NONLOCALITY OF CLASSICAL ELECTRODYNAMICS OF POINT PARTICLES, AND VIOLATION OF BELL'S INEQUALITIES

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ABSTRACT

We show that Bell's inequalities are violated in a model of two charged particles interacting with two potential barriers, which mimic the measuring instruments; the motion of each particle is described by the Abraham–Lorentz–Dirac equation in the nonrelativistic version, and the role of the hidden variables is played by the initial accelerations. The essential nonlocality property of the system is induced by the celebrated Dirac's nonrunaway condition, which makes the measuring instruments have a certain influence on the observed system, by determining the domain of definition of the hidden variable (the Bopp–Haag phenomenon). So this model strongly supports E. Nelson's suggestion, namely that nonlocality properties suited to violate Bell's inequalities appear in classical field theories when regularizing cutoffs are removed.

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Running title: BELL'S INEQUALITIES AND CLASSICAL ELECTRODYNAMICS

1. **Introduction** It is very well known that the proof of Bell's inequalities relies essentially on the assumption of some sort of locality, for example on what Bell calls the "principle of local causality" [1,2]. So, as experiments seem to violate Bell's inequalities in the way predicted by quantum mechanics, one concludes on the one hand that it is nature itself that seems to present nonlocal properties, and on the other hand that the formalism of quantum mechanics implicitly contains in itself some appropriate nonlocal features. Finally, one concludes that classical hidden–variable theories cannot correctly describe nature, at least if they are conceived, as is usually done, as local theories.

But clearly the same cannot be said of classical theories of nonlocal type. Now, it is obvious that one could produce some strange model with just the nonlocal properties required to violate Bell's inequalities, but this would be too artificial. In Bell's words: "*On the other hand, if no restrictions whatever are imposed on the hidden variables, ..., it is trivially clear that such schemes can be found to account for any experimental results whatever*" [3]. So, the significant problem for hidden variable theories is rather whether nonlocal properties might occur in classical theories in some natural way. This was apparently first stressed by Nelson [4,5]. What he had in mind is that the appropriate nonlocal features might arise naturally in classical field theories in the limit in which one removes space and momentum cutoffs, previously introduced in order to regularize the theory. Nelson gave indeed in support of his thesis some heuristic and qualitative arguments, which led him to state that ([5], page 438) "*A discussion of Bell's theorem leads to the conclusion that it is no obstacle to the description of quantum phenomena by classical random fields*"; but neither a general proof nor a concrete example were available to him. However, in [4] he suggested that it should be of interest to study preliminarily some classical models describing the interactions of fields with particles; in such a case, the removal of momentum cutoffs would be equivalent to taking the limit of point particles.

In the present paper we point out that classical electrodynamics of point particles in the so–called dipole approximation is indeed nonlocal, just in virtue of the celebrated Dirac's nonrunaway condition [6] (see also [7] and [8]), which plays in fact a constitutive role in the definition of the point limit itself. We then show how such a nonlocality is effective in leading to a violation of Bell's inequalities, although not exactly in the way conjectured by Nelson; this is obtained in a particular model which exploits a striking phenomenon, namely that of Bopp [9] and Haag [10] (see also [11]), occurring in classical electrodynamics of point particles as a consequence of Dirac's condition.

Let us briefly recall what is meant by classical electrodynamics of point particles in the dipole approximation, and which are the main results for it. Classical electrodynamics of a point particle is in principle nothing but the familiar Maxwell–Lorentz system, i.e., that having for unknowns the electromagnetic field and the particle's position with minimal coupling, namely: the field obeys Maxwell equations having for source the current due to the particle, while the particle satisfies the relativistic Newton equation with the Lorentz force due to the field. But for a point particle the system is ill defined, because of the infinite "self–force" on the particle (think of the Coulomb self–force in the static case), and so needs a regularization. This can be performed by imposing space and momentum cutoffs, or just momentum cutoffs, and then one remains with the problem of studying the limit in which the cutoffs are removed. Such a program is still unaccopmplished; in

