\nabla_\mu=\partial_\mu$

-ieA_μ) was found to be impossible for these states because phase transformations which correspond to this "covariantization" would lead to the consequence that the spinors would *not* keep their property to be self/anti-self charge conjugate spinors.

- Simple dynamical equations for λ and ρ spinors have been obtained [2d] on the basis of a new form of the Ryder-Burgard relation (which connects the left- and right- parts of the bispinors in the frame

with zero momentum [6,7]). Here they are:

$$i\gamma^\mu\partial_\mu\lambda^S(x)-m\rho^A(x)=0\quad, \qquad (6a)$$
$$i\gamma^\mu\partial_\mu\rho^A(x)-m\lambda^S(x)=0\quad, \qquad (6b)$$
$$i\gamma^\mu\partial_\mu\lambda^A(x)+m\rho^S(x)=0\quad, \qquad (6c)$$
$$i\gamma^\mu\partial_\mu\rho^S(x)+m\lambda^A(x)=0\quad. \qquad (6d)$$

In fact they can be written in the eight-component form, see also the old works [8,9] and the recent works [10,11].

- The connection with the Dirac spinors has been found [2a,b]. For instance,

$$\lambda^A_\uparrow(p^\mu)\lambda^A_\downarrow(p^\mu) = \frac{1}{2}$$

$$1 \; i \; {-1} \begin{pmatrix} u_{+1/2}(p^\mu) \\ u_{-1/2}(p^\mu) \\ v_{+1/2}(p^\mu) \\ v_{-1/2}(p^\mu) \end{pmatrix} \quad .(7)$$

  See also ref. [10].

- The sets of λ spinors and of ρ spinors are claimed [1c] to be *bi-orthonormal* sets each in the mathematical sense, provided that overall phase factors of 2-spinors $\theta_1 + \theta_2 = 0$ or π. For instance, on the classical level $\bar{\lambda}^S_\uparrow \lambda^S_\downarrow = 2iN^2 \cos(\theta_1 + \theta_2)$. Corresponding commutation relations for this type of states have also been proposed.

- The Lagrangian for λ- and ρ-type $j = 1/2$ states was given [2d,formula(24)].

- While in the massive case there are four λ-type spinors, two λ^S and two λ^A (the ρ spinors are connected by certain relations with the λ spinors for any spin case), in a massless case $\lambda^S_\uparrow$ and $\lambda^A_\uparrow$ identically vanish, provided that one takes into account that $\phi_L^{\pm 1/2}$ are

  eigenspinors of $\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}$, the 2×2 helicity operator.

- The possibility of the generalization of the concept of the Fock space was noted. It leads to "doubling" the Fock space [10].

- There does not exist the self/anti-self charge conjugate "spinors" in the $(1,0) \oplus (0,1)$ representation. Therefore, $\Gamma^5 S^c_{[1]}$ self/anti-self conjugate objects have been defined there.

- The *commutator* of the operations $U^s_{[1/2]}$ and $U^c_{[1/2]}$ in the Fock space may be equal to zero when acting on the Majorana states. The parity operator of the Fock space is the function of the charge operator [12].

- Several explicit constructs of the Bargmann-Wightman-Wigner-type theories [13] have been presented in [7,1,2,10].

We continue this research in the area of the physics of neutral particles because the present-day standard models do not provide any adequate formalism for describing neutrino and photon. Among new results we now present:

