Questions in the Theory of the $(1,0)\oplus(0,1)$ Quantized Fields

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We find a mapping between antisymmetric tensor matter fields and the Weinberg's $2(2j+1)$- component "bispinor" fields. Equations which describe the $j=1$ antisymmetric tensor field coincide with the Hammer-Tucker equations entirely and with the Weinberg ones within a subsidiary condition, the Klein-Gordon equation. The new Lagrangian for the Weinberg theory is proposed which is scalar and Hermitian. It is built on the basis of the concept of the 'Weinberg doubles'. Origins of a contradiction between the classical theory, the Weinberg theorem $B-A=\lambda$ for quantum relativistic fields and the claimed 'longitudity' of the antisymmetric tensor field (transformed on the $(1,0)\oplus(0,1)$ Lorentz group representation) after quantization are clarified. Analogs of the $j=1/2$ Feynman-Dyson propagator are presented in the framework of the $j=1$ Weinberg theory. It is then shown that under the definite choice of field functions and initial and boundary conditions the massless $j=1$ Weinberg-Tucker-Hammer equations contain all information that the Maxwell equations for electromagnetic field have. Thus, the former appear to be of use in describing some physical processes for which that could be necessitated or be convenient.

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I. INTRODUCTION

In the sixties Joos [1], Weinberg [2], and Weaver, Hammer and Good [3] proposed very attractive formalism (called as the $2(2j+1)$ theory) for describing higher spin particles. For instance, as opposed to the Proca 4-vector potentials which transform according to the $(1/2,1/2)$ representation of the Lorentz group, in the $j=1$ case the "bispinor" functions are constructed via the $(1,0)\oplus(0,1)$ representation what is on an equal footing to the description of Dirac $j=1/2$ particles. The $2(2j+1)$- component analogs of the Dirac functions in the momentum space were earlier defined as

$$\mathcal{U}_\sigma(\mathbf{p})=\frac{m}{\sqrt{2}}\begin{pmatrix}D^J\left(\alpha(\mathbf{p})\right)\xi_\sigma\\ D^J\left(\alpha^{-1\,\dagger}(\mathbf{p})\right)\xi_\sigma\end{pmatrix}\quad,\tag{1}$$

for positive-energy states, and

$$\mathcal{V}_\sigma(\mathbf{p})=\frac{m}{\sqrt{2}}\begin{pmatrix}D^J\left(\alpha(\mathbf{p})\Theta_{[1/2]}\right)\xi^*_\sigma\\ D^J\left(\alpha^{-1\,\dagger}(\mathbf{p})\Theta_{[1/2]}\right)(-1)^{2J}\xi^*_\sigma\end{pmatrix}\quad,\tag{2}$$

for negative-energy states, e.g., ref. [4, p.107]. The following notations is used

$$\alpha(\mathbf{p}) = \frac{p_0 + m + (\boldsymbol{\sigma}\cdot\mathbf{p})}{\sqrt{2m(p_0+m)}}, \quad \Theta_{[1/2]} = -i\sigma_2 \quad . \tag{3}$$

For instance, in the case of spin $j=1$, one has

$$D^1\left(\alpha(\mathbf{p})\right) = 1 + \frac{(\mathbf{J}\cdot\mathbf{p})}{m} + \frac{(\mathbf{J}\cdot\mathbf{p})^2}{m(p_0+m)} \quad , \tag{4a}$$

$$D^1\left(\alpha^{-1\,\dagger}(\mathbf{p})\right) = 1 - \frac{(\mathbf{J}\cdot\mathbf{p})}{m} + \frac{(\mathbf{J}\cdot\mathbf{p})^2}{m(p_0+m)} \quad , \tag{4b}$$

$$D^1\left(\alpha(\mathbf{p})\Theta_{[1/2]}\right) = \left[1 + \frac{(\mathbf{J}\cdot\mathbf{p})}{m} + \frac{(\mathbf{J}\cdot\mathbf{p})^2}{m(p_0+m)}\right]\Theta_{[1]} \quad , \tag{4c}$$

$$D^1\left(\alpha^{-1\,\dagger}(\mathbf{p})\Theta_{[1/2]}\right) = \left[1 - \frac{(\mathbf{J}\cdot\mathbf{p})}{m} + \frac{(\mathbf{J}\cdot\mathbf{p})^2}{m(p_0+m)}\right]\Theta_{[1]} \quad ; \tag{4d}$$

$\Theta_{[1/2]}, \Theta_{[1]}$ are the Wigner time-reversal operators for spin 1/2 and 1, respectively. Their forms depend on the choice of the basis for spin matrices. For instance in ref. [14] spin-1 Wigner operator was chosen in the anti-diagonal form. These definitions lead to the formulation in which the physical content given by positive and negative-energy "bispinors" is the same (like in the papers of Weinberg and in the further consideration of Tucker and Hammer [5]). In spite of the extensive elaboration of the Weinberg $2(2j+1)$- component theory since the sixties, *e.g.*, refs. [6–12] those researches did not provide us new significant insights in the particle physics.

Recently, a physically different construct in the $(1,0)\oplus(0,1)$ representations has been proposed [13]. Its remarkable feature is: a boson and its antiboson can possess opposite intrinsic parities. The author of those papers wrote: "...purely by accident, in an attempt to understand an old work of Weinberg [2] and to investigate the possible kinamatical origin for the violation of P, CP, and other discrete symmetries [14], a Wigner-type quantum field theory [15] was constructed for a spin-one boson." [1] The definition of the negative-energy solutions in this construct is similar to the Dirac construct for the spin-1/2 case:

$$\mathcal{V}_\sigma(\mathbf{p}) = \gamma_5 \mathcal{U}_\sigma(\mathbf{p}) = (-1)^{1-\sigma} S^c_{[1]} \mathcal{U}_{-\sigma}(\mathbf{p}) \quad , \tag{5}$$

with $S^c_{[1]}$ being the charge conjugation matrix in the $(1,0)\oplus(0,1)$ representation [13a,14]. They can be built by means of the same procedure like used in Eqs. (1) and (2) but with taking into account the possibility of an additional phase factor for up- (down-) components in the bispinorial $j=1/2$ basis, see, e.g., [14,17–19].

On the other hand, the interest in antisymmetric tensor fields, *e.g.*, [20–27], exists for a long time and even grows in connection with recent discoveries of tensor couplings in the π^- and K^+-meson decays. These fields also should transform according to the $(1,0)\oplus(0,1)$ representation.

In the present paper we give a mapping between antisymmetric tensor fields and Weinberg $j=1$ "bispinors", hence propose the Lagrangian formalism for a particular model in the $(1,0)\oplus(0,1)$ representation and emphasize consequences relevant to the present situation in the fundamental physics. This paper comprises ideas presented in [18,28–32].

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[1] Let us note that some steps in this direction have beed made earlier [16] but, unfortunately, the author of the papers of 1965 did not realize all possible physical consequences following from his equation.

II. MAPPING BETWEEN ANTISYMMETRIC TENSOR AND WEINBERG FORMULATIONS

Let us begin with the Proca equations for a $j=1$ massive particle

$$\partial_\mu F_{\mu\nu} = m^2 A_\nu \quad , \tag{6}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{7}$$

in the form given by [16,33]. The Euclidean metric, $x_\mu = (\vec{x}, x_4 = it)$ and notation $\partial_\mu = (\vec{\nabla}, -i\partial/\partial t)$, $\partial_\mu^2 = \vec{\nabla}^2 - \partial_t^2$, are used. By means of the choice of $F_{\mu\nu}$ components as the physical variables one can rewrite the set of equations to

$$m^2 F_{\mu\nu} = \partial_\mu \partial_\alpha F_{\alpha\nu} - \partial_\nu \partial_\alpha F_{\alpha\mu} \tag{8}$$

and

$$\partial_\lambda^2 F_{\mu\nu} = m^2 F_{\mu\nu} \quad . \tag{9}$$

It is easy to show that they can be represented in the form ($F_{44} = 0$, $F_{4i} = iE_i$ and $F_{jk} = \epsilon_{jki} B_i$; $p_\alpha = -i\partial_\alpha$):

$$\begin{cases} (m^2 + p_4^2) E_i + p_i p_j E_j + i\epsilon_{ijk} p_4 p_j B_k = 0 \\ \\ (m^2 + \vec{p}^{\,2}) B_i - p_i p_j B_j + i\epsilon_{ijk} p_4 p_j E_k = 0 \quad , \end{cases} \tag{10}$$

or

$$\begin{cases} \left[m^2 + p_4^2 + \vec{p}^{\,2} - (\vec{J}\vec{p})^2\right]_{ij} E_j + p_4 (\vec{J}\vec{p})_{ij} B_j = 0 \\ \left[m^2 + (\vec{J}\vec{p})^2\right]_{ij} B_j + p_4 (\vec{J}\vec{p})_{ij} E_j = 0 \quad . \end{cases} \tag{11}$$

Adding and subtracting the obtained equations yield

$$\begin{cases} m^2 (\vec{E} + i\vec{B})_i + p_\alpha p_\alpha \vec{E}_i - (\vec{J}\vec{p})^2_{ij} (\vec{E} - i\vec{B})_j + p_4 (\vec{J}\vec{p})_{ij} (\vec{B} + i\vec{E})_j = 0 \\ \\ m^2 (\vec{E} - i\vec{B})_i + p_\alpha p_\alpha \vec{E}_i - (\vec{J}\vec{p})^2_{ij} (\vec{E} + i\vec{B})_j + p_4 (\vec{J}\vec{p})_{ij} (\vec{B} - i\vec{E})_j = 0 \quad , \end{cases} \tag{12}$$

with $(\vec{J}_i)_{jk} = -i\epsilon_{ijk}$ being the $j=1$ spin matrices. Equations are equivalent (within a constant factor) to the Hammer-Tucker equation [5], see also [11,7]

$$(\gamma_{\alpha\beta} p_\alpha p_\beta + p_\alpha p_\alpha + 2m^2)\psi_1 = 0 \quad , \tag{13}$$

in the case of the choice $\chi = \vec{E} + i\vec{B}$ and $\varphi = \vec{E} - i\vec{B}$, $\psi_1 = \mathrm{column}(\chi, \quad \varphi)$. Matrices $\gamma_{\alpha\beta}$ are the covariantly defined matrices of Barut, Muzinich and Williams [34]. The equation (13) for massive particles is characterized by positive- and negative-energy solutions with a physical dispersion only $E_p = \pm\sqrt{\vec{p}^{\,2} + m^2}$, the determinant is equal to

$$\mathrm{Det}\left[\gamma_{\alpha\beta} p_\alpha p_\beta + p_\alpha p_\alpha + 2m^2\right] = -64 m^6 (p_0^2 - \vec{p}^{\,2} - m^2)^3 \quad , \tag{14}$$

but some points concerned with a massless limit should be clarified properly.[2] Following to the analysis of ref. [35b,p.1972][3] and in accordance with the Dirac technique for obtaining wave equations [36] one can conclude that other equations with the physical dispersion could be obtained from

$$(\gamma_{\alpha\beta}p_\alpha p_\beta + a p_\alpha p_\alpha + b m^2)\psi = 0 \quad , \tag{15}$$

with a and b being some numerical constants. As a result of taking into account $E^2 - \vec{p}^{\,2} = m^2$ we draw that the infinity number of equations with the appropriate dispersion exists provided that b and a are connected as follows:

$$\frac{b}{a+1} = 1 \qquad \text{or} \qquad \frac{b}{a-1} = 1 \quad .$$

However, there are only two equations which do not have 'acausal' solutions. The second one (with $a = -1$ and $b = -2$) is[4]

$$(\gamma_{\alpha\beta}p_\alpha p_\beta - p_\alpha p_\alpha - 2m^2)\psi_2 = 0 \quad . \tag{16}$$

Thus, we have found the 'double' of the Hammer-Tucker equation. In the tensor form it leads to the equations which are dual to (10)

$$\begin{cases} (m^2 + \vec{p}^{\,2})C_i - p_i p_j C_j - i\epsilon_{ijk}p_4 p_j D_k = 0 \\ \\ (m^2 + p_4^2)D_i + p_i p_j D_j - i\epsilon_{ijk}p_4 p_j C_k = 0 \quad . \end{cases} \tag{17}$$

They can be rewritten in the form, *cf.* (8),

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[2]Questions of the correct relativistic dispersion relations of different $j = 1$ equations (both massive and massless) and of particle interpretations of these solutions were also discussed in ref. [35b]. For instance it was shown that the Maxwell's equations possess 'acausal' solution with the energy $E = 0$ and the Weinberg equation, while has common solutions with the solutions of the Maxwell's equations, does *not* reduce entirely to the set of Maxwell's equations in the massless limit. The author of [2b] felt some unsatisfaction when discussed this question (see the first line after Eqs. (4.21,4.22) of ref. [2b]) but he missed to indicate in a clear manner that the matrix $(\vec{J}\cdot\vec{p})$ has no the inverse one. Several groups proposed recently interpretations of the $E = 0$ solution. One of them can be connected with the 'action-at-a-distance' concept. If accept this viewpoint, the electromagnetic field has probably an essentially non-local origins and it is connected with the structure of space-time itself. But, the question of the possibility of experimental observation of such the 'action-at-a-distance' is obviously non-trivial, even from the conceptual viewpoint. So, this comments is rather speculative one.

[3]I mean that some fraction of the operator $\delta_{\alpha\beta}p_\alpha p_\beta$ acting on physically permittable states can be substituted as $m^2 \leftrightarrow -\delta_{\alpha\beta}p_\alpha p_\beta$. The general equation can also be obtained by means of setting up the generalized Ryder-Burgard relation [13,18,19].