Nelson's words ([4], page 65–66): *"Is it an exaggeration to say that nothing whatever is known about the behavior of this system as the cutoffs are removed, that there is not one single theorem that has been proved?"*. There are however partial results. First of all there are the classical old results for the nonrelativistic case which go back to Lorentz and Abraham and are concerned with the dipole approximation; this is a linearization in which the current $\mathbf{j}(\mathbf{x},t) = \dot{\mathbf{q}}\delta(\mathbf{x}-\mathbf{q}(t))$, where $\mathbf{q}(t)$ is the particle motion, is approximated by $\dot{\mathbf{q}}\delta(\mathbf{x})$, while in the Lorentz force the magnetic term is neglected. The limit was performed in a way which today might be considered as heuristic, but in any case the most relevant result was that the naive point limit (i.e. that with a fixed bare mass) leads to a trivial dynamics, with the particle and the field actually decoupled. So mass renormalization was first introduced, with the bare mass diverging to $-\infty$, and in such a way it was shown that the particle obeys in the limit the well known third–order Abraham–Lorentz–Dirac equation. The relativistic case was dealt with in the year 1938, still in a not completely rigorous way, by Dirac [6,10], who found the relativistic version of the particle's equation; we will refer to both equations (i.e. the relativistic and the nonrelativistic ones) generically as the Abraham–Lorentz–Dirac equation. Such classic results were fully confirmed, at least in the nonrelativistic case and in the dipole approximation, by the recent works [12], [13] and [14], where the problem was studied with the present standard of rigour; in particular, in [14] the limit equation for the field was found for the first time.

In any case, for the aim of the present paper the most striking feature of the point limit is the generic appearence of the absurd runaway solutions, which apparently were first discussed by Dirac in connection with his relativistic equation: for example, it turns out that the free particle accelerates exponentially fast as time increases, and analogous divergences occur also for ther field.

Now, faced with such an apparently absurd situation one might be tempted to simply throw classical electrodynamics away. Another possibility, suggested by Dirac himself, consists of imposing from outside for the limit system a new prescription, which consists in restricting the phase space to those initial data leading to nonrunaway motions; we recall that, in the typical case of scattering, such motions are defined by the property that the acceleration vanishes for $t \to +\infty$. It is thus clear that Dirac's prescription assumes a constitutive role in the definition itself of the the theory, radically changing its mathematical structure: namely, classical electrodynamics of point particles as defined by mass renormalization and Dirac's prescription is a new theory, structurally different from standard electrodynamics of macroscopic (as opposed to point) particles. From the mathematical point of view, the difference consists in the fact that the nonrunaway prescription leads to a problem which resembles more to a Sturm–Liouville than to a Cauchy problem. This is just the reason why many people appear to dislike the Dirac prescription, blaming it of being, as they say, acausal [15]. The point we make is instead that such a theory is rather non locally causal, more or less in the sense of Bell; this is indeed a characteristic nonlocal feature of classical electrodynamics of point particles which ultimately turns out to lead to a violation of Bell's inequalities, a fact that strangely enough seems not to have been noticed up to now (see however [16]).

This will be exhibited below in a very simple model, conceived within a setup typical of the *gedankenexperiments* related to Bell's inequalities. The model consists of two charged particles which, after having somehow interacted, separate away along two opposite directions, and proceed with no further mutual interaction. Then, each of the two particles interacts with a measuring instrument, which we take to be an external potential barrier, the measurement consisting in observing whether the particle crosses the barrier or is reflected from it. A dicothomic variable is thus defined, which takes for example the value $+1$ in the former, and -1 in the latter case. As usual in this kind of problems, we allow each measuring instrument to be prepared in one of some (typically two or three) different settings, which here are just three different heights of the barrier.

Now, the measurement would be trivial in the purely mechanical case, because the particle would certainly cross the barrier or be reflected from it, according to the value of its energy. But things are completely different if the self–interaction with the electromagnetic field is taken into account, what we do by assuming that the particle's motion is a solution of the nonrelativistic Abraham–Lorentz–Dirac equation in presence of the external potential barrier. Indeed, use is made here of a relevant, highly nontrivial, property of the Abraham–Lorentz–Dirac equation which, although being known already to Bopp [9] and Haag [10] and somehow adumbrated in a theorem of Hale and Stokes [17], was particularly appreciated quite recently [11]; namely, that in general the nonrunaway solutions of the Abraham–Lorentz–Dirac equation in presence of an external barrier are not uniquely defined by the initial mechanical state (position and velocity) of the particle. As a consequence, the initial acceleration has really to be assigned as an additional variable, not uniquely defined by the mechanical state, and thus plays here the role of the hidden variable, for which a probabilistic description is needed. It turns out, as proven in [11], that according to the value of the hidden variable the particle crosses the barrier or is reflected from it, which is a situation somehow reminiscent of the tunnel effect. Moreover, such a Bopp–Haag phenomenon occurs for initial mechanical states in a domain which actually depends on the setting of the measuring instrument (i.e. on the height of the barrier), and this is the feature that turns out to attribute indeed an essential nonlocal character to the system.