- It is shown that the covariant derivative (and, hence, the interaction) can be introduced in this construct in the following way:

where Ł$^5 = \mathrm{diag}(\gamma^5 \quad -\gamma^5)$, the 8×8 matrix. With respect to the transformations

$$\lambda'(x) \to (\cos\alpha - i\gamma^5 \sin\alpha)\lambda(x) \quad , \tag{9a}$$
$$\overline{\lambda}'(x) \to \overline{\lambda}(x)(\cos\alpha - i\gamma^5 \sin\alpha) \quad , \tag{9b}$$
$$\rho'(x) \to (\cos\alpha + i\gamma^5 \sin\alpha)\rho(x) \quad , \tag{9c}$$
$$\overline{\rho}'(x) \to \overline{\rho}(x)(\cos\alpha + i\gamma^5 \sin\alpha) \tag{9d}$$

the spinors retain their properties to be self/anti-self charge conjugate spinors and the proposed Lagrangian [2d, p. 1472] remains to be invariant. This tells us that while self/anti-self charge conjugate states has zero eigenvalues of the ordinary (scalar) charge operator but they can possess the axial charge (cf. with the discussion of [10] and the old idea of R. E. Marshak).

In fact, from this consideration one can recover the Feynman-Gell-Mann equation (and its charge-conjugate equation). It is re-written in the two-component form

$$\begin{cases} \left[\pi_\mu^- \pi^{\mu\,-} - m^2 - \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}\right]\chi(x) = 0 \quad , \\ \left[\pi_\mu^+ \pi^{\mu\,+} - m^2 + \frac{g}{2}\widetilde{\sigma}^{\mu\nu}F_{\mu\nu}\right]\phi(x) = 0 \quad , \end{cases} \tag{10}$$

where already one has $\pi_\mu^\pm = i\partial_\mu \pm gA_\mu$, $\sigma^{0i} = -\widetilde{\sigma}^{0i} = i\sigma^i$, $\sigma^{ij} = \widetilde{\sigma}^{ij} = \epsilon_{ijk}\sigma^k$ and $\nu^{DL}(x) = \mathrm{column}(\chi \quad \phi)$.

- Next, because the transformations

$$\lambda'_S(p^\mu) = \begin{pmatrix} \Xi & 0 \\ 0 & \Xi \end{pmatrix} \lambda_S(p^\mu)$$

$$\equiv \lambda^*_A(p^\mu) \quad , \quad (11a)$$

$$\lambda''_S(p^\mu) = \begin{pmatrix} i\Xi & 0 \\ 0 & -i\Xi \end{pmatrix} \lambda_S(p^\mu) \equiv -i\lambda^*_S(p^\mu) \quad , \quad (11b)$$

$$\lambda'''_S(p^\mu) = \begin{pmatrix} 0 & i\Xi \\ i\Xi & 0 \end{pmatrix} \lambda_S(p^\mu) \equiv i\gamma^0\lambda^*_A(p^\mu) \quad , \quad (11c)$$

$$\lambda^{IV}_S(p^\mu) = \begin{pmatrix} 0 & \Xi \\ -\Xi & 0 \end{pmatrix} \lambda_S(p^\mu) \equiv \gamma^0\lambda^*_S(p^\mu) \quad (11d)$$

with the 2×2 matrix Ξ defined as (ϕ is the azimuthal angle related to $\mathbf{p} \to \mathbf{0}$)

$$\Xi = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix} \quad , \quad \Xi\Lambda_{R,L}(\overset{\circ}{p}{}^\mu \leftarrow \mathrm{p}^\mu)\Xi^{-1} = \Lambda^*_{R,L}(\overset{\circ}{p}{}^\mu \leftarrow p^\mu) \ , \ (12)$$

and corresponding transformations for λ^A do *not* change the properties of bispinors to be in the

self/anti-self charge conjugate spaces, the Majorana-like field operator

($b^\dagger \equiv a^\dagger$) admits additional phase (and, in general,

normalization) transformations:

$$\nu^{ML\,\prime}(x^\mu) = \left[c_0 + i(\boldsymbol{\tau}\cdot\mathbf{c})\right]\nu^{ML\,\dagger}(x^\mu) \quad , \qquad (13)$$

where c_α are arbitrary parameters. The $\boldsymbol{\tau}$ matrices are defined over the field of 2×2 matrices[1] and the Hermitian

[1] This concept is closely related with the Wigner's concept of the *sign* spin, which was discussed recently by M. Moshinsky [14]. In general, this notation was used extensively in the earlier works of many researchers.