[4]The determinant of the matrix in the left side of the following equation is also given by the formula (14).

$$m^2 \widetilde{F}_{\mu\nu} = \partial_\mu \partial_\alpha \widetilde{F}_{\alpha\nu} - \partial_\nu \partial_\alpha \widetilde{F}_{\alpha\mu} \quad , \tag{18}$$

with $\widetilde{F}_{4i} = iD_i$ and $\widetilde{F}_{jk} = -\epsilon_{jki} C_i$. The vector C_i is an analog of E_i and D_i is an analog of B_i because in some cases it is convenient to equate $\widetilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma}$, $\epsilon_{1234} = -i$. The following properties of the antisymmetric Levi-Civita tensor

$$\epsilon_{ijk}\epsilon_{ijl} = 2\delta_{kl} \quad , \quad \epsilon_{ijk}\epsilon_{ilm} = (\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}) \quad ,$$

and

$$\epsilon_{ijk}\epsilon_{lmn} = \mathrm{Det} \begin{pmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{pmatrix}$$

have been used.

Comparing the structure of the Weinberg equation ($a = 0$, $b = 1$) with the Hammer-Tucker 'doubles' one can convince ourselves that the former can be represented in the tensor form:

$$m^2 F_{\mu\nu} = \partial_\mu \partial_\alpha F_{\alpha\nu} - \partial_\nu \partial_\alpha F_{\alpha\mu} + \frac{1}{2}(m^2 - \partial_\lambda^2) F_{\mu\nu} \quad , \tag{19}$$

that corresponds to Eq. (21). However, as we learnt, it is possible to build an equation — 'double':

$$m^2 \widetilde{F}_{\mu\nu} = \partial_\mu \partial_\alpha \widetilde{F}_{\alpha\nu} - \partial_\nu \partial_\alpha \widetilde{F}_{\alpha\mu} + \frac{1}{2}(m^2 - \partial_\lambda^2) \widetilde{F}_{\mu\nu} \quad , \tag{20}$$

that corresponds to Eq. (22). The Weinberg's set of equations is written in the form:

$$(\gamma_{\alpha\beta} p_\alpha p_\beta + m^2)\psi_1 = 0 \quad , \tag{21}$$
$$(\gamma_{\alpha\beta} p_\alpha p_\beta - m^2)\psi_2 = 0 \quad . \tag{22}$$

Thanks to the Klein-Gordon equation (9) these equations are equivalent to the Proca tensor equations (and to the Hammer-Tucker ones) in a free case. However, if interaction is included, one cannot say that. Thus, the general solution describing the $j = 1$ states can be presented as a superposition

$$\Psi^{(1)} = c_1 \psi_1^{(1)} + c_2 \psi_2^{(1)} \quad , \tag{23}$$

where the constants c_1 and c_2 are to be defined from the boundary, initial and normalization conditions. Let me note a surprising fact: while both the massive Proca equations (or the Hammer-Tucker ones) and the Klein-Gordon equation do not possess "non-physical" solutions, their sum, Eqs. (19,20), or the Weinberg equations (21,22), acquire tachyonic solutions. Next, equations (21) and (22) can recast in another form (index "T" denotes a transpose matrix):

$$\left[\gamma_{44} p_4^2 + 2\gamma_{4i}^T p_4 p_i + \gamma_{ij} p_i p_j - m^2\right] \psi_1^{(2)} = 0 \quad , \tag{24}$$
$$\left[\gamma_{44} p_4^2 + 2\gamma_{4i}^T p_4 p_i + \gamma_{ij} p_i p_j + m^2\right] \psi_2^{(2)} = 0 \quad , \tag{25}$$

respectively, if understand $\psi_1^{(2)} \sim \mathrm{column}(B_i + iE_i, \quad B_i - iE_i) = i\gamma_5\gamma_{44}\psi_1^{(1)}$ and $\psi_2^{(2)} \sim \mathrm{column}(D_i + iC_i, \quad D_i - iC_i) = i\gamma_5\gamma_{44}\psi_2^{(1)}$. The general solution is again a linear combination

$$\Psi^{(2)} = c_1\psi_1^{(2)} + c_2\psi_2^{(2)} \quad . \tag{26}$$

From, *e.g.*, Eq. (21), dividing $\psi_1^{(1)}$ into longitudinal and transversal parts one can come to the equations

$$\begin{aligned}&\left[E^2-\vec{p}^{\,2}\right](\vec{E}+i\vec{B})^{\parallel} - m^2(\vec{E}-i\vec{B})^{\parallel}+\\&+\left[E^2+\vec{p}^{\,2}-2E(\vec{J}\vec{p})\right](\vec{E}+i\vec{B})^{\perp} - m^2(\vec{E}-i\vec{B})^{\perp} = 0 \quad ,\end{aligned} \tag{27}$$

and

$$\begin{aligned}&\left[E^2-\vec{p}^{\,2}\right](\vec{E}-i\vec{B})^{\parallel} - m^2(\vec{E}+i\vec{B})^{\parallel}+\\&+\left[E^2+\vec{p}^{\,2}+2E(\vec{J}\vec{p})\right](\vec{E}-i\vec{B})^{\perp} - m^2(\vec{E}+i\vec{B})^{\perp} = 0 \quad .\end{aligned} \tag{28}$$

One can see that in the classical field theory antisymmetric tensor matter fields are the fields with the transversal components in massless limit. The longitudinal "parts" of the above equations do not contain the terms as $\vec{J}\cdot\vec{p}$) provided that the longitudinal modes are associated with the plane waves too. This can be easily seen on choosing the spin basis where $(S^i)_{jk} = -i\epsilon^{ijk}$ and on using the definition of the longitudinal modes, $\vec{p}\times(\vec{E}\pm i\vec{B})^{\parallel}\equiv 0$.

In the connection with the above discussion, the statements of the "longitudinal nature" of the antisymmetric tensor field after quantization, made by several authors [22,23,25] and [28a], are very surprising. For instance, M. Kalb and P. Ramond claimed explicitly [23c, p. 2283, the third line from below]: "thus, the massless $\phi_{\mu\nu}$ has one degree of freedom". If the interpretation is used when the Kalb-Ramond Lagrangian describes the $(1,0)\oplus(0,1)$ field only, these authors contradicted with the Correspondence Principle and with the principles of the relativistic theory. We discuss this question below and in subsequent publications. At least, they had to explain, whether the gauge field associated with the Kalb-Ramond gauge invariance has physical significance or not, and what are the transformation laws for the antisymmetric tensor field of the third rank and for "potentials" $\phi_{\mu\nu}$.

Under the transformations $\psi_1^{(1)}\rightarrow\gamma_5\psi_2^{(1)}$ or $\psi_1^{(2)}\rightarrow\gamma_5\psi_2^{(2)}$ the set of equations (21) and (22), or (24) and (25), leaves to be invariant. The origin of this fact is the dual invariance of the set of the Proca equations. In a matrix form dual transformations correspond to the chiral transformations (see for discussion, *e.g.*, ref. [37]).

Another equation has been proposed in refs. [16,13]

$$(\gamma_{\alpha\beta}p_\alpha p_\beta + \wp_{u,v}m^2)\psi = 0 \quad , \tag{29}$$

where $\wp_{u,v} = i(\partial/\partial t)/E$, what distinguishes u- (positive-energy) and v- (negative-energy) solutions. For instance, in [13a,footnote 4] it is claimed that

$$\psi_\sigma^{+}(x) = \frac{1}{(2\pi)^3}\int\frac{d^3p}{2\omega_p}u_\sigma(\vec{p})e^{ipx} \quad , \tag{30}$$

$\omega_p = \sqrt{m^2+\vec{p}^{\,2}}$, $p_\mu x_\mu = \vec{p}\vec{x} - Et$, must be described by the equation (21), in the meantime,

$$\psi_\sigma^{-}(x) = \frac{1}{(2\pi)^3}\int\frac{d^3p}{2\omega_p}v_\sigma(\vec{p})e^{-ipx} \quad , \tag{31}$$

by the equation (22). Nevertheless, calculating the determinants (14) of the equations (13,16) we convinced ourselves that the first one has the *negative-energy* solutions and the second one, *the positive-energy* solutions. The same is true for both Weinberg equations, they are also have these solutions and below we are going to give their explicit forms. The question of the choice of appropriate equations for different physical systems was discussed in refs. [14,17,18]. The answer depends on desirable particle properties with respect to discrete symmetries.

Let me consider the question of the 'double' solutions on the basis of spinorial analysis. In ref. [16a,p.1305] (see also [38, p.60-61]) relations between the Weinberg $j = 1$ "bispinor" (bivector, indeed) and symmetric spinors of $2j$- rank have been discussed. It was noted there: "The wave function may be written in terms of two three-component functions $\psi = \mathrm{column}(\chi \quad \varphi)$, that, for the continuous group, transform independently each of other and that are related to two symmetric spinors:

$$\chi_1 = \chi_{\dot{1}\dot{1}}, \quad \chi_2 = \sqrt{2}\chi_{\dot{1}\dot{2}}, \quad \chi_3 = \chi_{\dot{2}\dot{2}} \quad , \tag{32}$$

$$\varphi_1 = \varphi^{11}, \quad \varphi_2 = \sqrt{2}\varphi^{12}, \quad \varphi_3 = \varphi^{22} \quad , \tag{33}$$

when the standard representation for the spin-one matrices, with S_3 diagonal is used." Under the inversion operation we have the following rules [38, p.59]: $\varphi^\alpha \to \chi_{\dot\alpha}$, $\chi_{\dot\alpha} \to \varphi^\alpha$, $\varphi_\alpha \to -\chi^{\dot\alpha}$ and $\chi^{\dot\alpha} \to -\varphi_\alpha$. Hence, one can deduce (if one understand $\chi_{\dot\alpha\dot\beta} = \chi_{\{\dot\alpha}\chi_{\dot\beta\}}$, $\varphi^{\alpha\beta} = \varphi^{\{\alpha}\varphi^{\beta\}}$)

$$\chi_{\dot{1}\dot{1}} \to \varphi^{11} \quad , \quad \chi_{\dot{2}\dot{2}} \to \varphi^{22} \quad , \quad \chi_{\{\dot{1}\dot{2}\}} \to \varphi^{\{12\}} \quad , \tag{34}$$

$$\varphi^{11} \to \chi_{\dot{1}\dot{1}} \quad , \quad \varphi^{22} \to \chi_{\dot{2}\dot{2}} \quad , \quad \varphi^{\{12\}} \to \chi_{\{\dot{1}\dot{2}\}} \quad . \tag{35}$$

However, this definition of symmetric spinors of the second rank χ and φ is ambiguous. We are also able to define, *e.g.*, $\tilde\chi_{\dot\alpha\dot\beta} = \chi_{\{\dot\alpha}H_{\dot\beta\}}$ and $\tilde\varphi^{\alpha\beta} = \varphi^{\{\alpha}\Phi^{\beta\}}$, where $H_{\dot\beta} = \varphi^*_\beta$, $\Phi^\beta = (\chi^{\dot\beta})^*$. It is straightforward to show that in the framework of the second definition we have under the space-inversion operation:

$$\tilde\chi_{\dot{1}\dot{1}} \to -\tilde\varphi^{11} \quad , \quad \tilde\chi_{\dot{2}\dot{2}} \to -\tilde\varphi^{22} \quad , \quad \tilde\chi_{\{\dot{1}\dot{2}\}} \to -\tilde\varphi^{\{12\}} \quad , \tag{36}$$

$$\tilde\varphi^{11} \to -\tilde\chi_{\dot{1}\dot{1}} \quad , \quad \tilde\varphi^{22} \to -\tilde\chi_{\dot{2}\dot{2}} \quad , \quad \tilde\varphi^{\{12\}} \to -\tilde\chi_{\{\dot{1}\dot{2}\}} \quad . \tag{37}$$

The Weinberg "bispinor" $(\chi_{\dot\alpha\dot\beta} \quad \varphi^{\alpha\beta})$ corresponds to the equations (24) and (25) , meanwhile $(\tilde\chi_{\dot\alpha\dot\beta} \quad \tilde\varphi^{\alpha\beta})$, to the equations (21) and (22). Similar conclusions can be achieved in the case of the parity definition as $P^2 = -1$. Transformation rules are then $\varphi^\alpha \to i\chi_{\dot\alpha}$, $\chi_{\dot\alpha} \to i\varphi^\alpha$, $\varphi_\alpha \to -i\chi^{\dot\alpha}$ and $\chi^{\dot\alpha} \to -i\varphi_\alpha$, ref. [38, p.59] . Hence, $\chi_{\dot\alpha\dot\beta} \leftrightarrow -\varphi^{\alpha\beta}$ and $\tilde\chi_{\dot\alpha\dot\beta} \leftrightarrow -\tilde\varphi^{\alpha\beta}$, but $\varphi_\alpha{}^\beta \leftrightarrow \chi^\alpha{}_\beta$ and $\tilde\varphi_\alpha{}^\beta \leftrightarrow \tilde\chi^\alpha{}_\beta$.

Next, in the previous formulations of the Weinberg theory the following Lagrangian was proposed [8,9] and [11b,28a,b]:

$$\mathcal{L}^W = -\partial_\mu\overline{\psi}\gamma_{\mu\nu}\partial_\nu\psi - m^2\overline{\psi}\psi \quad , \tag{38}$$

$\gamma_{\mu\nu}$ are the Barut-Muzinich-Williams matrices which are chosen to be Hermitian. It is scalar, *cf.* [28a], and Hermitian[5], *cf.* [8] and it contains only first-order time derivatives. Again implying interpretation of the "6-spinor" as[6]

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[5]When the Euclidean metric is used the only inconvenience must be taken in mind where it is necessary: we need imply that $\partial^\dagger_\mu = (\vec\nabla, -\partial/\partial x_4)$, provided that $\partial_\mu = (\vec\nabla, \partial/\partial x_4)$, ref. [33].

