About the Claimed Longitudinal Nature of the Antisymmetric Tensor Field After Quantization

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Abstract

It has long been claimed that the antisymmetric tensor field of the second rank is longitudinal after quantization. In my opinion, such a situation produces speculations about the violation of the Correspondence Principle. On the basis of the Lagrangian formalism I calculate the Pauli-Lubanski vector of relativistic spin for this field. Even at the classical level it can be equal to zero after applications of well-known constraints. The correct quantization procedure permits us to propose solution of this puzzle in the modern field theory. Obtained results develop the previous consideration [*Physica A*214 (1995) 605-618].

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Quantum electrodynamics (QED) is a construct which found overwhelming experimental confirmations (for recent reviews see, *e.g.*, refs. [17,18]). Nevertheless, a number of theoretical aspects of this theory deserves more attention. First of all, they are: the problem of "fictious photons of helicity other than $\pm j$, as well as the indefinite metric that must accompany them"; the renormalization idea, which "would be sensible only if it was applied with finite renormalization factors, not infinite ones (one is not allowed to neglect [and to subtract] infinitely large quantities)"; contradictions with the

Weinberg theorem "that no symmetric tensor field of rank j can be constructed from the creation and annihilation operators of massless particles of spin j", *etc.* They were shown at by Dirac [15,16] and by Weinberg [47]. Moreover, it appears now that we do not yet understand many specific features of classical electromagnetism, first of all, the problems of longitudinal modes, of the gauge and of the Coulomb action-at-a-distance, refs. [30,32,45,23–26,12]. Secondly, the standard model, which has been constructed on the basis of ideas, which are similar to QED, appears to be no able to explain many puzzles in neutrino physics.

In my opinion, all these shortcomings can be the consequences of ignoring several important questions. "In the classical electrodynamics of charged particles, a knowledge of $F^{\mu\nu}$ completely determines the properties of the system. A knowledge of A^μ is redundant there, because it is determined only up to gauge transformations, which do not affect $F^{\mu\nu}$... Such is not the case in quantum theory..." [36]. We learnt, indeed, about this fact from the Aharonov-Bohm [1] and the Aharonov-Casher effects [2]. However, recently several attempts have been undertaken to explain the Aharonov-Bohm effect classically [42]. These attempts have, in my opinion, logical basis. In the mean time, quantizing the antisymmetric tensor field led us to a new puzzle, which until now was not drawn much attention to. It was claimed that the antisymmetric tensor field of the second rank is longitudinal after quantization [41,35,13,7,38]. We know that the antisymmetric tensor field (electric and magnetic fields, indeed) is transversal in the Maxwellian classical electrodynamics. It is doubtfully that physically longitudinal components can be transformed into the physically transverse ones in the $\hbar \to 0$ limit.[1] How should we manage with the Correspondence Principle in this case? It is often concluded: one is not allowed to use the antisymmetric tensor field to represent the quantized

electromagnetic field in relativistic quantum mechanics. Nevertheless, we are convinced that a reliable theory should be constructed on the basis of a minimal number of ingredients ("Occam's Razor") and should have well-defined classical limit. Therefore, in this paper I undertake a detailed analysis of rotational properties of the antisymmetric tensor field, I calculate the Pauli-Lubanski operator of relativistic spin (which must define whether the quantum is in the left- or right- polarized states or in the unpolarized state) and then conclude, if it is possible to obtain the conventional electromagnetic theory with photon helicities $h = \pm 1$ provided that strengths (*not* potentials) are chosen to be physical variables. The particular case also exists when the Pauli-Lubanski vector for the antisymmetric tensor field

of the second rank is equal to zero, what corresponds to the claimed '*longitudity*' (helicity $h = 0$?) of this field.

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[1]It is interesting to compare this question with the group-theoretical consideration in ref. [34] which deals with the reduction of rotational degrees of freedom to gauge degrees of freedom in infinite-momentum/zero-mass limit. The only

mentions of the transversality of the quantized antisymmetric tensor field see in refs. [46,11].

Researches in this area from a viewpoint of the Weinberg's $2(2j+1)$ component theory have been started in refs. [3,4,19–21,23–26]. I would also like to point out that the problem at hand is directly connected with our understanding of the nature of neutral particles, including neutrinos [39,40,48,5,6,27–29]. ¿From a mathematical viewpoint theoretical content provided by the space-time structure and corresponding symmetries should not depend, what representation space, which field operators transform on, is chosen.

