Apartado Postal C-580, Zacatecas 98068, ZAC., México

Internet address: valeri@cantera.reduaz.mx

URL: http://cantera.reduaz.mx/˜ valeri/valeri.htm

PHOTON-NOTOPH EQUATIONS*

Valeri V. Dvoeglazov

(Received 1998)

In the sixties Ogievetskii and Polubarinov proposed the concept of a *notoph*, whose helicity properties are complementary to those of a *photon*. We analyze the theory of antisymmetric tensor fields in the view of the normalization problem. The obtained result is that it is possible to describe both photon and notoph degrees of freedom on the basis of the modified Bargmann-Wigner formalism for the symmetric second-rank spinor. Physical consequences are discussed.

---

*Contributed to "International Workshop on Fundamental Open Problems in Mathematics, Physics and Other Sciences at the Turn of the Millenium", Beijing, China, August 28, 1997 (Corresponding Participant).

In a recent series of the papers [1–5], which are the continuation of the Ahluwalia *et al* work [6–8] we tried to construct a self-consistent theory

of the quantized antisymmetric tensor (AST) field of the second rank and

of the 4-vector field. Previous published works [9–14], as well as textbooks [15–18] can *not* be considered as

the works which solved the main problems, whether the quantized AST field and the quantized 4-vector field are transverse or longitudinal fields (in the sense if the helicity $h = \pm 1$ or $h = 0$)? can the electromagnetic potential be a 4-vector in a quantized theory (cf. [19, p.251])? how should the massless limit be taken? and many other fundamental problems. The most rigorous works are refs. [20–22,19], but it is not easy to extract corresponding answers even from them. A lot of problems of rigorous description of the light is still opened. Ideas of this paper are based on three referee reports from "Foundation of Physics", which were very useful even though critical ones.

First of all, we note after the referee that 1) "...In natural units ($c = \hbar = 1$) ... a lagrangian density, since the action is dimensionless, has dimension of [energy]^4"; 2) One can always renormalize the lagrangian density and "one can obtain the same equations of motion... by substituting $L \to (1/M^N)L$, where M is an arbitrary energy scale", cf. [3]; 3) the right physical dimension of the field strength tensor $F^{\mu\nu}$ is [energy]^2; "the transformation

$F^{\mu\nu} \to (1/2m)F^{\mu\nu}$ [which was regarded in ref. [5]] ... requires a more detailed study ... [because] the transformation above changes its physical dimension: it is not a simple normalization transformation". Furthermore, in the first papers on the notoph [10–12][1] the authors

used the normalization of the 4-vector F^μ field[2] to

---

[1]It is also known as a longitudinal Kalb-Ramond field, but the consideration of Ogievetskii and Polubarinov seems to me to be more rigorous because it permits to study the $m \to 0$ procedure.

[2]It is well

known that it is related to a third-rank antisymmetric field tensor.

$[\text{energy}]^2$ and, hence, the antisymmetric tensor "potentials" $A^{\mu\nu}$, to $[\text{energy}]^1$.

After taking into account these observations let us repeat the procedure of the derivations of the Proca equations from the Bargmann-Wigner equations for a *symmetric* second-rank spinor. We set

$$\Psi_{\{\alpha\beta\}} = (\gamma^\mu R)_{\alpha\beta}(c_a m A_\mu + c_f F_\mu) + (\sigma^{\mu\nu} R)_{\alpha\beta}(c_A m \gamma^5 A_{\mu\nu} + c_F F_{\mu\nu})\,, \quad (1)$$

where

$$R = \begin{pmatrix} i\Theta & 0 \\ 0 & -i\Theta \end{pmatrix} \quad , \quad \Theta = -i\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\,. \quad (2)$$

Matrices γ^μ are chosen in the Weyl representation, i.e., γ^5 is assumed to be diagonal. Constants c_i are some numerical dimensionless coefficients. The reflection operator R has the properties

$$R^T = -R\,, \quad R^\dagger = R = R^{-1}\,, \quad (3a)$$
$$R^{-1}\gamma^5 R = (\gamma^5)^T\,, \quad (3b)$$
$$R^{-1}\gamma^\mu R = -(\gamma^\mu)^T\,, \quad (3c)$$
$$R^{-1}\sigma^{\mu\nu} R = -(\sigma^{\mu\nu})^T\,. \quad (3d)$$

They are necessary for the expansion (1) to be possible in such a form, i.e., in order the $\gamma^\mu R$, $\sigma^{\mu\nu} R$ and $\gamma^5\sigma^{\mu\nu} R$ to be *symmetrical* matrices.