[6]One can also choose

$$\psi^{(2)} = \begin{pmatrix} \vec E + i\vec B \\ -\vec E + i\vec B \end{pmatrix} = \gamma_5\psi \quad .$$

$$\begin{cases} \chi = \vec{E} + i\vec{B} & , \\ \phi = \vec{E} - i\vec{B} & , \end{cases} \tag{39}$$

$\psi = \text{column}(\chi \quad \phi)$, $\vec{E}$ and $\vec{B}$ are the real 3-vectors, the Lagrangian (38) can be re-written in the following way:

$$\mathcal{L}^{AT} = -(\partial_\mu F_{\nu\alpha})(\partial_\mu F_{\nu\alpha}) + 2(\partial_\mu F_{\mu\alpha})(\partial_\nu F_{\nu\alpha}) + 2(\partial_\mu F_{\nu\alpha})(\partial_\nu F_{\mu\alpha}) + m^2 F_{\mu\nu} F_{\mu\nu} \quad . \tag{40}$$

In a massless limit this form of the Lagrangian leads to the Euler-Lagrange equation

$$(\square - m^2) F_{\alpha\beta} - 2(\partial_\beta F_{\alpha\mu,\mu} - \partial_\alpha F_{\beta\mu,\mu}) = 0 \quad , \tag{41}$$

where $\square = \partial_\nu \partial_\nu$. After the application of the generalized Lorentz condition [22] the massless Lagrangian (40) becomes to be equivalent to the Lagrangian of a free massless skew-symmetric field given in ref. [22]:

$$\mathcal{L}^H = \frac{1}{8} F_k F_k \quad , \tag{42}$$

with $F_k = i\epsilon_{kjmn} F_{jm,n}$. It is re-written in $(m = 0)$:

$$\begin{aligned} \mathcal{L}^H &= -\frac{1}{4}(\partial_\mu F_{\nu\alpha})(\partial_\mu F_{\nu\alpha}) + \frac{1}{2}(\partial_\mu F_{\nu\alpha})(\partial_\nu F_{\mu\alpha}) = \\ &= \frac{1}{4}\mathcal{L}^{AT} - \frac{1}{2}(\partial_\mu F_{\mu\alpha})(\partial_\nu F_{\nu\alpha}) \quad , \end{aligned} \tag{43}$$

what proves the statement made above. After the application of the Fermi method *mutatis mutandis* as in ref. [22] (*cf.* with the quantization procedure for a 4-vector potential field) one achieved the result that the Lagrangians (38) and (42) describe massless particles possessing longitudinal physical components only. Transversal components are removed by means of the "gauge" transformation

$$F_{\mu\nu} \rightarrow F_{\mu\nu} + A_{[\mu\nu]} = F_{\mu\nu} + \partial_\nu \Lambda_\mu - \partial_\mu \Lambda_\nu \tag{44}$$

(or by the transformation similar to the above but applied to the Weinberg bivector). This is a contradiction to which has been paid attention in [28a,b]: the $j = 1$ antisymmetric tensor field was believed to possess the longitudinal component only, the helicity is therefore equal to $\lambda = 0$. In the meantime, they transform according to the $(1,0) + (0,1)$ representation of the Lorentz group (like a Helmoltz-Weinberg bivector). How is the Weinberg theorem, ref. [2], for the (A,B) representation to be treated in this case? [7] If we want to have well-defined creation and annihilation operators the antisymmetric tensor field should have helicities $\lambda = \pm 1$.[8] Moreover, do the claims of the "longitudinal nature" of the antisymmetric tensor field and, hence, the Weinberg $j = 1$ field signify that we must abandon

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Since $\overline{\psi}^{(2)} = -\overline{\psi}\gamma_5$ the dynamical term (38) is not changed. But the sign in the mass term would be inverse.

[7]Let me recall that the Weinberg theorem states: *The fields constructed from the massless particle operator* $a(\vec{p},\lambda)$ *of the definite helicity transform according to representation* (A,B) *such that* $B - A = \lambda$.

[8]Several authors indicated this from different viewpoints in refs. [20,21,24,13].

the Correspondence Principle: in the classical physics we know that an antisymmetric tensor field is with transversal components, see also Eqs. (27,28)?[9]

This contradiction has been analyzed in refs. [29–32,39,40] in detail. The result achieved is: transversal components are always linked with longitudinal spin components and can be decoupled only in particular cases. Using the Weinberg formalism we provide additional support to this conclusion in the following Section.

We conclude this Section: both the theory of Ahluwalia *et al.* [13,14] and the model based on the use of ψ_1 and ψ_2 are connected with the antisymmetric tensor matter field description. They have to be quantized consistently. Special attention should be paid to the translational and rotational invariance (the conservation of energy-momentum and angular momentum, indeed), the interaction representation, causality, locality and covariance of the theory, *i.e.* to all topics, which are the axioms of the modern quantum field theory [41,42]. A consistent theory has also to take into account the degeneracy of states: two dual functions ψ_1 and ψ_2 (or $F_{\mu\nu}$ and $\widetilde{F}_{\mu\nu}$, the 'doubles') are considered to yield the same spectrum.

III. WHAT PARTICLES ARE DESCRIBED BY THE WEINBERG THEORY?

In the previous Section the concept of the Weinberg $j=1$ field as a system of degenerate states has been proposed. As a matter of fact a model with the Weinberg 'doubles' is equivalent to dual electrodynamics with the antisymmetric tensor field $F_{\mu\nu}$ and its dual $\widetilde{F}_{\mu\nu}$. Unfortunately, many works concerned with the dual theories [24,37,43,44] did not worked out quantization issues in detail and many specific features of such a consideration have not been taken into account earlier.[10]

We begin with the Lagrangian which is similar to Eq. (38) but includes additional terms which respond to the Weinberg 'double'. Here it is:[11]

$$\mathcal{L}^{(1)} = -\partial_\mu \overline{\psi}_1 \gamma_{\mu\nu} \partial_\nu \psi_1 - \partial_\mu \overline{\psi}_2 \gamma_{\mu\nu} \partial_\nu \psi_2 - m^2 \overline{\psi}_1 \psi_1 + m^2 \overline{\psi}_2 \psi_2 \quad . \tag{45}$$

The Lagrangian (45) leads to the equations (21,22) which possess solutions with a "correct" bradyon physical dispersion and tachyonic solutions as well. The second equation coincides with the Ahluwalia *et al.* equation for v spinors (Eq. (13) ref. [13a]) or with Eq. (12) of ref. [16c]. If accept the concept of the Weinberg field as a set of degenerate states, one has to allow for possible transitions $\psi_1 \leftrightarrow \psi_2$ (or $F_{\mu\nu} \leftrightarrow \widetilde{F}_{\mu\nu}$). ¿From the first sight, one can propose the Lagrangian with the following dynamical part:

$$\mathcal{L}^{(2')} = -\partial_\mu \overline{\psi}_1 \gamma_{\mu\nu} \partial_\nu \psi_2 - \partial_\mu \overline{\psi}_2 \gamma_{\mu\nu} \partial_\nu \psi_1 \quad , \tag{46}$$

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[9] Concerning with this contradiction one of the referees made the comment:"...the contradictions following Eq. (44) arise from a confusion regarding the "transverse" nature of the $\vec{E}$ and $\vec{B}$ fields with respect to the $\vec{p}$ and at the same time "longitudinal" nature of helicity for the SAME $\vec{E}$ and $\vec{B}$ configuration". I agree.

[10] We wish to mention that dual formulations of the Dirac field, the $(1/2,0)\oplus(0,1/2)$ representation, have also been considered, *e. g.*, ref. [45–47,17,18]. The interaction of the Dirac field with the dual fields $F_{\mu\nu}$ and $\widetilde{F}_{\mu\nu}$ has been considered in ref. [48] (this implies the existence of the anomalous electric dipole moment of a fermion).

[11] Of course, one can use another form with substitutions: $\psi^{(1)}_{1,2} \rightarrow \psi^{(2)}_{2,1}$ and $\gamma_{\mu\nu} \rightarrow \widetilde{\gamma}_{\mu\nu}$, where $\widetilde{\gamma}_{\mu\nu} \equiv \gamma^T_{\mu\nu} \equiv \gamma_{44}\gamma_{\mu\nu}\gamma_{44}$.

where ψ_1 and ψ_2 are defined by the equations (21,22). But, this form appears not to admit a mass term in a usual manner. From a mathematical viewpoint one can find solution: set m^2 to be pure imaginary quantity (or in the operator formulation, the anti-Hermitian operator). We touched this case earlier [30]. More logical approach seems to be in regarding all four states described by Eqs. (21,22,24,25). The following Lagrangian can be proposed in this case:

$$\mathcal{L}^{(2)} = -\partial_\mu \psi_1^{(1)\,\dagger} \tilde{\gamma}_{\mu\nu} \partial_\nu \psi_2^{(2)} - \partial_\mu \psi_2^{(2)\,\dagger} \gamma_{\mu\nu} \partial_\nu \psi_1^{(1)} - \partial_\mu \psi_2^{(1)\,\dagger} \tilde{\gamma}_{\mu\nu} \partial_\nu \psi_1^{(2)} - \partial_\mu \psi_1^{(2)\,\dagger} \gamma_{\mu\nu} \partial_\nu \psi_2^{(1)} - \\ - m^2 \psi_2^{(2)\,\dagger} \psi_1^{(1)} - m^2 \psi_1^{(1)\,\dagger} \psi_2^{(2)} + m^2 \psi_2^{(1)\,\dagger} \psi_1^{(2)} + m^2 \psi_1^{(2)\,\dagger} \psi_2^{(1)} \quad . \tag{47}$$

Both the Lagrangian (45) and (47) are scalars,[12] Hermitian and they contain only first-order time derivatives. The both lead to similar equations for $\psi_1^{(1,2)}(x)$ and $\psi_2^{(1,2)}(x)$ but one should not forget about the difference in signs in mass terms when considering the equations for $\psi_i^{(k)}(x)$.

At this point I would like to regard the question of solutions in the momentum space. Using the plane-wave expansion[13]

$$\psi_1^{(k)}(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{1}{m\sqrt{2E_p}} \left[\mathcal{U}_1^{(k)\,\sigma}(\vec{p}) a_\sigma^{(k)}(\vec{p}) e^{ipx} + \mathcal{V}_1^{(k)\,\sigma}(\vec{p}) b_\sigma^{(k)\,\dagger}(\vec{p}) e^{-ipx}\right] \quad , \tag{48}$$

$$\psi_2^{(k)}(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{1}{m\sqrt{2E_p}} \left[\mathcal{U}_2^{(k)\,\sigma}(\vec{p}) c_\sigma^{(k)}(\vec{p}) e^{ipx} + \mathcal{V}_2^{(k)\,\sigma}(\vec{p}) d_\sigma^{(k)\,\dagger}(\vec{p}) e^{-ipx}\right] \quad , \tag{49}$$

$E_p = \sqrt{\vec{p}^{\,2} + m^2}$, one can see that the momentum-space 'double' equations

$$\left[-\gamma_{44} E^2 + 2iE\gamma_{4i}\vec{p}_i + \gamma_{ij}\vec{p}_i\vec{p}_j + m^2\right] \mathcal{U}_1^\sigma(\vec{p}) = 0 \quad (\text{or } \mathcal{V}_1^\sigma(\vec{p})) \quad , \tag{50}$$

$$\left[-\gamma_{44} E^2 + 2iE\gamma_{4i}\vec{p}_i + \gamma_{ij}\vec{p}_i\vec{p}_j - m^2\right] \mathcal{U}_2^\sigma(\vec{p}) = 0 \quad (\text{or } \mathcal{V}_2^\sigma(\vec{p})) \tag{51}$$

are satisfied by "bispinors"

$$\mathcal{U}_1^{(1)\,\sigma}(\vec{p}) = \frac{m}{\sqrt{2}} \begin{pmatrix} \left[1 + \frac{(\vec{J}\vec{p})}{m} + \frac{(\vec{J}\vec{p})^2}{m(E+m)}\right] \xi_\sigma \\ \left[1 - \frac{(\vec{J}\vec{p})}{m} + \frac{(\vec{J}\vec{p})^2}{m(E+m)}\right] \xi_\sigma \end{pmatrix} \quad , \tag{52}$$

and

$$\mathcal{U}_2^{(1)\,\sigma}(\vec{p}) = \frac{m}{\sqrt{2}} \begin{pmatrix} \left[1 + \frac{(\vec{J}\vec{p})}{m} + \frac{(\vec{J}\vec{p})^2}{m(E+m)}\right] \xi_\sigma \\ \left[-1 + \frac{(\vec{J}\vec{p})}{m} - \frac{(\vec{J}\vec{p})^2}{m(E+m)}\right] \xi_\sigma \end{pmatrix} \quad , \tag{53}$$

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[12] It is easy to verify this by means of taking into account proposed interpretations of $\psi_i^{(k)}(x)$ which are connected with the tensor $F^{\mu\nu}$ and its dual. There is also another way, on the basis of the use of explicit forms of momentum-space "6-spinors", see below.