I begin with the antisymmetric tensor field operator (in general, complex-valued):

$$F^{\mu\nu}(x) = \sum_\eta \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_p} \left[F^{\mu\nu}_{\eta\,(+)}(\mathbf{p})\, a_\eta(\mathbf{p})\, e^{-ip\cdot x} + \mathrm{F}^{\mu\nu}_{\eta\,(-)}(\mathbf{p})\, b_\eta^\dagger(\mathbf{p})\, e^{+ip\cdot x}\right. \tag{1}$$

and with the Lagrangian, including, in general, mass term:[2]

$$\mathcal{L} = \frac{1}{4}(\partial_\mu F_{\nu\alpha})(\partial^\mu F^{\nu\alpha}) - \frac{1}{2}(\partial_\mu F^{\mu\alpha})(\partial^\nu F_{\nu\alpha}) - \frac{1}{2}(\partial_\mu F_{\nu\alpha})(\partial^\nu F^{\mu\alpha}) + \frac{1}{4}m^2 F_{\mu\nu}F^{\mu\nu} \quad . \tag{3}$$

The Lagrangian leads to the equation of motion in the following form (provided that the appropriate antisymmetrization procedure has been taken into account):

$$\frac{1}{2}(\square + m^2)F_{\mu\nu} + (\partial_\mu F_{\alpha\nu}{}^{,\alpha} - \partial_\nu F_{\alpha\mu}{}^{,\alpha}) = 0 \quad , \tag{4}$$

where $\square = -\partial_\alpha\partial^\alpha$. It is this equation for antisymmetric-tensor-field components that follows from the Proca-Bargmann-Wigner consideration, which is characterized by the equations:[3]

$$\partial_\alpha F^{\alpha\mu} + \frac{m}{2}A^\mu = 0 \quad , \tag{5a}$$

$$2mF^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad , \tag{5b}$$

provided that $m \neq 0$ and in the final expression one takes into account the Klein-Gordon equation $(\square - m^2)F_{\mu\nu} = 0$. The latter expresses relativistic dispersion relations $E^2 - \mathbf{p}^2 = m^2$ and it follows from the coordinate Lorentz transformation laws [43, §2.3].

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[2]The massless limit ($m \to 0$) of the Lagrangian is connected with the Lagrangians used in the conformal field
theory and in the conformal supergravity by adding the total derivative:

$$\mathcal{L}_{CFT} = \mathcal{L} + \frac{1}{2}\partial_\mu\left(F_{\nu\alpha}\partial^\nu F^{\mu\alpha} - F^{\mu\alpha}\partial^\nu F_{\nu\alpha}\right) \quad . \tag{2}$$

The gauge-invariant form ($F_{\mu\nu} \to F_{\mu\nu} + \partial_\nu\Lambda_\mu - \partial_\mu\Lambda_\nu$), ref. [41], is obtained only if one uses the Fermi procedure *mutatis mutandis* by removing the additional "phase" field $\lambda(\partial_\mu F^{\mu\nu})^2$, with the appropriate coefficient λ, from the Lagrangian. This has certain analogy with the QED, where the question, whether the Lagrangian is gauge-invariant or not, is solved depending on the presence of the term $\lambda(\partial_\mu A^\mu)^2$. For details see ref. [35] and what is below.

[3]In the textbooks the equations with the "renormalized" potentials $A^\mu \to 2mA^\mu$ are usually used. This "renormalization" can change the asymptotic $m \to 0$ behaviour of classical potentials. Therefore, until the complete investigation of this question one should use the form (5a,5b) which follows from the Dirac equations satisfied by the symmetric spinor of the second rank.

Following the variation procedure given, *e.g.*, in refs. [14,8,10] one can obtain that for rotations $x^{\mu'} = x^\mu + \omega^{\mu\nu} x_\nu$ the corresponding variation of the wave function is found from the formula:

$$\delta F^{\alpha\beta} = \frac{1}{2}\omega^{\kappa\tau} {\cal T}_{\kappa\tau}^{\alpha\beta,\mu\nu} F_{\mu\nu} \quad . \tag{6}$$

The generators of infinitesimal transformations are then defined as

$$\begin{aligned}{\cal T}_{\kappa\tau}^{\alpha\beta,\mu\nu} &= \frac{1}{2} g^{\alpha\mu} (\delta_\kappa^\beta\, \delta_\tau^\nu \,-\, \delta_\tau^\beta\, \delta_\kappa^\nu) \,+\, \frac{1}{2} g^{\beta\mu} (\delta_\kappa^\nu \delta_\tau^\alpha \,-\, \delta_\tau^\nu\, \delta_\kappa^\alpha) + \\ &+ \frac{1}{2} g^{\alpha\nu} (\delta_\kappa^\mu\, \delta_\tau^\beta \,-\, \delta_\tau^\mu\, \delta_\kappa^\beta) \,+\, \frac{1}{2} g^{\beta\nu} (\delta_\kappa^\alpha\, \delta_\tau^\mu \,-\, \delta_\tau^\alpha\, \delta_\kappa^\mu) \quad .\end{aligned} \tag{7}$$