conjugation operation is assumed to act on the c- numbers as the complex conjugation. One can parametrize $c_0 = \cos\phi$ and $\mathbf{c} = \mathbf{n}$

$\sin\phi$ and, thus, define the $SU(2)$ group of phase transformations. One can select the Lagrangian which is composed from the both field operators (with λ spinors and ρ spinors) and which remains to be invariant with respect to this kind of transformations. The conclusion is: it is permitted a non-Abelian construct which is based on the spinors of the Lorentz group only (cf. with the old ideas of T. W. Kibble and R. Utiyama) . This is not surprising because both the $SU(2)$ group and $U(1)$ group are the sub-groups of the extended Poincaré group (cf. [6]). Another non-Abelian model was proposed in the $(1,0)\oplus(0,1)\oplus(1/2,1/2)$ by T. Barrett (e.g., ref. [15]) and, recently, by M. W. Evans and J.-P. Vigier [16].[2]

- The new construct has been presented in which the fermion and its antifermion may have the same intrinsic parities [1c,2f]. We can deduce

the following properties of creation (annihilation) operators in the Fock space:

$$U^s_{[1/2]} a_\uparrow(\mathbf{p}) (U^s_{[1/2]})^{-1} = -i a_\downarrow(-\mathbf{p})\,, \quad U^s_{[1/2]} a_\downarrow(\mathbf{p}) (U^s_{[1/2]})^{-1} = +i a_\uparrow(-\mathbf{p})\,, \tag{14a}$$

$$U^s_{[1/2]} b^\dagger_\uparrow(\mathbf{p}) (U^s_{[1/2]})^{-1} = +i b^\dagger_\downarrow(-\mathbf{p})\,, \quad U^s_{[1/2]} b^\dagger_\downarrow(\mathbf{p}) (U^s_{[1/2]})^{-1} = -i b_\uparrow(-\mathbf{p})\,, \tag{14b}$$

what signifies that the states created by the operators $a^\dagger(\mathbf{p})$ and $b^\dagger(\mathbf{p})$ have very different properties with respect to the space inversion operation, comparing to the Dirac states ($\pm$ stand for denoting the positive- (negative) energy states). Namely,

$$U^s_{[1/2]}|\mathbf{p},\uparrow>^+ \; = +i|-\mathbf{p},\downarrow>^+\,, \quad U^s_{[1/2]}|\mathbf{p},\uparrow>^- = +i|-\mathbf{p},\downarrow>^-\,, \tag{15a}$$

$$U^s_{[1/2]}|\mathbf{p},\downarrow>^+ \; = -i|-\mathbf{p},\uparrow>^+\,, \quad U^s_{[1/2]}|\mathbf{p},\downarrow>^- = -i|-\mathbf{p},\uparrow>^-\,. \tag{15b}$$

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[2]The Evans-Vigier construct got strong criticism, which is mainly reasonable. I want to add that one can find algebraic errors in their works. Nevertheless, nobody seems to be able to doubt the $\mathbf{B}$- cyclic relations.

For the charge conjugation operation in the Fock space we have two physically different possibilities. The first one

$$U^c_{[1/2]} a_\uparrow (\mathbf{p}) (U^c_{[1/2]})^{-1} = +b_\uparrow (\mathbf{p}) \,, \quad U^c_{[1/2]} a_\downarrow (\mathbf{p}) (U^c_{[1/2]})^{-1} = +b_\downarrow (\mathbf{p}) \,, \quad (16a)$$
$$U^c_{[1/2]} b^\dagger_\uparrow (\mathbf{p}) (U^c_{[1/2]})^{-1} = -a^\dagger_\uparrow (\mathbf{p}) \,, \quad U^c_{[1/2]} b^\dagger_\downarrow (\mathbf{p}) (U^c_{[1/2]})^{-1} = -a^\dagger_\downarrow (\mathbf{p}) \quad (16b)$$

is, in fact, reminiscent with the Dirac construct. The action of this operator on the physical states are