[13] I stress that to keep a mathematical approach as general as possible is in the aims of the present investigation. The relevance of different photon spin states to different forms of field operators will be studied in more detail in forthcoming publications.

respectively. The form (52) has been presented by Hammer, Tucker and Novozhilov in refs. [5,4], see also [11]. The bispinor normalization in the cited papers is chosen to unit. However, as mentioned in ref. [13] it is more convenient to work with bispinors normalized to the mass, *e.g.*, $\pm m^{2j}$ in order to make zero-momentum spinors to vanish in the massless limit. Here and below I keep the normalization of bispinors as in ref. [13]. Bispinors of Ahluwalia *et al.*, ref. [13], can be written in the more compact form:

$$u^\sigma_{AJG}(\vec{p}) = \begin{pmatrix} \left[m + \frac{(\vec{J}\vec{p})^2}{E+m}\right]\xi_\sigma \\ (\vec{J}\vec{p})\xi_\sigma \end{pmatrix} \quad , \quad v^\sigma_{AJG}(\vec{p}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} u^\sigma_{AJG}(\vec{p}) \quad . \tag{54}$$

They coincide with the Hammer-Tucker-Novozhilov bispinors within a normalization and a unitary transformation by $\mathcal{U}$ matrix:

$$u^\sigma{}_{[13]}(\vec{p}) = m \quad \cdot \mathbf{U}\mathcal{U}^\sigma{}_{[5,4]}(\vec{p}) = \frac{m}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \mathcal{U}^\sigma{}_{[5,4]}(\vec{p}) \quad , \tag{55}$$

$$v^\sigma{}_{[13]}(\vec{p}) = m \quad \cdot \mathbf{U}\gamma_5\mathcal{U}^\sigma{}_{[5,4]}(\vec{p}) = \frac{m}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \gamma_5\mathcal{U}^\sigma{}_{[5,4]}(\vec{p}) \quad . \tag{56}$$

But, as we found the Weinberg equations (with $+m^2$ and with $-m^2$) have solutions with both positive- and negative-energies. We have to propose the interpretation of the latter. In the framework of this paper one can consider that $\mathcal{V}^{(1,2)}_\sigma(\vec{p}) = (-1)^{1-\sigma}\gamma_5 S^c_{[1]}\mathcal{U}^{(1,2)}_{-\sigma}(\vec{p})$ and, thus, the explicit form of the negative-energy solutions would be the same as of the positive-energy solutions in accordance with definitions (1,2), see the discussion in the Section I. Thus, in the case of a choice $\mathcal{U}^{(1)\,\sigma}_1(\vec{p})$ and $\mathcal{V}^{(1)\,\sigma}_2 \sim \gamma_5\mathcal{U}^{(1)\,\sigma}_1(\vec{p})$ as physical bispinors we come to the Bargmann-Wightman-Wigner-type (BWW) quantum field model proposed by Ahluwalia *et al.* Of course, following to the same logic one can choose $\mathcal{U}^{(1)\,\sigma}_2$ and $\mathcal{V}^{(1)\,\sigma}_1$ bispinors and come to yet another version of the BWW theory. While in this case parities of a boson and its antiboson are opposite, we have -1 for $\mathcal{U}-$ bispinor and $+1$ for $\mathcal{V}-$ bispinor, *i.e.* different in the sign from the model of Ahluwalia *et al.*.[14] In the meantime, the construct proposed by Weinberg [2] and developed in this paper is also possible. I do not agree with the claim of the authors of ref. [13a,footnote 4] which states $\mathcal{V}^{(1)\,\sigma}_1(\vec{p})$ are not solutions of the equation (21). The origin of the possibility that the $\mathcal{U}_i$- and $\mathcal{V}_i$- bispinors in Eqs. (50,51) can coincide each other is the following: the Weinberg equations are of the second order in time derivatives. The Bargmann-Wightman-Wigner construct presented by Ahluwalia [13] is not the only construct in the $(1,0)\oplus(0,1)$ representation and one can start with the earlier definitions of the $2(2j+1)$ bispinors.

Next, in the Section II we gave two additional equations (24,25). Their solutions can also be useful because of the possibility of the use of the Lagrangian form (47). The solutions in the momentum representation are written as follows

$$\mathcal{U}^{(2)\,\sigma}_1(\vec{p}) = \frac{m}{\sqrt{2}} \begin{pmatrix} \left[1 - \frac{(\vec{J}\vec{p})}{m} + \frac{(\vec{J}\vec{p})^2}{m(E+m)}\right]\xi_\sigma \\ \left[-1 - \frac{(\vec{J}\vec{p})}{m} - \frac{(\vec{J}\vec{p})^2}{m(E+m)}\right]\xi_\sigma \end{pmatrix} \quad , \tag{57}$$

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[14] At the present level of our knowledge this mathematical difference has no physical significance, because it is believed that "ONLY relative intrinsic parities of the particles are physically observable". But we want to stay in the most general frameworks and, perhaps, some forms of interactions can lead to the observed physical difference between these models.

$$\mathcal{U}_2^{(2)\,\sigma}(\vec{p}) == \frac{m}{\sqrt{2}}\begin{pmatrix}\left[1-\frac{(\vec{J}\vec{p})}{m}+\frac{(\vec{J}\vec{p})^2}{m(E+m)}\right]\xi_\sigma\\ \left[1+\frac{(\vec{J}\vec{p})}{m}+\frac{(\vec{J}\vec{p})^2}{m(E+m)}\right]\xi_\sigma\end{pmatrix} \quad . \tag{58}$$

Therefore, one has $\mathcal{U}_2^{(1)}(\vec{p}) = \gamma_5\mathcal{U}_1^{(1)}(\vec{p})$ and $\overline{\mathcal{U}}_2^{(1)}(\vec{p}) = -\overline{\mathcal{U}}_1^{(1)}(\vec{p})\gamma_5$; $\mathcal{U}_1^{(2)}(\vec{p}) = \gamma_5\gamma_{44}\mathcal{U}_1^{(1)}(\vec{p})$ and $\overline{\mathcal{U}}_1^{(2)} = \overline{\mathcal{U}}_1^{(1)}\gamma_5\gamma_{44}$; $\mathcal{U}_2^{(2)}(\vec{p}) = \gamma_{44}\mathcal{U}_1^{(1)}(\vec{p})$ and $\overline{\mathcal{U}}_2^{(2)}(\vec{p}) = \overline{\mathcal{U}}_1^{(1)}\gamma_{44}$. In fact, they are connected by transformations of the inversion group.

Let me now repeat the quantization procedure for antisymmetric tensor field presented, *e.g.*, in ref. [22], however, it will be applied to the Weinberg field. Let me trace contributions of $\mathcal{L}^{(1)}$ to dynamical invariants. From the definitions [33]:

$$\mathcal{T}_{\mu\nu} = -\sum_i\left\{\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\partial_\nu\phi_i + \partial_\nu\overline{\phi}_i\frac{\partial\mathcal{L}}{\partial(\partial_\mu\overline{\phi}_i)}\right\} + \mathcal{L}\delta_{\mu\nu} \quad , \tag{59}$$

$$P_\mu = \int\mathcal{P}_\mu(x)d^3x = -i\int\mathcal{T}_{4\mu}d^3x \tag{60}$$

one can find the energy-momentum tensor[15]

[15]Finding the classical dynamical invariants from the Lagrangian $\mathcal{L}^{(2)}$ does not present any difficulties. Here they are:

$$\begin{aligned}\mathcal{T}_{\mu\nu}^{(2)} &= \partial_\alpha\psi_1^{(1)\,\dagger}\widetilde{\gamma}_{\alpha\mu}\partial_\nu\psi_2^{(2)} + \partial_\alpha\psi_1^{(2)\,\dagger}\gamma_{\alpha\mu}\partial_\nu\psi_2^{(1)} + \partial_\alpha\psi_2^{(1)\,\dagger}\widetilde{\gamma}_{\alpha\mu}\partial_\nu\psi_1^{(2)} + \partial_\alpha\psi_2^{(2)\,\dagger}\gamma_{\alpha\mu}\partial_\nu\psi_1^{(1)} + \\ &+ \partial_\nu\psi_1^{(1)\,\dagger}\widetilde{\gamma}_{\mu\alpha}\partial_\alpha\psi_2^{(2)} + \partial_\nu\psi_1^{(2)\,\dagger}\gamma_{\mu\alpha}\partial_\alpha\psi_2^{(1)} + \partial_\nu\psi_2^{(1)\,\dagger}\widetilde{\gamma}_{\mu\alpha}\partial_\alpha\psi_1^{(2)} + \partial_\nu\psi_2^{(2)\,\dagger}\gamma_{\mu\alpha}\partial_\alpha\psi_1^{(1)} + \mathcal{L}^{(2)}\delta_{\mu\nu} \quad ;\end{aligned} \tag{61}$$

$$\begin{aligned}\mathcal{H}^{(2)} = \int\Big[&-\partial_4\psi_1^{(1)\,\dagger}\gamma_{44}\partial_4\psi_2^{(2)} + \partial_i\psi_1^{(1)\,\dagger}\gamma_{ij}\partial_j\psi_2^{(2)} - \partial_4\psi_1^{(2)\,\dagger}\gamma_{44}\partial_4\psi_2^{(1)} + \partial_i\psi_1^{(2)\,\dagger}\gamma_{ij}\partial_j\psi_2^{(1)} - \\ &- \partial_4\psi_2^{(1)\,\dagger}\gamma_{44}\partial_4\psi_1^{(2)} + \partial_i\psi_2^{(1)\,\dagger}\gamma_{ij}\partial_j\psi_1^{(2)} - \partial_4\psi_2^{(2)\,\dagger}\gamma_{44}\partial_4\psi_1^{(1)} + \partial_i\psi_2^{(2)\,\dagger}\gamma_{ij}\partial_j\psi_1^{(1)} + \\ &+ m^2\psi_1^{(1)\,\dagger}\psi_2^{(2)} - m^2\psi_1^{(2)\,\dagger}\psi_2^{(1)} - m^2\psi_2^{(1)\,\dagger}\psi_1^{(2)} + m^2\psi_2^{(2)\,\dagger}\psi_1^{(1)}\Big]\,d^3x \quad .\end{aligned} \tag{62}$$

The charge operator and the spin tensor are

$$\begin{aligned}\mathcal{J}_\mu^{(2)} = i\Big[&\partial_\alpha\psi_1^{(1)\,\dagger}\widetilde{\gamma}_{\alpha\mu}\psi_2^{(2)} + \partial_\alpha\psi_1^{(2)\,\dagger}\gamma_{\alpha\mu}\psi_2^{(1)} + \partial_\alpha\psi_2^{(1)\,\dagger}\widetilde{\gamma}_{\alpha\mu}\psi_1^{(2)} + \partial_\alpha\psi_2^{(2)\,\dagger}\gamma_{\alpha\mu}\psi_1^{(1)} - \\ &- \psi_1^{(1)\,\dagger}\widetilde{\gamma}_{\mu\alpha}\partial_\alpha\psi_2^{(2)} - \psi_1^{(2)\,\dagger}\gamma_{\mu\alpha}\partial_\alpha\psi_2^{(1)} - \psi_2^{(1)\,\dagger}\widetilde{\gamma}_{\mu\alpha}\partial_\alpha\psi_1^{(2)} - \psi_2^{(2)\,\dagger}\gamma_{\mu\alpha}\partial_\alpha\psi_1^{(1)}\Big] \quad ;\end{aligned} \tag{63}$$

$$\begin{aligned}S_{\mu\nu,\lambda}^{(2)} = i\Big[&\partial_\alpha\psi_1^{(1)\,\dagger}\widetilde{\gamma}_{\alpha\lambda}N_{\mu\nu}^{\psi_2^{(2)}}\psi_2^{(2)} + \partial_\alpha\psi_1^{(2)\,\dagger}\gamma_{\alpha\lambda}N_{\mu\nu}^{\psi_2^{(1)}}\psi_2^{(1)} + \\ &+\partial_\alpha\psi_2^{(1)\,\dagger}\widetilde{\gamma}_{\alpha\lambda}N_{\mu\nu}^{\psi_1^{(2)}}\psi_1^{(2)} + \partial_\alpha\psi_2^{(2)\,\dagger}\gamma_{\alpha\lambda}N_{\mu\nu}^{\psi_1^{(1)}}\psi_1^{(1)} + \\ &+ \psi_1^{(1)\,\dagger}N_{\mu\nu}^{\psi_1^{(1)\,\dagger}}\widetilde{\gamma}_{\lambda\alpha}\partial_\alpha\psi_2^{(2)} + \psi_1^{(2)\,\dagger}N_{\mu\nu}^{\psi_1^{(2)\,\dagger}}\gamma_{\lambda\alpha}\partial_\alpha\psi_2^{(1)} + \psi_2^{(1)\,\dagger}N_{\mu\nu}^{\psi_2^{(1)\,\dagger}}\widetilde{\gamma}_{\lambda\alpha}\partial_\alpha\psi_1^{(2)} + \psi_2^{(2)\,\dagger}N_{\mu\nu}^{\psi_2^{(2)\,\dagger}}\gamma_{\lambda\alpha}\partial_\alpha\psi_1^{(1)}\Big] \quad .\end{aligned} \tag{64}$$

Questions of the translational invariance, the choice of bispinors answering the physical states, the renormalizability of the theory based on the $\mathcal{L}^{(2)}$, the possibility of existence of the chiral charge for this system (like for the Majorana states in the $(1/2,0)\oplus(0,1/2)$ representation, what has been shown in the previous papers of the author) are required detailed elaboration in a separate paper.

$$\mathcal{T}^{(1)}_{\mu\nu} = \partial_\alpha \overline{\psi}_1 \gamma_{\alpha\mu} \partial_\nu \psi_1 + \partial_\nu \overline{\psi}_1 \gamma_{\mu\alpha} \partial_\alpha \psi_1 + \\ + \partial_\alpha \overline{\psi}_2 \gamma_{\alpha\mu} \partial_\nu \psi_2 + \partial_\nu \overline{\psi}_2 \gamma_{\mu\alpha} \partial_\alpha \psi_2 + \mathcal{L}^{(1)} \delta_{\mu\nu} \quad . \tag{65}$$