In section 2 the relevant notions concerning the Abraham–Lorentz–Dirac equation and the Bopp–Haag nonuniqueness phenomenon are recalled, and it is discussed how the particle's aceleration plays the role of the hidden variable; in section 3 the two–particle model is discussed, and it is shown that Bell's inequalities are violated for certain unfactorized probability distributions of the hidden variables; some further discussions of a general type are deferred to section 4, and the conclusions then follow.

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2. **Relevant features of the Abraham–Lorentz–Dirac equation: Dirac's condition and the Bopp–Haag nonuniqueness phenomenon**. Limiting our attention to the case considered below in our model, namely that of a particle moving on a line (the x axis) in the presence of an external potential energy V in the nonrelativistic approximation, the Abraham–Lorentz–Dirac equation has the form

$$\ddot{x} = -\frac{1}{m}V'(x) + \varepsilon \dddot{x} \, , \tag{1}$$

where dot and prime denote derivatives with respect to time t and position coordinate x respectively, m is the particle's mass, while $\varepsilon = \frac{2}{3} e^2/mc^3$ is a parameter (with the dimensions of a time) depending on the speed c of light and on the charge e of the particle. The natural phase space for the equation is $\mathbf{R}^3$, referred to coordinates (x, v, a) defining the particle's position x, velocity $v = \dot{x}$ and acceleration $a = \dot{v}$. But it turns out that generic initial data in such a space give rise to runaway motions. This is immediately seen in the simplest example, i.e. that of the free particle characterized by $V = 0$, because equation (1) then reduces to a closed equation for the acceleration, namely $\epsilon \dot{a} = a$; the general solution $a(t) = a_0 \exp(t/\epsilon)$ thus leads to absurd self–accelerating motions for all initial data a_0, with the only exception of the initial data on the manifold $a = 0$, which lead to the natural motions $a(t) = 0$. Such an invariant subset of phase space, defined by $a = 0$ and constituted by orbits not having runaway character, can be called the *physical manifold* or *Dirac manifold.*

More in general, let us consider a scattering problem, with the force vanishing sufficiently fast at infinity. The problem of existence of the Dirac manifold can be stated in the following way: given an initial mechanical state (x_0, v_0), one asks whether there exists an initial acceleration a_0 such that the corresponding motion, with initial data (x_0, v_0, a_0), has a nonrunay character, i.e. satisfies the condition $a(t) \to 0$ for $t \to +\infty$. This clearly is a kind of Sturm–Liouville problem. Existence was proven by Hale and Stokes for a large class of potentials, but they couldn't prove uniqueness; mathematically, this is due to the circumstance that the existence problem turns out to be reduced to a fixed point problem not involving a contraction. Thus, for a given mechanical state one can a priori expect that there exist several allowed values for the initial acceleration; in other terms, the Dirac manifold is not a priori the graph of a function $a = f(x, v)$, and can in general be folded above the mechanical (x, v) plane.

As a matter of fact, a case of nonuniqueness was already known to Bopp and Haag, who could find by elementary methods two solutions corresponding to the same mechanical state in a certain domain, for a potential step. But such a nonuniqueness property did not arouse much interest, and apparently was not even known to Hale and Stokes. In addition, the presence of just two nonrunaway solutions (for a given mechanical state) in the case discussed by Bopp and Haag led some authors (see the discussion in [18]) to conceive that only one of them should be retained as physically meaningful, while the other one should be discarded, although it is not obvious which criterion of selection should be adopted.