$$U^c_{[1/2]} |\mathbf{p}, \uparrow>^+ \; = + \,|\mathbf{p}, \uparrow>^- \quad , \quad U^c_{[1/2]} |\mathbf{p}, \downarrow>^+ = + \,|\mathbf{p}, \downarrow>^- \quad , \qquad (17a)$$
$$U^c_{[1/2]} |\mathbf{p}, \uparrow>^- \; = - \,|\mathbf{p}, \uparrow>^+ \quad , \quad U^c_{[1/2]} |\mathbf{p}, \downarrow>^- = - \,|\mathbf{p}, \downarrow>^+ \quad .$$
(17b)

But, one can also build the charge conjugation operator in the Fock space which acts, *e.g.*, in the following manner:

$$\widetilde{U}^c_{[1/2]} a_\uparrow (\mathbf{p}) (\widetilde{U}^c_{[1/2]})^{-1} = -b_\downarrow (\mathbf{p}) \,, \quad \widetilde{U}^c_{[1/2]} a_\downarrow (\mathbf{p}) (\widetilde{U}^c_{[1/2]})^{-1} = -b_\uparrow (\mathbf{p}) \,, \quad (18a)$$
$$\widetilde{U}^c_{[1/2]} b^\dagger_\uparrow (\mathbf{p}) (\widetilde{U}^c_{[1/2]})^{-1} = +a^\dagger_\downarrow (\mathbf{p}) \,, \quad \widetilde{U}^c_{[1/2]} b^\dagger_\downarrow (\mathbf{p}) (\widetilde{U}^c_{[1/2]})^{-1} = +a^\dagger_\uparrow (\mathbf{p}) \,, \quad (18b)$$

and, therefore,

$$\widetilde{U}^c_{[1/2]} |\mathbf{p}, \uparrow>^+ \; = - \,|$$
p, $\downarrow>^-$, $\widetilde{U}^c_{[1/2]} |\mathbf{p}, \downarrow>^+ = - \,|\mathbf{p}, \uparrow>^-$, (19a)

$\widetilde{U}^c_{[1/2]} |\mathbf{p}, \uparrow>^- = + \,|\mathbf{p}, \downarrow>^+$, $\widetilde{U}^c_{[1/2]} |\mathbf{p}, \downarrow>^- = + \,|\mathbf{p}, \uparrow>^+$.(19b)

One can convince ourselves by straightforward verification in the correctness of the assertions made in [1] (see also the old paper [12]) that

the situation is possible when the operators of the space inversion and the charge conjugation commute in the Fock space. For instance,

$$U^c_{[1/2]} U^s_{[1/2]} |\mathbf{p}, \uparrow>^+ \; = +i U^c_{[1/2]} | -\mathbf{p}, \downarrow>^+ =$$
+ i $| -\mathbf{p}, \downarrow>^-$, (20a)

U$^s_{[1/2]} U^c_{[1/2]} |\mathbf{p}, \uparrow>^+ = U^s_{[1/2]} |\mathbf{p}, \uparrow>^- = +i| -\mathbf{p}, \downarrow>^-$.(20b)

The second choice of the charge conjugation operator answers for the case when the $\widetilde{U}^c_{[1/2]}$ and $U^s_{[1/2]}$ operations anticommute:

$$
\begin{aligned}
\widetilde{U}^c_{[1/2]} U^s_{[1/2]} |\mathbf{p},\uparrow>^+ &= +i\widetilde{U}^c_{[1/2]} |-\mathbf{p},\downarrow>^+ = -i\,|-\mathbf{p},\uparrow>^- \quad , &(21a)\\
U^s_{[1/2]} \widetilde{U}^c_{[1/2]} |\mathbf{p},\uparrow>^+ &= -U^s_{[1/2]} |\mathbf{p},\downarrow>^- = +i\,|-\mathbf{p},\uparrow>^- \quad . &(21b)
\end{aligned}
$$