As a result the Hamiltonian is[16]

$$\mathcal{H}^{(1)} = \int \Big[-\partial_4 \overline{\psi}_2 \gamma_{44} \partial_4 \psi_2 + \partial_i \overline{\psi}_2 \gamma_{ij} \partial_j \psi_2 - \\ - \partial_4 \overline{\psi}_1 \gamma_{44} \partial_4 \psi_1 + \partial_i \overline{\psi}_1 \gamma_{ij} \partial_j \psi_1 + m^2 \overline{\psi}_1 \psi_1 - m^2 \overline{\psi}_2 \psi_2 \Big] \, d^3x \quad . \tag{66}$$

The quantized Hamiltonian

$$\mathcal{H}^{(1)} = \sum_\sigma \int \frac{d^3p}{(2\pi)^3} E_p \, \left[a^\dagger_\sigma(\vec p) a_\sigma(\vec p) + b_\sigma(\vec p) b^\dagger_\sigma(\vec p) + c^\dagger_\sigma(\vec p) c_\sigma(\vec p) + d_\sigma(\vec p) d^\dagger_\sigma(\vec p) \right] \quad , \tag{67}$$

is obtained after using the plane-wave expansion following the procedure of, *e.g.*, refs. [41,42]. Acknowledging the suggestion of one critical collegue I regard the matters of translational invariance and positive-definiteness of the energy in the theory based on the $\mathcal{L}^{(1)}$ in more detail. I proceed step by step to the fermionic consideration of ref. [41, p.145].[17] The condition of the translational invariance imposes the constraints:

$$\psi_1(x+a) = e^{-iP_\mu a_\mu} \psi_1(x) e^{iP_\mu a_\mu} \quad , \quad \psi_2(x+a) = e^{-iP_\mu a_\mu} \psi_2(x) e^{iP_\mu a_\mu} \quad , \tag{68}$$

or in the differential form

$$\partial_\mu \psi_1(x) = -i \left[P_\mu, \psi_1(x) \right]_- \quad , \quad \partial_\mu \overline{\psi}_1(x) = -i \left[P_\mu, \overline{\psi}_1(x) \right]_- \quad , \tag{69}$$

$$\partial_\mu \psi_2(x) = -i \left[P_\mu, \psi_2(x) \right]_- \quad , \quad \partial_\mu \overline{\psi}_2(x) = -i \left[P_\mu, \overline{\psi}_2(x) \right]_- \quad . \tag{70}$$

These constraints are satisfied provided that

$$\left[P_\mu, a_\sigma(\vec p) \right]_- = -p_\mu a_\sigma(\vec p) \quad , \quad \left[P_\mu, b_\sigma(\vec p) \right]_- = -p_\mu b_\sigma(\vec p) \quad , \tag{71}$$

$$\left[P_\mu, a^\dagger_\sigma(\vec p) \right]_- = +p_\mu a^\dagger_\sigma(\vec p) \quad , \quad \left[P_\mu, b^\dagger_\sigma(\vec p) \right]_- = +p_\mu b^\dagger_\sigma(\vec p) \quad . \tag{72}$$

Analogous relations exist for operators $c_\sigma(\vec p)$ and $d_\sigma(\vec p)$. Replacing P_μ by its expansion, this is equivalent to

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[16]The Hamiltonian can also be obtained from the second-order Lagrangian presented in [13b,Eq.(18)] by means of the procedure developed by M. V. Ostrogradsky [49] long ago (see also the Weinberg's remark on the page B1325 of the first paper [2]). The Ostrogradsky's procedure seems not to have been applied in [13] to obtain conjugate momentum operators.

[17]In order not to darken the essence of the question I assume that transitions $\psi_1 \leftrightarrow \psi_2$ and transitions between states of different signs of energy (like in [41]) are irrelevant at the moment. Otherwise, the only correction should be taken into account where necessary, namely, the commutators (77,78) should be generalized, see ref. [30].

$$a^\dagger_\sigma(\vec{k})\left[a_\sigma(\vec{k}), a_{\sigma'}(\vec{p})\right]_- + \left[a^\dagger_\sigma(\vec{k}), a_{\sigma'}(\vec{p})\right]_- a_\sigma(\vec{k}) = -(2\pi)^3\delta^{(3)}(\vec{p}-\vec{k})a_{\sigma'}(\vec{p}) \tag{73}$$

$$b_\sigma(\vec{k})\left[b^\dagger_\sigma(\vec{k}), b_{\sigma'}(\vec{p})\right]_- + \left[b_\sigma(\vec{k}), b_{\sigma'}(\vec{p})\right]_- b^\dagger_\sigma(\vec{k}) = -(2\pi)^3\delta^{(3)}(\vec{p}-\vec{k})b_{\sigma'}(\vec{p}) \tag{74}$$

$$a^\dagger_\sigma(\vec{k})\left[a_\sigma(\vec{k}), a^\dagger_{\sigma'}(\vec{p})\right]_- + \left[a^\dagger_\sigma(\vec{k}), a^\dagger_{\sigma'}(\vec{p})\right]_- a_\sigma(\vec{k}) = (2\pi)^3\delta^{(3)}(\vec{p}-\vec{k})a^\dagger_{\sigma'}(\vec{p}) \tag{75}$$

$$b_\sigma(\vec{k})\left[b^\dagger_\sigma(\vec{k}), b^\dagger_{\sigma'}(\vec{p})\right]_- + \left[b_\sigma(\vec{k}), b^\dagger_{\sigma'}(\vec{p})\right]_- b^\dagger_\sigma(\vec{k}) = (2\pi)^3\delta^{(3)}(\vec{p}-\vec{k})b_{\sigma'}(\vec{p}) \tag{76}$$

We can list very similar formulas for the states defined by the field function $\psi_2(x)$. Therefore, we deduce the commutation relations

$$\left[a_\sigma(\vec{p}), a^\dagger_{\sigma'}(\vec{k})\right]_- = \left[c_\sigma(\vec{p}), c^\dagger_{\sigma'}(\vec{k})\right]_- = (2\pi)^3\delta_{\sigma\sigma'}\delta(\vec{p}-\vec{k}) \quad , \tag{77}$$

$$\left[b_\sigma(\vec{p}), b^\dagger_{\sigma'}(\vec{k})\right]_- = \left[d_\sigma(\vec{p}), d^\dagger_{\sigma'}(\vec{k})\right]_- = (2\pi)^3\delta_{\sigma\sigma'}\delta(\vec{p}-\vec{k}) \quad . \tag{78}$$

It is easy to see that the Hamiltonian is positive-definite and the translational invariance still keeps in the framework of this description (*cf.* with ref. [13]). Please pay attention here: *I did never apply the indefinite metric*, which is regarded to be a rather obscure concept.

Analogously, from the definitions

$$\mathcal{J}_\mu = -i\sum_i\left\{\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\phi_i - \overline{\phi}_i\frac{\partial\mathcal{L}}{\partial(\partial_\mu\overline{\phi}_i)}\right\} \quad , \tag{79}$$

$$Q = -i\int\mathcal{J}_4(x)d^3x \quad , \tag{80}$$

and

$$\begin{aligned}\mathcal{M}_{\mu\nu,\lambda} &= x_\mu\mathcal{T}_{\lambda\nu} - x_\nu\mathcal{T}_{\lambda\mu} - \\ &- i\sum_i\left\{\frac{\partial\mathcal{L}}{\partial(\partial_\lambda\phi_i)}N^{\phi_i}_{\mu\nu}\phi_i + \overline{\phi}_i N^{\overline{\phi}_i}_{\mu\nu}\frac{\partial\mathcal{L}}{\partial(\partial_\lambda\overline{\phi}_i)}\right\} \quad ,\end{aligned} \tag{81}$$

$$M_{\mu\nu} = -i\int\mathcal{M}_{\mu\nu,4}(x)d^3x \quad , \tag{82}$$

one can find the current operator

$$\begin{aligned}\mathcal{J}^{(1)}_\mu &= i\left[\partial_\alpha\overline{\psi}_1\gamma_{\alpha\mu}\psi_1 - \overline{\psi}_1\gamma_{\mu\alpha}\partial_\alpha\psi_1 + \right. \\ &\left. + \partial_\alpha\overline{\psi}_2\gamma_{\alpha\mu}\psi_2 - \overline{\psi}_2\gamma_{\mu\alpha}\partial_\alpha\psi_2\right] \quad ,\end{aligned} \tag{83}$$

and using (81) the spin momentum tensor

$$\begin{aligned}S^{(1)}_{\mu\nu,\lambda} &= i\left[\partial_\alpha\overline{\psi}_1\gamma_{\alpha\lambda}N^{\psi_1}_{\mu\nu}\psi_1 + \overline{\psi}_1 N^{\overline{\psi}_1}_{\mu\nu}\gamma_{\lambda\alpha}\partial_\alpha\psi_1 + \right. \\ &\left. + \partial_\alpha\overline{\psi}_2\gamma_{\alpha\lambda}N^{\psi_2}_{\mu\nu}\psi_2 + \overline{\psi}_2 N^{\overline{\psi}_2}_{\mu\nu}\gamma_{\lambda\alpha}\partial_\alpha\psi_2\right] \quad .\end{aligned} \tag{84}$$

If the Lorentz group generators (a $j=1$ case) are defined from

$$\overline{\Lambda}\gamma_{\mu\nu}\Lambda a_{\mu\alpha}a_{\nu\beta} = \gamma_{\alpha\beta} \quad , \tag{85}$$

$$\overline{\Lambda}\Lambda = 1 \quad , \tag{86}$$

$$\overline{\Lambda} = \gamma_{44}\Lambda^\dagger\gamma_{44} \quad . \tag{87}$$

then in order to keep the Lorentz covariance of the Weinberg equations and of the Lagrangian (45) one should use the following generators:

$$N_{\mu\nu}^{\psi_1,\psi_2(j=1)} = -N_{\mu\nu}^{\overline{\psi}_1,\overline{\psi}_2(j=1)} = \frac{1}{6}\gamma_{5,\mu\nu} \quad , \tag{88}$$

The matrix $\gamma_{5,\mu\nu} = i\left[\gamma_{\mu\lambda},\gamma_{\nu\lambda}\right]_-$ is defined to be Hermitian. Let me note that the matters of the choice of generators for Lorentz transformations have also been regarded in [16]. Due to the fact that the set of the Weinberg states is a degenerate set one can also consider the situation when a Weinberg equation (*e.g.*, Eq. (21)) transfers over another one (*e.g.*, Eq. (24)). This case corresponds to the possibility of combining pure Lorentz transformations with transformations of the inversion group; the corresponding rules are different from (85)-(87) .

The quantized charge operator and the quantized spin operator follow immediately from (83) and (84):

$$Q^{(1)} = \sum_\sigma \int \frac{d^3p}{(2\pi)^3}\left[a_\sigma^\dagger(\vec p)a_\sigma(\vec p) - b_\sigma(\vec p)b_\sigma^\dagger(\vec p) + c_\sigma^\dagger(\vec p)c_\sigma(\vec p) - d_\sigma(\vec p)d_\sigma^\dagger(\vec p)\right] \quad , \tag{89}$$

$$\begin{aligned}(W^{(1)}\cdot n)/m = \sum_{\sigma\sigma'}\int \frac{d^3p}{(2\pi)^3}\frac{1}{m^2E_p}\overline{u}_1^\sigma(\vec p)(E_p\gamma_{44} - i\gamma_{4i}p_i)\; I\otimes(\vec J\vec n)\, u_1^{\sigma'}(\vec p)\times \\ \times\left[a_\sigma^\dagger(\vec p)a_{\sigma'}(\vec p) + c_\sigma^\dagger(\vec p)c_{\sigma'}(\vec p) - b_\sigma(\vec p)b_{\sigma'}^\dagger(\vec p) - d_\sigma(\vec p)d_{\sigma'}^\dagger(\vec p)\right]\end{aligned} \tag{90}$$

(provided that the frame is chosen in such a way that $\vec n \parallel \vec p$ is along the third axis). It is easy to verify the eigenvalues of the charge operator are ± 1, and of the Pauli-Lyuban'sky spin operator are

$$\xi_\sigma^*(\vec J\vec n)\xi_{\sigma'} = +1,\, 0\, -1 \tag{91}$$

in a massive case and ± 1 in a massless case.[18] Now we can answer the question: why "a queer reduction of degrees of freedom" did happen in the previous papers [22,23,25]? The origin of this surprising fact follows from the Hayashi (1973) paper, ref. [22, p.498]: The requirement of *"that the physical realizable state satisfies a quantal version of the generalized Lorentz condition"*, formulas (18) of ref. [22],[19] permits one to eliminate upper (or down)

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[18]See the discussion of the massless limit of the Weinberg bispinors in ref. [35,51]. While in a massless limit $W_\mu n_\mu = 0$ this does not signify that W_μ would be always equal to zero; in this case we already cannot define a normalized space-like vector n_μ whose space part is parallel to the vector $\vec p$. It becomes light-like.