It is ${\cal T}_{\kappa\tau}^{\alpha\beta,\mu\nu}$, the generators of infinitesimal transformations, that enter in the formula for the relativistic spin tensor:

$$J_{\kappa\tau} = \int d^3\mathbf{x} \left[\frac{\partial {\cal L}}{\partial(\partial F^{\alpha\beta}/\partial t)} {\cal T}_{\kappa\tau}^{\alpha\beta,\mu\nu} F_{\mu\nu} \right] \quad . \tag{8}$$

As a result one obtains:

$$\begin{aligned}J_{\kappa\tau} = \int d^3\mathbf{x}\, [(\partial_\mu F^{\mu\nu}) (g_{0\kappa} F_{\nu\tau} - g_{0\tau} F_{\nu\kappa}) - (\partial_\mu F^\mu{}_\kappa) F_{0\tau} + (\partial_\mu F^\mu{}_\tau) F_{0\kappa} + \\ + \, F^\mu{}_\kappa (\partial_0 F_{\tau\mu} + \partial_\mu F_{0\tau} + \partial_\tau F_{\mu 0}) - F^\mu{}_\tau (\partial_0 F_{\kappa\mu} + \partial_\mu F_{0\kappa} + \partial_\kappa F_{\mu 0})] \quad .\end{aligned} \tag{9}$$

If agree that the orbital part of the angular momentum

$$L_{\kappa\tau} = x_\kappa \Theta_{0\,\tau} - x_\tau \Theta_{0\,\kappa} \quad , \tag{10}$$

with $\Theta_{\tau\lambda}$ being the energy-momentum tensor, does not contribute to the Pauli-Lubanski operator when acting on the one-particle free states (as in the Dirac $j=1/2$ case), then the Pauli-Lubanski 4-vector is constructed as follows [37, Eq.(2-21)]

$$W_\mu = -\frac{1}{2} \epsilon_{\mu\kappa\tau\nu} J^{\kappa\tau} P^\nu \quad , \tag{11}$$

with $J^{\kappa\tau}$ defined by Eqs. (8,9). The 4-momentum operator P^ν can be replaced by its eigenvalue when acting on the plane-wave eigenstates. One should choose space-like normalized vector $n^\mu n_\mu = -1$, for example $n_0 = 0$, $\mathbf{n} = \widehat{\mathbf{p}} = \mathbf{p}/\vert\mathbf{p}\vert$.[4]

After lengthy calculations in a spirit of [37, p.58,147] one can find the explicit form of the relativistic spin:

$$(W_\mu \cdot n^\mu) = -(\mathbf{W}\cdot\mathbf{n}) = -\frac{1}{2}\epsilon^{ijk} n^k J^{ij} p^0 \quad , \tag{12a}$$

$$\mathbf{J}^k = \frac{1}{2}\epsilon^{ijk} J^{ij} =$$

[4]Let me remind that the helicity operator is connected with the Pauli-Lubanski vector in the following manner $(\mathbf{J}\cdot\widehat{\mathbf{p}}) = (\mathbf{W}\cdot\widehat{\mathbf{p}})/E_p$, see ref. [44]. The choice of ref. [37, p.147], $n^\mu = \left(t^\mu - p^\mu \frac{p\cdot t}{m^2}\right) \frac{m}{\vert\mathbf{p}}$ $\vert$, with $t \equiv (1,0,0,0)$ being a time-like vector, is also possible but it leads to some oscurities in the procedure of taking the massless limit. These oscurities will be clarified in a separate paper.

$\epsilon^{ijk} \int d^3\mathbf{x} \left[F^{0i}(\partial_\mu F^{\mu j}) + F_\mu{}^j(\partial^0 F^{\mu i} + \partial^\mu F^{i0} + \partial^i\right.$
$\mathrm{F}^{0\mu}) \quad .(12b)$

Now it becomes obvious that the application of the generalized Lorentz conditions (which are quantum versions of free-space dual Maxwell's equations) leads in such a formulation to the absence of electromagnetism in a conventional sense. The resulting Kalb-Ramond field is longitudinal (helicity $h = 0$). All the components of the angular momentum tensor for this case are identically equated to zero. The discussion of this fact can also be found in ref. [35,24]. This situation can occur in the particular choice of the normalization of the operators $J_{\mu\nu}$ and $g_\mu \equiv J_{\mu\nu}P^\nu$ only.