On the other hand, in [11] it was shown that such a nonuniqueness phenomenon is indeed a common fact for a large class of potentials, and was in particular proven to occur essentially for all potential barriers with a sufficiently sharp maximum; moreover, it was found that there occur in general not just two, but an arbitrary number of solutions. This goes as follows. For a given barrier and an initial position x_0, there exists an interval of initial velocities, and thus an interval I of initial energies (located about the maximum of the barrier), such that the corresponding initial acceleration leading to nonrunaway motion is not unique. More precisely, for any positive integer n there exists an interval $I_n \subset I$ with n nonrunaway solutions crossing the barrier and n nonrunaway solutions reflected from it. It is thus clear that the allowed initial accelerations corresponding to a given mechanical state are all apparently on the same footing, and there is no hope to find a natural criterion for selecting a particular one among them as privileged. It rather appears that, given an initial mechanical state with energy in the interval I_n, one should instead more naturally be led to assign some probability distribution to the allowed values for the "nonmechanical variable", i.e. for the acceleration.

Such a qualification of nonmechanical variable for the initial particle's acceleration seems to be appropriate. Indeed, in [13] and [14] it was shown that the initial acceleration to be inserted in the Cauchy data for the Abraham–Lorentz–Dirac equation is a certain definite function of the initial data of the original Maxwell–Lorentz system describing the complete system particle plus field; in other words, the particle's initial acceleration in the Cauchy problem for the Abraham–Lorentz–Dirac equation is just a trace of the initial data for the field in the complete Maxwell–Lorentz system.

Furthermore, a property of the allowed initial accelerations (or of the initial field in the complete system, according to what just said) which naturally favours the interpretation of the acceleration as the hidden variable with respect to the mechanical ones is the dependence of the allowed initial accelerations on the initial position x_0 as $|x_0|$ is taken farther and farther away from the barrier, which is the case of interest for the description of scattering processes. Indeed, while the energy strip I (where the nonuniqueness phenomenon occurs) becomes essentially independent of x_0, it turns out that the allowed initial accelerations corresponding to a given mechanical state collapse to zero exponentially fast as $|x_0| \to +\infty$; and this makes the different accelerations, leading to nonrunaway motions for a given mechanical state, essentially undistinguishable, as should be expected of variables to be qualified as hidden. In physical terms, with reference to the complete Maxwell–Lorentz system, it is thus actually impossible to prepare the initial state with a concrete control of the initial electromagnetic field required to discriminate whether the particle will cross the barrier or not. This seems indeed a property to be expected of a hidden variable, namely of a variable that, according to Bell, should be rather called *uncontrolled*, *"for these variables, by hypothesis, for the time being, cannot be manipulated at will by us"*[19].

3. **The model, and the violation of Bell's inequalities**. In all *gedankenexperiments* concerned with Bell's inequalities, one deals first of all, following Einstein, Podolsky and Rosen themselves, with two equal subsystems which initially interact in some way and then separate away along two opposite directions, evolving as free subsystems; then, measurements of some dicothomic physical quantity are separately performed on each of them by some instrument which can be prepared in a certain number of different settings. In our model, the system is constituted of two equal charged particles which, after having initially interacted in some way that doesn't concern us here, separate away along opposite directions on a straight line, and then proceed, say for $x_1 \geq L$ and $x_2 \leq -L$, with no mutual interaction; the measuring instruments are just two potential barriers located on opposite sides with respect to the origin (i.e. with respect to the source) very far away from it, with heights that can assume three different values, and the measurement consists in observing whether each particle crosses its barrier or is reflected from it. The dicothomic variable is defined as taking the value $+1$ in case of crossing and the value -1 in the opposite case. The motion of each independent particle is described as a solution of the nonrelativistic

Abraham–Lorentz–Dirac equation with the given potential. It is assumed that the heights of the barriers are such as to allow for the nonuniqueness phenomenon described above to occur; so the mechanical state (x, v) of a particle does not uniquely define its motion, and the role of the hidden variable uniquely defining the motion is played by the acceleration, which takes values in a domain depending on the height of the barrier, i.e. on the setting of the measuring instrument.