The substitution of the above expansion into the Bargmann-Wigner set [15]

$$[i\gamma^\mu\partial_\mu - m]_{\alpha\beta}\, \Psi_{\{\beta\gamma\}}(x) = 0\,, \quad (4a)$$
$$[i\gamma^\mu\partial_\mu - m]_{\gamma\beta}\, \Psi_{\{\alpha\beta\}}(x) = 0\,. \quad (4b)$$

gives us the new "Proca" equations:

$$c_a m(\partial_\mu A_\nu - \partial_\nu A_\mu) + c_f(\partial_\mu F_\nu - \partial_\nu F_\mu) = i c_A m^2 \epsilon_{\alpha\beta\mu\nu} A^{\alpha\beta} + 2 m c_F F_{\mu\nu} \quad (5a)$$
$$c_a m^2 A_\mu + c_f m F_\mu = i c_A m \epsilon_{\mu\nu\alpha\beta}\partial^\nu A^{\alpha\beta} + 2 c_F \partial^\nu F_{\mu\nu}\,. \quad (5b)$$

In the case $c_a = 1$, $c_F = \frac{1}{2}$ and $c_f = c_A = 0$ they are reduced to the ordinary

Proca equations.[3] In the general case we obtain dynamical equations which connect the photon, the notoph and their potentials. Divergent (in $m \to 0$) parts of field functions and of dynamical variables should be removed by corresponding gauge (or Kalb-Ramond gauge) transformations. It is well known that the notoph massless field is considered to be the pure longitudinal field after one takes into account $\partial_\mu A^{\mu\nu} = 0$. Apart from these dynamical equations we can obtain the set of constraints by means of the subtraction of the equations of the Bargmann-Wigner set (instead of the addition as for (5a,5b)). It reads

$$m c_a \partial^\mu A_\mu + c_f \partial^\mu f_\mu = 0\,, \tag{6a}$$

$$m c_A \partial^\alpha A_{\alpha\mu} + \frac{i}{2} c_F \epsilon_{\alpha\beta\nu\mu} \partial^\alpha F^{\beta\nu} = 0\,. \tag{6b}$$

that suggests $\widetilde{F}^{\mu\nu} \sim imA^{\mu\nu}$ and $f^\mu \sim mA^\mu$, as in [10].

Thus, after the suitable choice of the dimensionless coefficients c_i the lagrangian density for the photon-notoph field can be proposed:

$$\mathcal{L} = \mathcal{L}^{Proca} + \mathcal{L}^{Notoph} = -\frac{1}{8} F_\mu F^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \\ + \frac{m^2}{2} A_\mu A^\mu + \frac{m^2}{4} A_{\mu\nu} A^{\mu\nu}\,, \tag{7}$$

The limit $m \to 0$ may be taken for dynamical variables, in the end of calculations only.

---

[3]We still note that the division by m in the first equation is *not* a well-defined operation in the case if someone is interested in the subsequent limiting procedure $m \to 0$. Probably, in order to avoid this obscure point one may wish to write the Dirac equations in the form $[(i\gamma^\mu \partial_\mu)/m - \mathbb{1}]\,\psi(x) = 0$ which follows straightforwardly in the derivation of the Dirac equation on the basis of the Ryder-Burgard relation [7] and the Wigner rules for boosting the field function from the zero-momentum frame.

Furthermore, it is logical to introduce the normalization scalar field $\varphi(x)$ and consider the expansion:

$$\Psi_{\{\alpha\beta\}} = (\gamma^\mu R)_{\alpha\beta}(\varphi A_\mu) + (\sigma^{\mu\nu} R)_{\alpha\beta} F_{\mu\nu}\,. \tag{8}$$

Then, we arrive at the following set

$$2m F_{\mu\nu} = \varphi(\partial_\mu A_\nu - \partial_\nu A_\mu) + (\partial_\mu \varphi) A_\nu - (\partial_\nu \varphi) A_\mu\,, \tag{9a}$$
$$\partial^\nu F_{\mu\nu} = \frac{m}{2}(\varphi A_\mu)\,, \tag{9b}$$

which in the case of the constant scalar field $\varphi = 2m$ also can be reduced to the set of the Proca equations. The additional constraints are

$$(\partial^\mu \varphi) A_\mu + \varphi(\partial^\mu A_\mu) = 0\,, \tag{10a}$$
$$\partial_\mu \widetilde{F}^{\mu\nu} = 0\,. \tag{10b}$$

At the moment it is not yet obvious how can we account for other equations in the $(1,0)\oplus(0,1)$ representation, e.g. [7b]. One can wish to seek the generalization of the Proca set on the basis of the introduction of two mass parameters m_1 and m_2. But, when we apply the BW procedure to the Dirac equation we cannot obtain new physical content. Another equation in the