[19]Read: "a quantal version" of the Maxwell equations imposed on the state vectors in the Fock space. Applying them leads to the case when Eq. (90) is equal to zero *identically.* Nonetheless, such a procedure should be taken cautiously, see, *e.g.*, ref. [13, Table 2], for the discussion of the acausal physical dispersion of the equations (4.19) and (4.20) of ref. [2b], "which *are just Maxwell's free-space equations for left- and right- circularly polarized radiation.*" See also the footnote # 1 in ref. [28c]. Let me mention, the fact of existence of 'acausal' solutions is probably connected with the indefinite metric problem, with the appearence of the ghost states in the gauge models and with the concept of 'action-at-a-distance', ref. [67].

part of the Weinberg "bispinor" and to remove transversal components of the remained part by means of the "gauge" transformation (44), what *"ensures the massless skew-symmetric field is longitudinal"*. The reader can convince himself in this "obvious fact" by looking at the explicit form of the Pauli-Lyuban'sky operator, Eq. (90). Taking into account both positive- and negative- energy solutions (*cf.* with [25]) in the Lagrangian (40) and *not* applying the generalized Lorentz condition (*cf.* with [22,23]) we are able to account for both transversal and longitudinal components, *i.e.*, to describe a $j = 1$ particle. Furthermore, one can say even simpler: the application of the generalized Lorentz condition may be successful to the non-zero energy states of helicities ± 1,[20] so in earlier works, as a matter of fact, the authors implied the existence of such states. On the other hand, longitudinal components of the Weinberg fields are directly linked with the mass of a $j = 1$ particle, see [51] and, possibly, with the concept of the $\mathbf{B}^{(3)}$ Evans-Vigier field [39]. This fact can provide deeper understanding of relations between Casimir invariants of a particle field and space-time structures. The presented wisdom does *not* contradict with neither the Weinberg theorem nor the classical limit, Eqs. (27,28) of the previous Section. Thanks to the mapping between the antisymmetric tensor and Weinberg formulations the conclusion is valid for both the Weinberg $2(2j+1)$ component "bispinor" and the antisymmetric (skew-symmetric) tensor field. Thus, we have now proven that a photon (a $j = 1$ massless particle) can possess spin degrees of freedom, what is in accordance with experiment. The contradictory claims of several collegues about the pure "longitudinal nature" of quantized antisymmetric fields, which they have been making since the sixties and which are repeating until the present, are incredible and unreasonable. We can suggest an analogy considering the modified electrodynamics recently proposed by Evans and Vigier. In fact, the authors of the earlier "longitudinal" papers "align themselves" with the concept of the $\mathbf{B}^{(3)}$ field (named it as the Kalb-Ramond field), but, surprisingly, they reject transversal modes (after quantization)!? By the way, it is obviously from the consideration of the similar construct in the $(1/2,0)\oplus(0,1/2)$ representation that on an equal footing those authors could claim that a $j = 1/2$ massless neutrino field would be pure longitudinal too... Simply speaking, such claims are absurdity...

Finally, for the sake of completeness let me re-write Lagrangians presented above into the 12-component form:

$$\mathcal{L}^{(1)} = -\partial_\mu \overline{\Psi} \Gamma_{\mu\nu} \partial_\nu \Psi - m^2 \overline{\Psi}\Psi \quad , \tag{92}$$

where

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad , \quad \overline{\Psi} = \begin{pmatrix} \psi_1^\dagger & \psi_2^\dagger \end{pmatrix} \cdot \begin{pmatrix} \gamma_{44} & 0 \\ 0 & -\gamma_{44} \end{pmatrix} \tag{93}$$

are the doublet wave functions,

$$\Gamma_{\mu\nu} = \begin{pmatrix} \gamma_{\mu\nu} & 0 \\ 0 & -\gamma_{\mu\nu} \end{pmatrix} \quad , \quad \Gamma^5 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \quad , \quad \Gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \quad . \tag{94}$$

The Lagrangian $\mathcal{L}^{(2)}$ can be written in a similar fashion:

$$\begin{aligned} \mathcal{L}^{(2)} = &-\partial_\mu^\dagger \Psi^{(1)\,\dagger} \Gamma_{\mu\nu} \Gamma^5 \Gamma^0 \partial_\nu^\dagger \Psi^{(2)} - \partial_\mu \Psi^{(2)\,\dagger} \Gamma_{\mu\nu} \Gamma^5 \Gamma^0 \partial_\nu \Psi^{(1)} - \\ &- m^2 \Psi^{(1)\,\dagger} \Gamma^5 \Gamma^0 \Psi^{(2)} + m^2 \Psi^{(2)\,\dagger} \Gamma^5 \Gamma^0 \Psi^{(1)} \quad . \end{aligned} \tag{95}$$

[20]If the energy is equal to zero,in my opinion, there is no any sense to speak about helicity at all.

One can conclude this Section: the generalized Lorentz condition can be incompatible with the specific properties of the antisymmetric tensor field deduced from the ordinary approach of the classical physics. I mean that its application can lead (and did lead in the earlier papers) to the loss of information about either transversal or longitudinal modes of the antisymmetric tensor field. The connection of the presented model with the Bargmann-Wightman-Wigner-type quantum field theories deserves further elaboration. As a matter of fact the presented model develops Weinberg and Ahluwalia ideas of the Dirac-like description of bosons on an equal footing with fermions, *i.e.*, on the ground of the $(j,0)\oplus(0,j)$ representation of the Lorentz group.

IV. WEINBERG PROPAGATORS

Accordingly to the Feynman-Dyson-Stueckelberg ideas, a causal propagator has to be constructed by using the formula (e. g., ref. [41, p.91])

$$S_F(x_2,x_1)=\int\frac{d^3k}{(2\pi)^3}\frac{m}{E_k}\Big[\theta(t_2-t_1)\,a\;u^\sigma(k)\otimes\overline{u}^\sigma(k)e^{-ikx}+ \\ +\,\theta(t_1-t_2)\,b\;v^\sigma(k)\otimes\overline{v}^\sigma(k)e^{ikx}\Big]\quad, \tag{96}$$

$x=x_2-x_1$. In the $j=1/2$ Dirac theory it results to

$$S_F(x)=\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\frac{\hat{k}+m}{k^2-m^2+i\epsilon}\quad, \tag{97}$$

provided that the constant a and b are determined by imposing

$$(i\hat{\partial}_2-m)S_F(x_2,x_1)=\delta^{(4)}(x_2-x_1)\quad, \tag{98}$$

namely, $a=-b=1/i$.

However, in the framework of the Weinberg theory, ref. [2], which is a generalization of the Dirac ideas to higher spins, the attempts of constructing a covariant propagator in such a way have been fallen. For example, on the page B1324 of ref. [2a] Weinberg writes: "Unfortunately, the propagator arising from Wick's theorem is *not* equal to the covariant propagator except for $j=0$ and $j=1/2$. The trouble is that the derivatives act on the $\epsilon(x)=\theta(x)-\theta(-x)$ in $\Delta^C(x)$ as well as on the functions[21] Δ and Δ_1. This gives rise to extra terms proportional to equal-time δ functions and their derivatives... The cure is well known: ...compute the vertex factors using only the original covariant part of [the Hamiltonian] $\mathcal{H}(x)$; do not use [the Wick propagator] for internal lines; instead use the covariant propagator, [the formula (5.8) in ref. [2a]]." The propagator, recently proposed in ref. [35c,d] (see also other papers of the same author), is the causal propagator. "Only the physically acceptable causal solutions of the Weinberg equations enter these propagators." However, it does not satisfy us down to the ground since the old problem remains: the Feynman-Dyson propagator is not the Green's function of the Weinberg equation. The covariant propagator presented in [5], while a Green's function of the $(1,0)\oplus(0,1)$ equation, would propagate kinematically spurious solutions [35c]... The aim of the following work is

[21]In the cited paper the following notation has been used: $\Delta_1(x)\equiv i\left[\Delta_+(x)+\Delta_+(-x)\right]$, $\Delta(x)\equiv\Delta_+(x)-\Delta_+(-x)$ and $i\Delta_+(x)\equiv\frac{1}{(2\pi)^3}\int\frac{d^3p}{2E_p}\exp(ipx)$.

to consider the problem of constructing propagators in the framework of the model proposed in the previous Sections.

The set of four equations has been proposed in Section II. We consider the most general case. Let us check, if the sum of four equations ($x = x_2 - x_1$)

$$\left[\gamma_{\mu\nu}\partial_\mu\partial_\nu - m^2\right]\int \frac{d^3p}{(2\pi)^3 2E_p}\Big[\theta(t_2-t_1)\, a\ \mathcal{U}_1^{\sigma\,(1)}(\vec p)\otimes\overline{\mathcal{U}}_1^{\sigma\,(1)}(\vec p)e^{ipx}+$$
$$+\theta(t_1-t_2)\, b\ \mathcal{V}_1^{\sigma\,(1)}(\vec p)\otimes\overline{\mathcal{V}}_1^{\sigma\,(1)}(\vec p)e^{-ipx}\Big]+$$
$$+\left[\gamma_{\mu\nu}\partial_\mu\partial_\nu + m^2\right]\int \frac{d^3p}{(2\pi)^3 2E_p}\Big[\theta(t_2-t_1)\, c\ \mathcal{U}_2^{\sigma\,(1)}(\vec p)\otimes\overline{\mathcal{U}}_2^{\sigma\,(1)}(\vec p)e^{ipx}+$$
$$+\theta(t_1-t_2)\, d\ \mathcal{V}_2^{\sigma\,(1)}(\vec p)\otimes\overline{\mathcal{V}}_2^{\sigma\,(1)}(\vec p)e^{-ipx}\Big]+$$
$$+\left[\widetilde{\gamma}_{\mu\nu}\partial_\mu\partial_\nu + m^2\right]\int \frac{d^3p}{(2\pi)^3 2E_p}\Big[\theta(t_2-t_1)\, e\ \mathcal{U}_1^{\sigma\,(2)}(\vec p)\otimes\overline{\mathcal{U}}_1^{\sigma\,(2)}(\vec p)e^{ipx}+$$
$$+\theta(t_1-t_2)\, f\ \mathcal{V}_1^{\sigma\,(2)}(\vec p)\otimes\overline{\mathcal{V}}_1^{\sigma\,(2)}(\vec p)e^{-ipx}\Big]+$$
$$+\left[\widetilde{\gamma}_{\mu\nu}\partial_\mu\partial_\nu - m^2\right]\int \frac{d^3p}{(2\pi)^3 2E_p}\Big[\theta(t_2-t_1)\, g\ \mathcal{U}_2^{\sigma\,(2)}(\vec p)\otimes\overline{\mathcal{U}}_2^{\sigma\,(2)}(\vec p)e^{ipx}+$$
$$+\theta(t_1-t_2)\, h\ \mathcal{V}_2^{\sigma\,(2)}(\vec p)\otimes\overline{\mathcal{V}}_2^{\sigma\,(2)}(\vec p)e^{-ipx}\Big] = \delta^{(4)}(x_2-x_1) \qquad (99)$$

can be satisfied by the definite choice of the constant a, b *etc.* In the process of calculations I assume that the set of the analogs of the "Pauli spinors" in the $(1,0)$ or $(0,1)$ spaces is the complete set and it is normalized to $\delta_{\sigma\sigma'}$.

The simple calculations yield

$$\begin{aligned}&\partial_\mu^{x_2}\partial_\nu^{x_2}\left[a\,\theta(t_2-t_1)\,e^{ip(x_2-x_1)} + b\,\theta(t_1-t_2)\,e^{-ip(x_2-x_1)}\right] =\\
&= -\left[a\,p_\mu p_\nu \theta(t_2-t_1)\exp\left[ip(x_2-x_1)\right] + b\,p_\mu p_\nu\theta(t_1-t_2)\exp\left[-ip(x_2-x_1)\right]\right]+\\
&+ a\left[-\delta_{\mu4}\delta_{\nu4}\delta\,'(t_2-t_1) + i(p_\mu\delta_{\nu4}+p_\nu\delta_{\mu4})\delta(t_2-t_1)\right]\exp\left[i\vec p(\vec x_2-\vec x_1)\right]+\\
&+ b\,\left[\delta_{\mu4}\delta_{\nu4}\delta\,'(t_2-t_1) + i(p_\mu\delta_{\nu4}+p_\nu\delta_{\mu4})\delta(t_2-t_1)\right]\exp\left[-i\vec p(\vec x_2-\vec x_1)\right] \quad ;\end{aligned} \qquad (100)$$

and

$$\mathcal{U}_1^{(1)}\overline{\mathcal{U}}_1^{(1)} = \frac{1}{2}\begin{pmatrix} m^2 \mathbb{1} & S_p\otimes S_p\\ \overline{S}_p\otimes\overline{S}_p & m^2\mathbb{1}\end{pmatrix} \quad , \quad \mathcal{U}_2^{(1)}\overline{\mathcal{U}}_2^{(1)} = \frac{1}{2}\begin{pmatrix} -m^2 \mathbb{1} & S_p\otimes S_p\\ \overline{S}_p\otimes\overline{S}_p & -m^2\mathbb{1}\end{pmatrix} \quad , \qquad (101)$$

$$\mathcal{U}_1^{(2)}\overline{\mathcal{U}}_1^{(2)} = \frac{1}{2}\begin{pmatrix} -m^2 \mathbb{1} & \overline{S}_p\otimes \overline{S}_p\\ S_p\otimes S_p & -m^2\mathbb{1}\end{pmatrix} \quad , \quad \mathcal{U}_2^{(2)}\overline{\mathcal{U}}_2^{(2)} = \frac{1}{2}\begin{pmatrix} m^2 \mathbb{1} & \overline{S}_p\otimes \overline{S}_p\\ S_p\otimes S_p & m^2\mathbb{1}\end{pmatrix} \quad , \qquad (102)$$

where

$$S_p = m + (\vec J\vec p) + \frac{(\vec J\vec p)^2}{E+m} \quad , \qquad (103)$$

$$\overline{S}_p = m - (\vec J\vec p) + \frac{(\vec J\vec p)^2}{E+m} \quad . \qquad (104)$$

Due to the fact that

$$\left[E - (\vec{J}\vec{p})\right] S_p \otimes S_p = m^2 \left[E + (\vec{J}\vec{p})\right] \quad , \tag{105}$$
$$\left[E + (\vec{J}\vec{p})\right] \overline{S}_p \otimes \overline{S}_p = m^2 \left[E - (\vec{J}\vec{p})\right] \tag{106}$$

after simplifying the left side of (99) and comparing it with the right side we find: the causal propagator is admitted by using the "Wick's formula" for the time-ordered particle operators provided that the constants are equal to $1/4im^2$. It is necessary to consider all four equations, Eqs. (21,22,24,25).