Consider a time, which following Einstein–Podolsky–Rosen we call T, at which the two particles are outside the interaction region, i.e. have positions $x_1(T) = x_1^* > L$, $x_2(T) = x_2^* < -L$, and assume that the velocities $v_1(T) = v_1^* > 0$, $v_2(T) = v_2^* < 0$ are such that the corresponding initial energies of the two particles belong to the intervals where the nonuniqueness phenomenon occurs, for all the three possible heights of the barrier; assume moreover that the barriers are so far away from the interaction region that the location of the nonuniqueness intervals is practically independent of the precise values of the positions x_1^*, x_2^*. Fix then the heights of the barriers in the following way: choose three positive numbers n_μ ($\mu = 1, 2, 3$), and fix the height of the first barrier in such a way that the allowed values for the acceleration $a_1(T)$ of the first particle are in number of n_μ if the barrier is at μ–th height; analogously for the second particle, having chosen three positive numbers m_ν ($\nu = 1, 2, 3$). Denote by $a_1^{i,\mu}$ ($i = 1, \cdots, n_\mu$) and by $a_2^{j,\nu}$ ($j = 1, \cdots, m_\nu$) the allowed values of the hidden variables $a_1(T)$, $a_2(T)$, when the instruments are in settings μ and ν respectively ($\mu, \nu = 1, 2, 3$).

For a given setting of the barriers, i.e. of the measuring instruments, the physical or Dirac manifold of the two–particle system turns out to be a well defined four–dimensional manifold in the six–dimensional phase space $\mathbf{R}^3 \times \mathbf{R}^3$ with coordinates $(x_1, v_1, a_1, x_2, v_2, a_2)$. Indeed, in the non–interaction region the Dirac manifold is just the product of the two–dimensional Dirac manifolds of the uncoupled particles, while in the interaction region the Dirac manifold is simply defined by prolongation, i.e. by letting the system evolve backward in time according to the coupled dynamics, whose precise definition is not of interest here. Concerning the global Dirac manifold, notice in particular that changing even just one of the barriers produces a change in the complete manifold itself, and that the manifold does not have a product structure in the interaction region, which is the one where the initial data are in principle asssigned.

According to the three possible choices for each of the barriers we have nine distinct Dirac manifolds, say $D_{\mu\nu}$ ($\mu, \nu = 1, 2, 3$), and the section of each such manifold $D_{\mu\nu}$ with the two–dimensional plane $x_1 = x_1^*$, $x_2 = x_2^*$, $v_1 = v_1^*$, $v_2 = v_2^*$ is just a finite set of points, namely the set of points $(x_1^*, v_1^*, a_1^{i,\mu}, x_2^*, v_2^*, a_2^{j,\nu})$, with $i = 1, \cdots, n_\mu$; $j = 1, ..., m_\nu$; $\mu, \nu = 1, 2, 3$. So, for any choice of the heights of the two barriers one has a discrete space of events $\Omega_{\mu\nu}$, i.e. the space of the pairs of allowed initial accelerations $(a_1^{i,\mu}, a_2^{j,\nu})$, whose cardinality depends on the height of both barriers. Now, in order to discuss the outcomes of our *gedankenexperiment*, we have to assign a probability to the initial states, i.e. we have to assign a probability measure $\mathrm{Pr}_{\mu\nu}$ on each space $\Omega_{\mu\nu}$ ($\mu, \nu = 1, 2, 3$). It is clear that in such a way one thus assigns an invariant probability measure on each Dirac manifold $D_{\mu\nu}$, and conversily that every invariant probability measure on $D_{\mu\nu}$ defines a probability measure $\mathrm{Pr}_{\mu\nu}$ on $\Omega_{\mu\nu}$. On the other hand there seems to be no reason to privilege any particular invariant probability measure on $D_{\mu\nu}$, and consequently any particular probability measure on $\Omega_{\mu\nu}$; in particular, as the Dirac manifolds don't have a product structure, there is no reason to privilege the factorized measures, i.e. the measures assigning independent probabilities to the accelerations of the two particles. In consideration of this, we seem to be authorized to assume that all possible choices of the probability measures $\mathrm{Pr}_{\mu\nu}$ on $\Omega_{\mu\nu}$ are on the same footing. In particular, the nine probability measures $\mathrm{Pr}_{\mu\nu}$ (each defined on the corresponding space $\Omega_{\mu\nu}$) can be assigned independently from each other. This is what we do here; further comments comments will be given below.