Next, one can compose states which would have somewhat similar properties to those which we have become accustomed. The states $|\mathbf{p},\uparrow>^+ \pm i|\mathbf{p},\downarrow>^+$ correspond to the positive (negative) parity, respectively. But, what is important, *the antiparticle states* (moving backward in time) have the same properties with respect to the operation of space inversion as the corresponding *particle states* (as opposed to $j = 1/2$ Dirac particles). This is again in accordance with the analysis of Nigam and Foldy [12], and Ahluwalia [1c]. The states which are eigenstates of the charge conjugation operator in the Fock space are

$$
U^c_{[1/2]} \left(|\mathbf{p},\uparrow>^+ \pm i\,|\mathbf{p},\uparrow>^- \right) = \mp i \left(|\mathbf{p},\uparrow>^+ \pm i\,|\mathbf{p},\uparrow>^- \right) \quad . \qquad (22)
$$

There is no a simultaneous set of states which were "eigenstates" of the operator of the space inversion and of the charge conjugation $U^c_{[1/2]}$.

- We found the Majorana representation of the Barut-Muzinich-Williams matrices and the spinors of the (modified) Weinberg formulation (the momentum-space functions in the $(1,0)\oplus(0,1)$ representation space). In this representation the $\gamma_{\mu\nu}$ matrices are the real matrices; the γ_5 matrix is the pure imaginary matrix. The matrix of the unitary transformation is:

$$
U = \frac{1}{2\sqrt{2}} \begin{pmatrix} (1-i)+(1+i)\Theta & -(1-i)+(1+i)\Theta \\ (1+i)+(1-i)\Theta & -(1+i)+(1-i)\Theta \end{pmatrix} \quad , \qquad (23a)
$$

$$
U^\dagger = \frac{1}{2\sqrt{2}} \begin{pmatrix} (1+i)+(1-i)\Theta & (1-i)+(1+i)\Theta \\ -(1+i)+(1-i)\Theta & -(1-i)+(1+i)\Theta \end{pmatrix} \quad . \qquad (23b)
$$

As a result we arrive, $\gamma_{\mu\nu}^{MR} = U\gamma_{\mu\nu}^{CR}$
U†:

$$\gamma_{00}^{MR} = \begin{pmatrix} 0 & \Theta \\ \Theta & 0 \end{pmatrix}, \quad \gamma_{01}^{MR} = \gamma_{10}^{MR} = \begin{pmatrix} 0 & -J_1\Theta \\ -J_1\Theta & 0 \end{pmatrix}, \tag{24a}$$

$$\gamma_{02}^{MR} = \gamma_{20}^{MR} = \begin{pmatrix} iJ_2\Theta & 0 \\ 0 & -iJ_2\Theta \end{pmatrix}, \quad \gamma_{03}^{MR} = \gamma_{30}^{MR} = \begin{pmatrix} 0 & -J_3\Theta \\ -J_3\Theta & 0 \end{pmatrix}, \tag{24b}$$

$$\gamma_{ij}^{MR} = \gamma_{ji}^{MR} = \frac{1}{2}\begin{pmatrix} i(J_{ij}^* - J_{ij})\Theta & (J_{ij}^* + J_{ij})\Theta \\ (J_{ij}^* + J_{ij})\Theta & -i(J_{ij}^* - J_{ij})\Theta \end{pmatrix}, \tag{24c}$$

$$\text{and} \quad \gamma_5^{MR} = \begin{pmatrix} 0 & i\mathbb{1} \\ -i\mathbb{1} & 0 \end{pmatrix}. \tag{24d}$$