One of the possible ways to obtain helicities $h = \pm 1$ is a modification of the electromagnetic field tensor like ref. [30q], *i.e.*, introducing the non-Abelian electrodynamics [32]:

$$F_{\mu\nu} \quad \Rightarrow \quad \mathbf{G}^a_{\mu\nu} = \partial_\mu A^{(a)\,*}_\nu - \partial_\nu A^{(a)\,*}_\mu - i\frac{e}{\hbar}[A^{(b)}_\mu, A^{(c)}_\nu] \quad , \tag{13}$$

where (a), (b), (c) are the vector components in the (1), (2), (3) circular basis [30,32]. In the other words, one can add some ghost field (the $\mathbf{B}^{(3)}$ field) to the antisymmetric tensor $F_{\mu\nu}$. As a matter of fact this induces hypotheses on a massive photon and/or an additional displacement current. I can agree with the *possibility* of the $\mathbf{B}^{(3)}$ field concept (while it is required *rigorous* elaboration in the terminology of the modern quantum field theory), but, at the moment, I prefer to avoid any auxiliary constructions (even they are valuable in intuitive explanations and generalizations). If these non-Abelian constructions exist they should be deduced from a more general theory on the basis of some fundamental postulates, *e.g.*, in a spirit of refs. [4,6,22]. In the procedure of the quantization one can reveal the important case, when the transversality (in the meaning of existence of $h = \pm 1$) of the antisymmetric tensor field is preserved. This conclusion is related with existence of the dual tensor $\widetilde{F}^{\mu\nu}$, with possibility of the Bargmann-Wightman-Wigner-type quantum field theory revealed in ref. [4][5] and with normalization questions.

I choose the field operator, Eq. (1), such that:

$$F^{i0}_{(+)}(\mathbf{p}) = E^i(\mathbf{p}) \quad , \quad F^{jk}_{(+)}(\mathbf{p}) = -\epsilon^{jkl}B^l(\mathbf{p}) \quad ; \tag{14a}$$

$$F^{i0}_{(-)}(\mathbf{p}) = \tilde{F}^{i0}(\mathbf{p}) = B^i(\mathbf{p}) \quad , \quad F^{jk}_{(-)}(\mathbf{p}) = \tilde{F}^{jk}(\mathbf{p}) = \epsilon^{jkl}E^l(\mathbf{p}) \quad , \tag{14b}$$

where $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is the tensor dual to $F^{\mu\nu}$; and $\epsilon^{\mu\nu\rho\sigma} = -\epsilon_{\mu\nu\rho\sigma}$, $\epsilon^{0123} = 1$ is the totally antisymmetric Levi-Civita tensor. After lengthy but standard calculations one achieves:[6]

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[5]The remarkable feature of the Ahluwalia *et al.* consideration is: boson and its antiboson can possess opposite relative parities.

[6]Of course, the question of the behaviour of vectors $\mathbf{E}_\eta$ and $\mathbf{B}_\eta$ and/or of creation and ahhihilation operators with respect to the parity operation in this particular case deserves detailed elaboration.

$$\mathbf{J}^k = \sum_{\eta\eta'} \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_p} \left\{ \frac{i\epsilon^{ijk}\mathbf{E}^i_\eta(\mathbf{p})\mathbf{B}^j_{\eta'}(\mathbf{p})}{2} \left[a_\eta(\mathbf{p}) b^\dagger_{\eta'}(\mathbf{p}) + a_{\eta'}(\mathbf{p}) b^\dagger_\eta(\mathbf{p}) + b^\dagger_{\eta'}(\mathbf{p}) a_\eta(\mathbf{p}) + b^\dagger_\eta(\mathbf{p}) a_{\eta'}(\mathbf{p}) \right] - \right.$$

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p)) - i $\mathbf{B}^k_{\eta'}(\mathbf{p})(\mathbf{p}\cdot\mathbf{B}_\eta(\mathbf{p}))2E_p \times$

$\times \left[a_\eta(\mathbf{p}) b^\dagger_{\eta'}(\mathbf{p}) + b^\dagger_\eta(\mathbf{p}) a_{\eta'}(\mathbf{p}) \right] \Big\}$ (15)

One should choose normalization conditions. For instance, one can use the analogy with the (dual) classical electrodynamics:[7]