The $j = 1$ analogs of the formula (97) for the Weinberg propagators follows from the formula (3.6) of ref. [35d] immediately:

$$S_F^{(1)}(p) = -\frac{1}{i(2\pi)^4(p^2 + m^2 - i\epsilon)} \left[\gamma_{\mu\nu} p_\mu p_\nu - m^2\right] \quad , \tag{107}$$

$$S_F^{(2)}(p) = -\frac{1}{i(2\pi)^4(p^2 + m^2 - i\epsilon)} \left[\gamma_{\mu\nu} p_\mu p_\nu + m^2\right] \quad , \tag{108}$$

$$S_F^{(3)}(p) = -\frac{1}{i(2\pi)^4(p^2 + m^2 - i\epsilon)} \left[\tilde{\gamma}_{\mu\nu} p_\mu p_\nu + m^2\right] \quad , \tag{109}$$

$$S_F^{(4)}(p) = -\frac{1}{i(2\pi)^4(p^2 + m^2 - i\epsilon)} \left[\tilde{\gamma}_{\mu\nu} p_\mu p_\nu - m^2\right] \quad . \tag{110}$$

The conclusions are: one can construct an analog of the Feynman-Dyson propagator for the $2(2j+1)$ model and, hence, a "local" theory provided that the Weinberg states are "quadrupled" in the $j = 1$ case. They cannot propagate separately each other (*cf.* with the Dirac $j = 1/2$ case).

V. MASSLESS LIMIT: CAN THE 6-COMPONENT WEINBERG-TUCKER-HAMMER EQUATIONS DESCRIBE THE ELECTROMAGNETIC FIELD?

In previous Sections the equivalence of the Weinberg-Tucker-Hammer approach and the Proca approach for describing $j = 1$ states has been found. The $2(2j+1)$ component wave functions are given by Eq. (39) and by the formulas obtained after applying inversion group operations to (39). The aim of the present Section is to consider the question, under which conditions the Weinberg-Tucker-Hammer $j = 1$ equations can be transformed to Eqs. (4.21) and (4.22) of ref. [2b] :

$$\nabla \times \left[\vec{E} - i\vec{B}\right] + i(\partial/\partial t) \left[\vec{E} - i\vec{B}\right] = 0 \quad , \tag{4.21}$$
$$\nabla \times \left[\vec{E} + i\vec{B}\right] - i(\partial/\partial t) \left[\vec{E} + i\vec{B}\right] = 0 \quad . \tag{4.22}$$

By using the bivector interpretation of ψ (in the chiral representation) and the explicit forms of the Barut-Muzinich-Williams matrices, We are able to recast the $j = 1$ Tucker-Hammer equation (13) which is free of tachyonic solutions, or the Proca equation, Eq. (8) of the Section II, to the form

$$m^2 E_i = -\frac{\partial^2 E_i}{\partial t^2} + \epsilon_{ijk}\frac{\partial}{\partial x_j}\frac{\partial B_k}{\partial t} + \frac{\partial}{\partial x_i}\frac{\partial E_j}{\partial x_j} \quad , \tag{111}$$

$$m^2 B_i = \epsilon_{ijk}\frac{\partial}{\partial x_j}\frac{\partial E_k}{\partial t} + \frac{\partial^2 B_i}{\partial x_j^2} - \frac{\partial}{\partial x_i}\frac{\partial B_j}{\partial x_j} \quad . \tag{112}$$

The Klein-Gordon equation (the D'Alembert equation in the massless limit)

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_i^2}\right) F_{\mu\nu} = -m^2 F_{\mu\nu} \tag{113}$$

is implied ($c = \hbar = 1$). Introducing vector operators we write equations in the following form:

$$\frac{\partial}{\partial t}\operatorname{curl}\vec{B} + \operatorname{grad}\operatorname{div}\vec{E} - \frac{\partial^2 \vec{E}}{\partial t^2} = m^2 \vec{E} \quad , \tag{114}$$

$$\nabla^2 \vec{B} - \operatorname{grad}\operatorname{div}\vec{B} + \frac{\partial}{\partial t}\operatorname{curl}\vec{E} = m^2 \vec{B} \quad . \tag{115}$$

Taking into account the definitions:

$$\rho_e = \operatorname{div}\vec{E} \quad , \quad \vec{J}_e = \operatorname{curl}\vec{B} - \frac{\partial \vec{E}}{\partial t} \quad , \tag{116}$$

$$\rho_m = \operatorname{div}\vec{B} \quad , \quad \vec{J}_m = -\frac{\partial \vec{B}}{\partial t} - \operatorname{curl}\vec{E} \quad , \tag{117}$$

relations of the vector algebra ($\vec{X}$ is an arbitrary vector):

$$\operatorname{curl}\operatorname{curl}\vec{X} = \operatorname{grad}\operatorname{div}\vec{X} - \nabla^2 \vec{X} \quad , \tag{118}$$

and the Klein-Gordon equation (113) we obtain two equivalent sets of equations, which complete the Maxwell's set. The first one is

$$\frac{\partial \vec{J}_e}{\partial t} + \operatorname{grad}\rho_e = m^2 \vec{E} \quad , \tag{119}$$

$$\frac{\partial \vec{J}_m}{\partial t} + \operatorname{grad}\rho_m = 0 \quad ; \tag{120}$$

and the second one is

$$\operatorname{curl}\vec{J}_m = 0 \tag{121}$$

$$\operatorname{curl}\vec{J}_e = -m^2 \vec{B} \quad . \tag{122}$$

One can obtain the equations in different unit systems after one recalls, *e.g.*, relations of the Appendix of ref. [52]. I would also like to remind that the Weinberg set of equations (and, hence, the equations (119-122)[22]) can be obtained on the basis of a very few number

[22] Beginning with the dual massive equations and setting $\vec{C} \equiv \vec{E}$, $\vec{D} \equiv \vec{B}$ we could obtain

of postulates; in fact, by using the Lorentz transformation rules for the Weinberg bivector (or for the antisymmetric tensor field) and the Ryder-Burgard relation [13,14,17–19].

In a massless case the situation is different. Firstly, the set of equations (117), with the left side are chosen to be zero, is "an identity satisfied by certain space-time derivatives of $F_{\mu\nu}\ldots$, namely, refs. [53–55].

$$\frac{\partial F_{\mu\nu}}{\partial x^\sigma}+\frac{\partial F_{\nu\sigma}}{\partial x^\mu}+\frac{\partial F_{\sigma\mu}}{\partial x^\nu}=0 \quad ." \tag{127}$$

I believe that the similar consideration for the dual field $\widetilde{F}_{\mu\nu}$ as in refs. [53,54] can reveal that the same is true for the first equations (116). So, in the massless case we come across the problem of interpretation of the charge and currents.

Secondly, in order to satisfy the massless equations (121,122) one should assume that the currents are represented in the gradient forms of some scalar fields $\chi_{e,m}$. What physical significance have these chi-functions? In the massless case the charge densities are (see equations (119,120))

$$\rho_e=-\frac{\partial\chi_e}{\partial t}+const \quad , \quad \rho_m=-\frac{\partial\chi_m}{\partial t}+const \quad , \tag{128}$$

what tells us that ρ_e and ρ_m are constants provided that the primary functions $\chi_{e,m}$ are linear functions in time (decreasing or increasing?). It is useful to compare the resulting equations for $\rho_{e,m}$ and $\vec{J}_{e,m}$ and the fact of appearence of the functions $\chi_{e,m}$ with the 5-potential formulation of electromagnetic theory [54], see also refs. [24,55–59]. I believe, this concept can also be useful for explanation of the $E=0$ solutions in higher-spin equations [60,61,35] which have been "baptized" by Moshinsky and Del Sol in [62] as 'relativistic cockroach nest'. Next, I would like to note the following. We can obtain the Maxwell's free-space equations, in the definite choice of the χ_e and χ_m, namely, in the case they are constants. In ref. [56] it was mentioned that solutions of Eqs. (4.21,4.22) of ref. [2b] satisfy the equations of the type (111,112), *"but not always vice versa"*. Interpretation of this statement and investigations of Eq. (13) with different initial and boundary conditions (or of the functions χ) deserve further elaboration (both theoretical and experimental).

The question also arises on the transformation of the field function (39) from one to another frame. I would like to draw your attention at the remarkable fact which follows from a consideration of the problem in the momentum representation. For the first sight, one could conclude that under a transfer from one to another frame one has to describe the field by the Lorentz transformed function $\psi'(\mathbf{p})=\Lambda(\mathbf{p})\psi(\mathbf{p})$. However, if take into account the possibility of combining the Lorentz, dual (chiral) and parity transformations

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$$\frac{\partial\vec{J}_e}{\partial t}+\mathrm{grad}\,\rho_e=0 \quad , \tag{123}$$

$$\frac{\partial\vec{J}_m}{\partial t}+\mathrm{grad}\,\rho_m=m^2\vec{B} \quad ; \tag{124}$$

and

$$\mathrm{curl}\,\vec{J}_e=0 \quad , \tag{125}$$

$$\mathrm{curl}\,\vec{J}_m=m^2\vec{E} \quad . \tag{126}$$

This would signify that the physical content spanned by massive dual fields would be different. The reader can easily reveal parity-conjugated equations from Eqs. (24,25).

in the case of higher spin equations[23] and that all the equations for the four functions (21), (22), (24) and (25) reduce to the equations for $\mathbf{E}$ and $\mathbf{B}$, which appear to be the same in a massless limit, one could come to a different situation. The four bispinors $\mathcal{U}_1^{\sigma\,(1)}(\mathbf{p})$, $\mathcal{U}_2^{\sigma\,(1)}(\mathbf{p})$, $\mathcal{U}_1^{\sigma\,(2)}(\mathbf{p})$ and $\mathcal{U}_2^{\sigma\,(2)}(\mathbf{p})$, see Eqs. (52), (53), (57) and (58), form a complete set (as well as the transformed ones $\Lambda(\mathbf{p})\mathcal{U}_i^{\sigma\,(k)}(\mathbf{p})$) for each value of σ.

Namely,

$$\begin{aligned} &a_1\mathcal{U}_1^{\sigma\,(1)}(\mathbf{p})\overline{\mathcal{U}}_1^{\sigma\,(1)}(\mathbf{p}) + a_2\mathcal{U}_2^{\sigma\,(1)}(\mathbf{p})\overline{\mathcal{U}}_2^{\sigma\,(1)}(\mathbf{p}) + \\ &+ a_3\mathcal{U}_1^{\sigma\,(2)}(\mathbf{p})\overline{\mathcal{U}}_1^{\sigma\,(2)}(\mathbf{p}) + a_4\mathcal{U}_2^{\sigma\,(2)}(\mathbf{p})\overline{\mathcal{U}}_2^{\sigma\,(2)}(\mathbf{p}) = 1\!\!1 \quad . \end{aligned} \tag{129}$$

Constants a_i are defined by the choice of the normalization of bispinors. In any other frame we are able to obtain the primary wave function by choosing appropriate coefficients c_i^k of the expansion of the wave function (in fact, using appropriate dual rotations and inversions)

$$\Psi(\vec{p}) = \sum_{i,k=1,2} c_i^k \mathcal{U}_i^{(k)}(\vec{p}) \quad . \tag{130}$$

The same statement should be valid for negative-energy solutions, since their explicit forms coincide with the ones of positive-energy bispinors in the case of the Hammer-Tucker formulation for a $j=1$ boson, ref. [5]. Using the plane-wave expansion one can prove this conclusion in the coordinate representation. Thus, the question of what we observe in the experiment would be solved depending on the fixing of the relative phase factor between left- and right- parts of the field function (between $\vec{E}$ and $\vec{B}$, indeed) by appropriate physical conditions we are interested.

At last, I have to note that the massless case reveals a very strange thing.[24] The massless equations (121,122) written in the integral form lead to a conclusion about $\oint \vec{J}_{e,m}\cdot d\vec{l} = 0$. This is obviously unacceptable from a viewpoint of experiment. Thus, we have to conclude that either the $j=1$ field cannot be massless or there exist hidden parameters which all field functions (and, probably, space-time characteristics) depend on.

Finally, let me mention that in the nonrelativistic limit $c\to\infty$ one obtains the dual Levi-Leblond's "Galilean Electrodynamics", refs. [63,64].

The main conclusion of the paper is:[25] The Weinberg-Tucker-Hammer massless equations (or the Proca equations for $F_{\mu\nu}$), see also (111) and (112), are equivalent to the Maxwell's equations in the definite choice of the initial and boundary conditions, what proves their consistency. Their massless limit were shown in ref. [35] to be free of kinematical acausalities as opposed to Eqs. (4.21) and (4.22) of ref. [2b]. The Weinberg-Tucker-Hammer approach permits us to clarify the question of the claimed 'longitudinal nature' of the antisymmetric tensor field. It is free of the problem of the indefinite metric in the Fock space. The $j=1$ bosons are considered in a very similar fashion as fermions in the Dirac approach. This provides a convenient mathematical formalism for discussing properties of the $j=1$ bosons with respect to discrete symmetries operations. Therefore, we have to agree with S. Weinberg who spoke out about the equations (4.21) and (4.22): *"The fact that these field equations are*

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[23]This possibility has been discovered earlier and investigated in [13].

[24]I am grateful to Dr. A. E. Chubykalo for pointing out this fact and for discussions.