The 3×3 matrix Θ corresponds to the Wigner operator in the spin-1 representation

$$\Theta = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad . \tag{25}$$

If one writes

$$u^{MR}(p^\mu) = \frac{1}{2}\begin{pmatrix} \phi_L + \Theta\phi_R \\ \phi_L + \Theta\phi_R \end{pmatrix} + \frac{i}{2}\begin{pmatrix} -\phi_L + \Theta\phi_R \\ \phi_L - \Theta\phi_R \end{pmatrix} = \mathcal{U}^+ + i\mathcal{V}^+, \tag{26a}$$

$$v^{MR}(p^\mu) = \frac{1}{2}\begin{pmatrix} -\phi_L + \Theta\phi_R \\ -\phi_L + \Theta\phi_R \end{pmatrix} + \frac{i}{2}\begin{pmatrix} \phi_L + \Theta\phi_R \\ -\phi_L - \Theta\phi_R \end{pmatrix} = \mathcal{U}^- + i\mathcal{V}^-. \tag{26b}$$

(with $\mathcal{U}$ are the *real* parts and $\mathcal{V}$, the imaginary parts of the "bispinors") one can see that

$$v^{MR}(p^\mu) = \gamma_5^{MR} u^{MR}(p^\mu) = i\gamma_5^{WR}\gamma_0^{WR} u^{MR}(p^\mu) = \begin{pmatrix} 0 & i\mathbb{1} \\ -i\mathbb{1} & 0 \end{pmatrix} u^{MR}(p^\mu). \tag{27}$$

Surprisingly, we have

$$\mathcal{U}^{+}_{\uparrow}(p^{\mu}) = \mathcal{U}^{+}_{\downarrow}(p^{\mu})\,, \quad \mathcal{V}^{+}_{\uparrow}(p^{\mu}) = -\mathcal{V}^{+}_{\downarrow}(p^{\mu}) \quad , \tag{28a}$$

$$\text{but} \quad \mathcal{U}^{+}_{\rightarrow}(p^{\mu}) = 0\,, \quad \mathcal{V}^{+}_{\rightarrow}(p^{\mu}) \neq 0\,. \tag{28b}$$

While the "longitudinal" bispinor $u_{\rightarrow}(p^{\mu})$ has only the imaginary part in this representation, the negative-energy bispinor $v_{\rightarrow}(p^{\mu})$ has only the real part.

Finally, it is interesting to note that the $\lambda^{S(A)}(p^{\mu})$ and

$\rho^{S(A)}(p^{\mu})$ spinors become the pure real (pure imaginary) spinors in the momentum space representation for both $j = 1/2$ and $j = 1$ case.

- Furthermore, we have found some connections between the Dirac field operator and the Majorana-like operator composed of $\lambda^{S,A}$ spinors. If one uses relations (7) between the self/anti-self charge conjugate spinors and the Dirac spinors (together with the identities between λ and ρ spinors) one can deduce:

$$\Psi^{Dirac}(x^{\mu}) = (1 + \frac{i\gamma^{\mu}\partial_{\mu}}{m})\nu^{^{ML}}(x^{\mu}) \quad . \tag{29}$$

The commutation relations (for the creation/annihilation operators of self/anti-self charge conjugate states) may be slightly different comparing to those presented in [1c] but the set of the states

is still *bi-orthonormal.*

Finally, it is interesting to note that

$$\left[\nu^{^{ML}}(x^{\mu}) + \mathcal{C}\nu^{^{ML\,\dagger}}(x^{\mu})\right]/2 = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p}\sum_{\eta}\left[\begin{pmatrix} i\Theta\phi^{*\,\eta}_{_L}(p^{\mu})\\ 0\end{pmatrix} a_{\eta}(p^{\mu})e^{-ipx}\right.$$
$$\left. + \begin{pmatrix} 0\\ \phi^{\eta}_{_L}(p^{\mu})\end{pmatrix} a^{\dagger}_{\eta}(p^{\mu})e^{ipx}\right]\,, \tag{30a}$$

$$\left[\nu^{^{ML}}(x^{\mu}) - \mathcal{C}\nu^{^{ML\,\dagger}}(x^{\mu})\right]/2 = \int \tfrac{d^3\mathbf{p}}{(2\pi)^3}\tfrac{1}{2E_p}\sum_{\eta}\left[\begin{pmatrix} 0\\ \phi^{\eta}_{_L}(p^{\mu})\end{pmatrix} a_{\eta}(p^{\mu})e^{-ipx}\right.$$
$$\left. + \begin{pmatrix} -i\Theta\phi^{*\,\eta}_{_L}(p^{\mu})\\ 0\end{pmatrix} a^{\dagger}_{\eta}(p^{\mu})e^{ipx}\right]\,. \tag{30b}$$

thus naturally leading to the Ziino-Barut scheme of massive chiral fields, ref. [10].