Now Bell's theorem, which we take in Nelson's version (see [5], page 445), says that, with suitable assumptions of locality to be recalled in the next section: *There do not exist random variables α_μ and β_ν (for $\mu,\ \nu = 1,2,3$) such that α_μ and β_ν is equal ± 1 and*

$$\mathrm{Pr}_{\mu\mu}(\alpha_\mu \beta_\mu = -1) = 1 \tag{2}$$

$$\mathrm{Pr}_{\mu\nu}(\alpha_\mu \beta_\nu = -1) < \frac{1}{2} \quad (\mu \neq \nu) \tag{3}$$

(think of α_μ as the dichotomic variable corresponding to the first particle crossing or not the barrier at μ–th height, and analogously of β_ν for the second particle).

So a violation of Bell's inequalities occurs if one finds random variables α_μ, β_ν satisfying relations (2) and (3). We show now that our dichotomic variables do indeed satisfy them, for certain probability measures $\mathrm{Pr}_{\mu\nu}$. To be concrete, make the following choice: for barriers of the same height (i.e. for $\mu = \nu$) assign zero probability to all events in which both particles are reflected or both particles cross their barriers, so that (2) is satisfied; instead, for barriers of different heights (i.e. for $\mu \neq \nu$) assign arbitrary probabilities to all events of the set in which both particles are reflected or both particles cross their barriers, with the only constraint that their sum be less than 1/2, and distribute arbitrarily the remaining probabilities in the complementary set. It would be a rather simple exercise to take into consideration Bell's inequalities in their usual version involving correlations, and prove that they can be violated too.

Thus the counterexample to Bell's theorem was obtained here in a completely trivial way, just by exploiting the complete arbitrariness of the probability measures $\mathrm{Pr}_{\mu\nu}$. This seems to be in agreement with the quotation from Bell reported at the beginning of the introduction (*it is trivially clear ...*). However, the point we make is that the violation was obtained here for a model which is not a strange *ad hoc* one, but just classical electrodynamics of point particles in the dipole approximation, when due consideration is given to mass renormalization and to its main manifestation, namely the generic occurrence of runaway solutions, which is dealt with through Dirac's nonrunaway condition.

We now add some further comments, trying to emphasize the relevant features of our model wich led to the violation of Bell's inequalities.

1. We start by pointing out that nonlocality comes about in our model in two ways. The first one, which we discuss presently, manifests itself already in the case of just one particle, when the interaction with the measuring instrument is taken into account. This is related to the fact that the domain of definition of the hidden variable depends on the setting of the instrument, no matter how far away it is situated, and is ultimately a consequence of the nonlocal character of Dirac's nonrunaway condition (the Bopp–Haag phenomenon). Then, when the two–particle system is considered, this first nonlocality

property leads to the fact that the probability spaces $\Omega_{\mu\nu}$ themselves depend on the settings of both instruments; so one should expect that the probability distributions of the hidden variables too depend on the settings of both instruments. And this is completely at variance with the hypotheses used by Bell ("... *we supposed that the experimental settings could be changed without changing the probability distribution of the hidden parameter*", see [1], page 154, and note 21). In particular, the fact that the domain of definition of the hidden variable depends on the settings of both instruments is sufficient to completely invalidate the argument by which, following Mermin [20], Nelson showed (see [4] sec. 23, especially page 120) that classical hidden variables should be ruled out.

By the way, it seems to be of interest to point out that the situation described here for classical electrodynamics, namely that the setting of the instruments has a certain "influence" on the observed system, is quite similar to that occurring in quantum mechanics. In the words of Bell: "*Since quantum phenomena indicate that the experimental devices must be regarded as integral parts of the whole experimental situation, not separable from the system being studied, there is no reason to expect that there should be any quantities that can be held fixed as the experiments are changed.*" (see also the interesting remark at page 154 of [1]).

2. Let us now come to the second nonlocality property; this refers altogether to the global two–particle system (each particle being in the presence of the corresponding measuring instrument), and is related to the unfactorization of the initial state. Let us describe this point in a greater detail. As indicated above, when the measuring instruments are in settings μ, ν, one has to consider for the complete system the probability space $\Omega_{\mu\nu}$, a point of which is constituted by the pair of hidden variables (the accelerations at the "initial" time T), and an initial state is just a probability measure $\Pr_{\mu\nu}$ on $\Omega_{\mu\nu}$. Now, there exist first of all the states which are factorized (i.e. assign independent probabilities to each particle's acceleration), and one immediately proves (see the appendix) that for them one has