$(\mathbf{E}_\eta(\mathbf{p}) \cdot \mathbf{E}_{\eta'}(\mathbf{p}) + \mathbf{B}_\eta(\mathbf{p})\cdot$

$\mathbf{B}_{\eta'}) = 2E_p \delta_{\eta\eta'}$, (16a)

$\mathbf{E}_\eta \times \mathbf{B}_{\eta'} = \mathbf{p}\delta_{\eta\eta'} - \mathbf{p}\delta_{\eta,-\eta'}$.(16b)

These conditions still imply that $\mathbf{E} \perp \mathbf{B} \perp \mathbf{p}$. Finally, one obtains

$$\mathbf{J}^k = -i \sum_\eta \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\mathbf{p}^k}{2E_p} \left[a_\eta(\mathbf{p}) b^\dagger_{-\eta}(\mathbf{p}) + b^\dagger_\eta(\mathbf{p}) a_{-\eta}(\mathbf{p}) \right] \quad . \qquad (17)$$

If we want to describe states with the definite helicity quantum number (photons) we should assume that $b^\dagger_\eta(\mathbf{p}) = ia^\dagger_\eta(\mathbf{p})$ what is reminiscent with the Majorana-like theories [5,6,9].[8] One can take into account the prescription of the normal ordering and set up the commutation relations in the form:

$$\left[a_\eta(\mathbf{p}), a^\dagger_{\eta'}(\mathbf{k}) \right]_- = (2\pi)^3 \delta(\mathbf{p}-\mathbf{k}) \delta_{\eta,-\eta'} \quad . \qquad (18)$$

After acting

the operator (17) on the physical states, *i.e.*, $a^\dagger_h(\mathbf{p})|0>$, we are convinced that antisymmetric tensor field can describe particles with helicities to be equal to ± 1). One can see that the origins of this conclusion are the possibilities of different definitions of the field operator (and its normalization), the existence

of the '*antiparticle*' for the particle described by antisymmetric

tensor field. The latter statement is related with the Weinberg discussion of the connection between helicity and representations of the

Lorentz group [47a]. Next, I would like to point out that the Proca-like equations for antisymmetric tensor field with *mass*, *e.g.*, Eq. (4) can possess tachyonic solutions, see for the discussion in ref. [23]. Therefore, in a massive case the states can be "partly" tachyonic states mathematically. We then deal always with the problem of the choice of normalization conditions which could permit us to describe both transversal and longitudinal *physical* modes of the $j = 1$ field.

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[7]Different choices of the normalization could still lead to equating the spin operator to zero or even to the other values of helicity, which differ from ± 1. The question is: what cases are realized in the Nature and what processes do correspond to every case?

[8]Of course, the imaginary unit can be absorbed by the corresponding re-definition of negative-frequency solutions.

In conclusion, I calculated the Pauli-Lubanski vector of relativistic spin on the basis of the Nötherian symmetry method [14,8,10]. Let me remind that it is a part of the angular momentum vector, which is conserved as a consequence of the rotational invariance. After explicit [35] (or implicit [7]) applications of the constraints (the generalized Lorentz condition) in the Minkowski space, the antisymmetric tensor field becomes '*longitudinal*' in the meaning that the angular momentum operator is equated to zero (the sense which was attached by the authors

of the works [41,35,38,7]). I proposed one of possible ways to resolve this contradiction with the Correspondence Principle in refs. [23–26]. Another hypothesis has been proposed by Evans [30–32], in which the component of the Pauli-Lubanski vector generalized to the isovector space $(1),(2),(3)$ has been identified with the new $\mathbf{B}^{(3)}$ field of electromagnetism.[9] The present article continues these researches. The conclusion achieved is: the antisymmetric tensor

field can describe both the Maxwellian $j=1$ field and the Kalb-Ramond $j=0$ field. Nevertheless, I still think that the physical nature of the $E=0$ solution re-discovered in refs. [33,3], its connections

with the Evans-Vigier $\mathbf{B}^{(3)}$ field, ref. [30,32], with Avdeev-Chizhov δ'- type transversal solutions [7b], which cannot be interpreted as relativistic particles, as well as with my concept of χ boundary functions, ref. [26] are not completely explained until now. Finally, while I do not have any intention to doubt theoretical results of the ordinary quantum electrodynamics I am sure that the questions put forth in this note (as well as in previous papers of both mine and other groups) should be explained properly.

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[9]See also the paper of Chubykalo and Smirnov-Rueda [12]. The paper on connections between the Chubykalo and Smirnov-Rueda '*action-at-a-distance*' construct and the $B(3)$ theory was submitted (private communication from A. Chubykalo).

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