[25]This conclusion also follows from the results of the paper [13,35,28–32,39,40] and ref. [2b] provided that the fact that $(\vec{J}\vec{p})$ has no the inverse matrix has been taken into account.

of first order for any spin seems to me to be of no great significance... ” [2b,p. B888]. In the meantime, I would not like to darken theories based on the use of the vector potentials, *i.e.*, of the $D(1/2,1/2)$ representation of the Lorentz group. While the description of the $j=1$ massless field using this representation contradicts with the Weinberg theorem $B-A=\lambda$, what signifies that we do not have well-defined creation and annihilation operators in the beginning of a quantization procedure, one cannot forget about significant achievements of these theories. The formalism proposed here could be helpful only if we shall necessitate to go beyond the framework of the Standard Model, *i.e.* if we shall come across the reliable experimental results which cannot have satisfactory explanation on the basis of the concept of a minimal coupling introduced in the conventional manner (see, *e.g.*, ref. [14] for a discussion of the neutrino model which forbids such a form of the interaction).

Many questions related with the problem of longitudinal modes of the $j=1$ field, their relations with tachyonic models (particularly, with the concept of the Action At a Distance and the Recami's Extended Relativity), with the problem of the interpretations of mass and spin, with the problem of gauge degrees of freedom as well remain in the field of our future researches.

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A considerable part of this paper has been written in August, 1994. The distribution of the ideas, presented above, in many simposiums and in private correspondence has provided with additional confirmations that indicated contradictions in the accustomed models presents and ideas presented here have scientific basis. Recently I learnt about the papers of other authors published in the recent years in many journals over the world. The papers of H. A. Múnera and O. Guzmán on the longitudinal solutions of relativistic wave equations, of A. E. Chubykalo and R. Smirnov-Rueda [67] on the ‘action-at-a-distance’ concept in the classical electrodynamics (*cf.* with the researches of F. J. Belinfante in QED, refs. [68]), the papers of E. Recami, W. Rodrigues and J. Vaz on superluminal phenomena, which have recently been observed, also deserve steady attention and verifications as well. Furthermore, as I also learnt recently, some of issues considered by me in the present essay have been touched in the old works [65,66]. My papers present themselves some insights in the mentioned problems, which are though connected with the previous considerations but, in my opinion, more relevant to the modern field theory. The group-theoretical basis for our researches has been proposed in the old papers of E. Wigner, Y. S. Kim and S. Weinberg.

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REFERENCES

[1] H. Joos, Fortschr. Phys. **10** (1962) 65
[2] S. Weinberg, Phys. Rev. B**133** (1964) 1318; ibid B**134** (1964) 882; ibid **181** (1969) 1893
[3] D. L. Weaver, C. L. Hammer and R. H. Good, jr., Phys. Rev. B**135** (1964) 241
[4] Yu. V. Novozhilov, *Vvedenie v teoriyu elementarnykh chastitz.* (Moscow. Nauka, 1971) [English translation: *Introduction to Elementary Particle Theory.* (Pergamon Press. Oxford, 1975)]
[5] R. H. Tucker and C. L. Hammer, Phys. Rev. D**3** (1971) 2448
[6] C. L. Hammer, S. C. McDonald and D. L. Pursey, Phys. Rev. **171** (1968) 1349; D. Shay and R. H. Good, jr., Phys. Rev. **179** (1969) 1410
[7] P. A. Vlasov, Ukrain. Fiz. Zhurn. **18** (1973) 1238; ibid **22** (1977) 951, in Russian
[8] H. M. Ruck and W. Greiner, J. Phys. A**3** (1977) 657
[9] F. D. Santos, Phys. Lett. B**175** (1986) 110; F. D. Santos and H. Van Dam, Phys. Rev. C**34** (1991) 250
[10] L. Lukaszuk and L. Szimanowski, Phys. Rev. D**36** (1987) 2440
[11] V. V. Dvoeglazov and N. B. Skachkov, JINR Communications P2-84-199, Dubna:JINR, 1984; ibid P2-87-882 Dubna:JINR, 1987; Yadern. Fiz. **48** (1988) 1770 [English translation: Sov. J. Nucl. Phys. **48** (1988) 1065]
[12] V. K. Mishra *et al.*, Phys. Rev. C**43** (1991) 801
[13] D. V. Ahluwalia, M. B. Johnson and T. Goldman, Phys. Lett. B**316** (1993) 102; D. V. Ahluwalia and T. Goldman, Mod. Phys. Lett. A**8** (1993) 2623
[14] D. V. Ahluwalia, Int. J. Mod. Phys. **11** (1996) 1855
[15] E. P. Wigner, in *Group Theoretical Concepts and Methods in Elementary Particle Physics.* Ed. by F. Gürsey (Gordon and Breach, NY, 1965)
[16] A. Sankaranarayanan and R. H. Good, jr., Nuovo Cim. **36** (1965) 1303; Phys. Rev. **140** (1965) B509; A. Sankaranarayanan, Nuovo Cim. **38** (1965) 889
[17] V. V. Dvoeglazov, Rev. Mex. Fis. Suppl. (*Proc. XVIII Oaxtepec Symp. on Nucl. Phys. Jan 4-7, 1995*) **41** (1995) 159; Int. J. Theor. Phys. **34** (1995) 2467
[18] V. V. Dvoeglazov, Nuovo Cim. **111**B (1996) 483
[19] V. V. Dvoeglazov, Hadronic J. Suppl. **10** (1995) 349
[20] F. Chang and F. Gürsey, Nuovo Cim. A**63** (1969) 617
[21] Y. Takahashi and R. Palmer, Phys. Rev. D**1** (1970) 2974
[22] K. Hayashi, Phys. Lett. **44**B (1973) 497
[23] M. Kalb and P. Ramond, Phys. Rev. D**9** (1974) 2273
[24] O. M. Boyarkin, ZhETF **75** (1978) 26; Izvestiya VUZov:fiz. **24** No. 11 (1981) 29 [English translation: Sov. Phys. J. **24** (1981) 1003]; Vestzi AN BSSR **5** (1983) 94
[25] L. V. Avdeev and M. V. Chizhov, Phys. Lett. B**321** (1994) 212; *A queer reduction of degrees of freedom.* Preprint JINR E2-94-263 (hep-th/9407067), Dubna, 1994
[26] D. Birmingham *et al.*, Phys. Repts. **209** (1991) 129
[27] M. V. Chizhov, Mod. Phys. Lett. A**8** (1993) 2753; *New tensor interactions and the $K_L - K_S$ mass difference.* Preprint JINR E2-94-253 (hep-ph/9407237), Dubna:JINR, 1994; *New tensor interactions in μ decay.* Preprint JINR E2-94-254 (hep-ph/9407236), Dubna:JINR, 1994
[28] V. V. Dvoeglazov, Hadronic J. **16** (1993) 459; Rev. Mex. Fis. Suppl. (*Proc. XVII Oaxtepec Symp. on Nucl. Phys. Jan 4-7, 1994*) **40** (1994) 352; V. V. Dvoeglazov, Yu. N. Tyukhtyaev and S. V. Khudyakov, Izvestiya VUZov:fiz. **37**, No. 9 (1994) 110 [English translation: Russ. Phys. J. **37** (1994) 898]
[29] V. V. Dvoeglazov, Helv. Phys. Acta **70** (1997) 677
[30] V. V. Dvoeglazov, Helv. Phys. Acta **70** (1997) 686
[31] V. V. Dvoeglazov, Helv. Phys. Acta **70** (1997) 697
[32] V. V. Dvoeglazov, *Can the Weinberg-Tucker-Hammer Equations Describe the Electromagnetic Field?* Preprint EFUAZ FT-94-09-REV (hep-th/9410174), Zacatecas, Oct. 1994
[33] D. Lurie, *Particles and Fields.* (Interscience Publisher. New York, 1968)
[34] A. O. Barut, I. Muzinich and D. Williams, Phys. Rev. **130** (1963) 442
[35] D. V. Ahluwalia and D. J. Ernst, Phys. Lett. B**287** (1992) 18; Mod. Phys. Lett. A**7** (1992) 1967; Phys.

Rev. C**45** (1992) 3010; Int. J. Mod. Phys. E**2** (1993) 397
[36] P. A. M. Dirac, Proc. Roy. Soc. A **117** (1928) 610; ibid **118** (1928) 351
[37] V. I. Starzhev and S. I. Kruglov, Acta Phys. Polon. B**8** (1977) 807; V. I. Strazhev, ibid **9** (1978) 449; Int. J. Theor. Phys. **16** (1977) 111
[38] V. B. Berestetskiĭ, E. M. Lifshits and L. P. Pitaevskiĭ, *Relativistic Quantum Theory. Vol. I. – Landau Course of Theoretical Physics. Vol. IV.* (Moscow. Nauka, 1968) [English translation: (Oxford, Pergamon Press, 1979)]
[39] M. W. Evans, Physica A**214** (1995) 605
[40] V. V. Dvoeglazov, *About the Question of 'Longitudinal' Nature of the Antisymmetric Tensor Field After Quantization.* Preprint EFUAZ FT-95-16-REV (hep-th/9604148), Zacatecas, Jan. 1996
[41] C. Itzykson and J.-B. Zuber, *Quantum Field Theory.* (McGraw-Hill Book Co. New York, 1980)
[42] N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields.* (Moscow. Nauka, 1983) [English translation of the second edition: (John Wiley, 1980)]
[43] N. Cabbibo and E. Ferrari, Nuovo Cim. **23** (1962) 1147; M. Y. Han and L. C. Biedenharn, Nuovo Cim. A**2** (1971) 544; R. Mignani, Phys. Rev. D**13** (1976) 2437
[44] M. A. Defaria-Rosa, E. Recami and W. A. Rodrigues, jr., Phys. Lett. B**173** (1986) 233; ibid **188** (1987) 511E; M. A. Defaria-Rosa *et al.*, Phys. Lett. B**220** (1989) 195
[45] M. A. Markov, *On the Difference Between Muon and Electron Masses.* Preprint JINR D-1345, Dubna:JINR, 1963
[46] J. Brana and K. Ljolje, Fizika **12** (1980) 287
[47] A. O. Barut and G. Ziino, Mod. Phys. Lett. A**8** (1993) 1011; G. Ziino, Int. J. Mod. Phys. A**11** (1996) 2081
[48] P. M. Lavrov, J. Phys. A**18** (1985) 3455
[49] M. V. Ostrogradsky, Mem. Acad. St. Petersbourg **6** (1850) 385 [cited in ref. [50]]
[50] C. Grosse-Knetter, Phys. Rev. D**49** (1994) 6709; M. S. Rashid and S. S. Khalil, *Hamiltonian Description of Higher Order Lagrangians.* Preprint IC/93/420, Trieste, 1993
[51] D. V. Ahluwalia and M. Sawicki, Phys. Rev. D**47** (1993) 5161
[52] J. D. Jackson, *Classical Electrodynamics.* 2nd edition (John Wiley & Sons, Inc., 1975)
[53] F. J. Dyson, Am. J. Phys. **58** (1990) 209; N. Dombey, ibid **59** (1991) 85; R. W. Brehme, ibid **59** (1991) 85; J. L. Anderson, ibid **59** (1991) 86; I. E. Farquhar, ibid **59** (1991) 87; R. J. Hughes, ibid **60** (1992) 301; C. R. Lee, Phys. Lett. A**148** (1990) 146; I. E. Farquhar, ibid **151** (1991) 203; A. Vaidya and C. Farina, ibid **153** (1991) 265
[54] S. Tanimura, Ann. Phys. (USA) **220** (1992) 229
[55] D. Saad, L. P. Horwitz and R. I. Arshansky, Found. Phys. **19** (1989) 1126; M. C. Land, N. Shnerb and L. P. Horwitz, *Feynman's proof of the Maxwell equations as a way beyond the Standard Model.* Preprint TAUP-2076-93 (hep-th/9308003), Tel Aviv, 1993
[56] A. Gersten, *Conserved currents of the Maxwell equations in the presence of electric and magnetic sources.* Preprint CERN-TH.4687/87, Geneva:CERN, 1987; *Conserved currents of the Maxwell equations with electric and magnetic sources.* Preprint CERN-TH.4688/87, Geneva:CERN, 1987
[57] A. Bandyopadhyay, Int. J. Theor. Phys. **32** (1993) 1563
[58] R. P. Malik and T. Pradhan, Z. Phys. C**28** (1985) 525
[59] B. Sarel and L. P. Horwitz, *A Chiral Spin Theory in the Framework of an Invariant Evolution Parameter Formalism.* Preprint TAUP-2280-95; M. C. Land and L. P. Horwitz, *Off-shell Quantum Electrodynamics.* Preprint TAUP-2227-95, Tel Aviv, 1995
[60] J. R. Oppenheimer, Phys. Rev. **38** (1931) 725
[61] E. Gianetto, Lett. Nuovo Cim. **44** (1985) 140
[62] M. Moshinsky and A. Del Sol, Canadian J. Phys. **72** (1994) 453
[63] J.-M. Levy-Leblond, Comm. Math. Phys. **6** (1967) 286; M. Le Bellac and J.-M. Levy-Leblond, Nuovo Cim. B**14** (1973) 217
[64] F. S. Crawford, Am. J. Phys. **60** (1992) 109; ibid **61** (1993) 472; A. Horzela, E. Kapuścik and C. A.

Uzes, ibid **61** (1993) 471
[65] F. J. Belinfante, Physica **6** (1939) 887
[66] H. C. Ohanian, Am. J. Phys. **54** (1986) 500
[67] A. E. Chubykalo and R. Smirnov-Rueda, Phys. Rev. E**53** (1996) 5373; ibid. **55** (1997) 3793E
[68] F. J. Belinfante, Phys. Rev. **76** (1949) 226; D. I. Caplan and F. J. Belinfante, ibid **85** (1952) 721