The conclusion is: a lot of work is still required to make certain conclusions about the relevance of the presented construct to describing

the physical world and to the present situation in the neutrino physics.

But, it is important that this construct is permitted by the requirements of the extended Poincaré group symmetry; it is based on the very viable postulates: in fact, after imposing the conditions of the self/anti-self charge conjugacy we derived all consequences only on the basis of the Wigner rules for transformations of left- and right- handed

2-spinors and on the relations between these spinors in the frame with zero momentum.

Thus, as I was taught in the Gorbachev's epoch: "everything is permitted unless forbidden".

Note Added. I would like to express my gratitude to the anonymous referee of "Hadronic Journal" for the useful suggestion to include the discussion of the Santilli *isodual* conjugation [R. M. Santilli, Comm. Theor. Phys. **3** (1994) 153; R. M. Santilli, Hyperfine Interactions **109** (1997) 63; *Elements of Hadronic Mechanics, Vols. I and II* (Ukraine Academy of Sciences, Kiev, 1995); J. V. Kadeisvili, Math. Methods Appl. Sciences **19** (1996) 1349] into this paper. The Santilli *isoduality* is characterized by the map of conventional unit

$$+1 \to 1^d = -1^\dagger = -1\,.$$

Hence, the *isodual* images of the known fields are introduced. They are considered to be of the crucial importance in the description of the antimatter on the *first quantization level.* The need for new mathematics was also argued by F. Antonuccio [gr-qc/9311032, hep-th/9408166], who substantially developed new mathematical construct[3] in applications to the Poincaré group and to the differential calculus.

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[3]This branch of mathematics is now known as the *semi-complex* analysis. It

The author of the *isodual* mathematics writes "that isoduality on a Hilbert space is equivalent to charge conjugation", [R. M. Santilli, Hyperfine Interactions **109**, 63]; "the proof ... first appeared in paper of 1994 [R. M. Santilli, Comm. Theor. Phys. **3**,

153]" and in subsequent monographs. Since the classical representation of the antiparticles is *not* yet elaborated in the completely consistent form in the framework of the ordinary ideas (cf. refs. [E. C. G. Stueckelberg, Helv. Phys. Acta **14** (1941) 372, 588; ibid **15** (1942) 23; J. P. Costella, B. H. McKellar and A. A. Rawlinson, Am. J. Phys. **65** (1997) 835]), the Santilli's approach has certain logical bases and, seems, may serve as the alternative to the introduction of the common-used Fock representation space (the secondary

quantization scheme). Particularly, some advantages of this approach manifest themselves in the "axiomatically consistent inclusion of gravitation in the unified gauge theories of electroweak interactions". Furthermore, the author of the *isodual* mathematics made several predictions (particularly, the concept of *isodual* light), which can distinguish his theory from the 'old physics of antimatter'.

With respect to our ideas (and apart from the discussion of the Fock space) the Santilli approach may be useful in the construction of the self/anti-self charge conjugate states for spin $j = 1$. The difficulties of the construction of such $(1,0) \oplus (0,1)$ "bispinors" were discussed in the previous works [1c,2c] and above.

The complete investigation of possible connections between the Santilli's approach and the theories based on the *extended* Poincaré group, particularly, the comparison with the Barut-Ziino concept of the *doubling* of the Fock space [10]

will be presented in future works.

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is based on the some sort of "splitting" of the unit, $j^2 = 1$. The norm of the semicomplex numbers is *real*

but "unlike the complex case, it may take on *negative* values".

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