$$\Pr_{\mu\nu}(\alpha_\mu\beta_\nu = -1) > \frac{1}{2} \ ;$$

thus the violation of Bell's inequalities, as in the example above, can be obtained only for unfactorized states. In other words, the system decouples into two independent subsystems if the initial probability is factorized, while in the opposite case one has a correlation which is essential for the violation of the inequalities. Here too there is a strong analogy with quantum mechanics, where unfactorized states are required to violate Bell's inequalities; the corresponding correlation was discussed by Schroedinger [21], who called it *entanglement*. Now, why should the unfactorized (or entangled) states be preferred, as being the generic ones? To this we can give two answers. The first one is exactly the same given by Schroedinger in the case of quantum mechanics, namely that the initial (i.e. at time T) states are generically unfactorized (or entangled) just because of the previous mutual interaction of the particles. The second answer is related to the fact that, as one immedialtely proves (see again the appendix), the singlet (i.e. satisfying (2)) states which are factorized are necessarily trivial, i.e. are such that each particle either certainly crosses its barrier or is certainly reflected from it; consequently, in a sense "genuine" singlet states are necessarily unfactorized.

4. **Further comments.** So we have shown how nonrelativistic classical electrodynamics of point particles in the dipole approximation in general violates Bell's inequalities. On the other hand the fundamental problem raised by such inequalities is the connection between causality and relativity, and the interesting problem would be to know whether the inequalities are violated also for the relativistic (and nonlinear) version of our model. Now, in order to discuss this point we would first of all need a rigorous deduction of the dynamics of point particles interacting with the electromagnetic field in the relativistic case, which is still lacking. So we limit ourselves to express here our personal conjecture, which is as follows. We think that very probably such a rigorous discussion will eventually confirm the result of Dirac himself, namely that the particle obeys Dirac's relativistic version of the Abraham–Lorentz–Dirac equation. Now, for such an equation the situation with respect to runaway and nonrunaway motions is essentially the same as in the nonrelativistic case. Thus, all the requirements imposed by relativity should already be there, and Dirac's prescription of restricting the phase space to the nonrunaway solutions should be at all compatible with relativity and causality. As a consequence, no substantial changes should occur in the relativistic case, and the situation would be essentially the same as discussed in the present paper. To check whether this is the case or not is a very interesting open problem.

Now we address the following question: if it can be shown that the correct relativistic theory in the point limit is that of Dirac, with the essential ingredient of the nonrunaway prescription, with which form of locality would this be compatible ?

In this connection let us recall the properties of passive and active locality as introduced by Nelson, who verbally describes them as follows ([5], page 446). Let A and B be space–like separated bounded open sets in space–time, and A^+ the future cone of A; define by *slice* an open subset of space–time bounded by two parallel space–like hyperplanes, and let X be a slice disjoint from $A^+ \bigcup B^+$. Passive locality is the property: *if the field is known in the slice X, then an observation in one of A or B gives no additional information about an observation in the other.* Instead, active locality is: *an experiment in A affects the field only in A^+.*

Nelson shows (in the proof of the theorem quoted above) that at least one of the two locality properties has to be abandoned, if Bell's inequalities are to be violated, and says he is inclined to think that passive locality should be abandoned. We are rather inclined to think that neither active nor passive locality are imposed by relativity, at least if one takes for granted that the relativistic version of the Abraham–Lorentz–Dirac equation is correct. This is essentially due to the Bopp–Haag effect. Indeed, consider first active locality, which is in fact concerned with just one particle. This requires that an experiment (i.e. the setting of the instrument) in A affects the field only in its future cone A^+; but this is not the case (i.e. active locality does not hold) with the Abraham–Lorentz–Dirac equation, because the setting of the instrument affects also the past cone A^-, inasmuch as it determines by Dirac's condition, through the initial particle's acceleration, the domain of the possible initial data of the field (such a phenomenon is often referred to as the phenomenon of *preacceleration*). Neither does passive locality hold, again because the domain of definition of the field in $X \bigcap (A^- \bigcap B^-)$ is determined by the setting of both instruments.

These are the reasons that lead us to think that neither active nor passive locality in Nelson's sense should hold in relativistic theories. Which form of locality, in some suitable weak sense, should then be appropriate in relativistic theories is a very interesting question of principle, discussed by many authors (see for example [22], [23], [24]), on which we are unable to say anything conclusive at the moment.

5. **Conclusions.**

In conclusion, we have pointed out that, in classical electrodynamics of point particles in the nonrelativistic and dipole approximation, the setting of the measuring instruments has a certain "influence" on the observed system, inasmuch as it determines the possible range of the parameters playing the role of hidden variables; this is indeed the essence of the Bopp–Haag phenomenon, and is ultimately due to Dirac's nonrunawy condition. Then we have shown how this property leads in a particular model, for some unfactorized initial states, to a violation of Bell's inequalities. Furthermore, we have pointed out that analogous results might be expected to hold in the full relativistic nonlinear version of the model. In such a way we believe we have given a strong indication in favour of the correctness of the idea suggested by Nelson, namely that a nonlocality property suited to violate Bell's inequalities might appear in classical field theories when the regularization cutoffs are removed.

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APPENDIX

We prove here two simple lemmas concerning probabilities for factorized states. The formal description of the situation is the following one, referring to a given setting of the measurement instruments (so that the indices μ, ν will be omitted here). We have two random variables a_1 nad a_2 (in our model, the hidden variables, i.e. the accelerations of the first and of the second particle at the "initial" time T) whose possible values are a_{1i}, $a_{2.j}$, ($i = 1, \cdots, n$, $j = 1, \cdots, m$). The space of the elementary events is the set Ω of pairs (a_{1i}, a_{2j}). A state is a probability measure in Ω, i.e. a probability distribution p_{ij} (with $p_{ij} \geq 0, \sum_{ij} p_{ij} = 1$), and a state is factorized by definition if

$$p_{ij} = p_i q_j \ , \quad (p_i \geq 0 \ , \ q_j \geq 0 \ , \ \sum_i p_i = \sum_j q_j = 1) \ .$$

Let α, β be two dichotomic random variables, i.e. real valued functions on Ω, taking values $+1$ or -1; moreover, let α depend just on a_1 and β depend just on a_2 (in our case, whether the first particles crosses the barrier or not just depends on the value of a_1, and analogously for the second particle), i.e. assume

$$\alpha_{ij} = \alpha_i \ , \quad \beta_{ij} = \beta_j \ , \quad (i = 1, \cdots, n, \quad j = 1, \cdots, m) \ ,$$

Then, denoting by $\Pr(A)$ the probability of an event A, one has

Lemma 1: For factorized states it is

$$\Pr(\alpha\beta = -1) > \frac{1}{2} \ .$$

Proof. One has $\Pr(\alpha\beta = -1) = pq + (1-p)(1-q)$, where $p = \Pr(\alpha = 1)$, and $q = \Pr(\beta = -1)$. One thus has to look for the minimum, in the unit square, of the function $f(p,q) = pq + (1-p)(1-q)$, which is immediately found to be $\frac{1}{2}$.

Consider now a singlet state, i.e. one such that $\Pr(\alpha\beta = -1) = 1$. Say furthermore that a state is trivial if each of the variables α, β takes just one of the two possible values $+1$, -1 (i.e. each particle either certainly crosses the barrier or is certainly reflected from it). Then we have

Lemma 2: Singlet states which are factorized are trivial.
Proof. Let

$$\begin{aligned} &\alpha_i = +1 \quad \text{for} \quad i = 1, \cdots, n^* \ , \qquad \alpha_i = -1 \quad \text{for} \quad i = n^*+1, \cdots, n \ , \\ &\beta_j = +1 \quad \text{for} \quad j = 1, \cdots, m^* \ , \qquad \beta_j = -1 \quad \text{for} \quad j = m^*+1, \cdots, m \ . \end{aligned}$$

Then the singlet condition requires

$$\begin{aligned} p_i q_j &= 0 \quad \text{for} \quad i = 1, \cdots, n^* \ , \ j = 1, \cdots, m^* \\ p_i q_j &= 0 \quad \text{for} \quad i = n^*+1, \cdots, n \ , \ j = m^*+1, \cdots, m. \end{aligned}$$

One of the probabilitities $q_1, \cdots, q_n$ has to be nonvanishing, and assume for example $q_1 \neq 0$. Then necessarily one has $p_i = 0$ for $i = 1, \cdots, n^*$, which means that α takes only the value -1, and consequently β just the value $+1$.