$(1/2,0)\oplus(0,1/2)$ representation was obtained in ref. [23]. It has the form:

$$\left[i\gamma^\mu \partial_\mu - m_1 - \gamma^5 m_2\right] \Psi(x) = 0\,. \tag{11}$$

The Bargmann-Wigner procedure for the set of this kind of equations (which include the γ^5 matrix in the mass term) yields:

$$2m_1 F^{\mu\nu} + 2im_2 \widetilde{F}^{\mu\nu} = \varphi(\partial^\mu A^\nu - \partial^\nu A^\mu) + (\partial^\mu \varphi) A^\nu - (\partial^\nu \varphi) A^\mu\,, \tag{12a}$$
$$\partial^\nu F_{\mu\nu} = \frac{m_1}{2}(\varphi A_\mu) \tag{12b}$$

with the constraints

$$(\partial^\mu \varphi) A_\mu + \varphi(\partial^\mu A_\mu) = 0 \tag{13a}$$
$$\partial^\nu \widetilde{F}_{\mu\nu} = \frac{im_2}{2}(\varphi A_\mu)\,. \tag{13b}$$

The equality of mass factors ($m_1^{(1)} = m_1^{(2)}$ and $m_2^{(1)} = m_2^{(2)}$) in the set of the Dirac equations is obtained in the process of calculations as necessary conditions.

In fact, the results of this paper develop the old results of ref. [10]. We returned to this question due to recent interpretational controversies in claims of experimental observations of

the objects $\mathbf{E} \times \mathbf{E}^*$ and $\mathbf{A} \times \mathbf{A}^*$ in the non-linear optics [24].[4] In this connection one can consider that $\sim \mathbf{A} \times \mathbf{A}^*$ term can be regarded as the part of antisymmetric tensor potential and $\sim \mathbf{B} \times \mathbf{B}^*$, as the part of the 4-vector field (cf. the

[4]One can wish to

compare the *notoph* concept with theoretical works of M. W. Evans *et al.* [e.g., *The Enigmatic Photon*, Vols. I-IV, Kluwer Academic Publishers, 1994-97]. It is easy to see from the formulas (9,10) of ref. [10] that the Evans' proposal is *not any novelty.* The longitudinal field constructed from *polarization vectors* is nothing more than the *notoph* antisymmetric tensor potentials [10, Eq.(10)]. On the other hand, in the Evans' $\mathbf{B}$ cyclic relations we found that $\mathbf{B}^{(3)}$ field is *not* a part

of the *antisymmetric* tensor due to different Lorentz transformations [4]. While Evans refers often to $\mathbf{A} \times \mathbf{A}^*$ and $\mathbf{E} \times \mathbf{E}^*$ (or $\sim \mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$) as the same entities in *all cases*, this is not so. These Evans' claims are contradictory each other in the view of the Lorentz symmetry. While, in my opinion, Evans' works on the theory of longitudinal modes of electromagnetism are full of errors, the old work of V. I. Ogievetskii and I. V. Polubarinov shows that one can work rigorously with these concepts.

formulas (19a-c) in ref. [5]). According to [10, Eqs.(9,10)] we proceed in the construction of the "potentials" for the notoph as follows:

$$A_{\mu\nu}(\mathbf{p}) = N\left[\epsilon_\mu^{(1)}(\mathbf{p})\epsilon_\nu^{(2)}(\mathbf{p}) - \epsilon_\nu^{(1)}(\mathbf{p})\epsilon_\mu^{(2)}(\mathbf{p})\right] \tag{14}$$

On using explicit forms for the polarization vectors in the momentum space (e.g., refs. [19] and [5, formulas(15a,b)]) one obtains

$$A^{\mu\nu}(\mathbf{p}) = \frac{iN^2}{m}$$

-m - p$\frac{{}_r p_l}{p_0+m0-\frac{p_1p_3}{p_0+m}0-\frac{p_2p_3}{p_0+m}\frac{p_1p_3}{p_0+m}0}$,(15)

i.e., it coincides with the longitudinal components of the antisymmetric

tensor obtained in refs. [7a,Eqs.(2.14,2.17)] and [5, Eqs.(17b,18b)] within the normalization and different forms of the spin basis. The longitudinal states reduce to zero in the massless case under appropriate choice of the normalization and only if a $j = 1$ particle moves along with the third axis OZ. It is also useful to compare Eq. (15) with the formula (B2) in ref. [8] in order to realize the correct procedure for taking the massless limit.

Next, the Tam-Happer experiments [25] did not find satisfactory explanation in the framework of the ordinary QED (at least, their explanation is complicated by huge technical calculations).

On the other hand, in ref. [26] the very interesting model has

been proposed. It is based on gauging the Dirac field on using the coordinate-dependent parameters $\alpha_{\mu\nu}(x)$ in

$$\psi(x) \to \psi'(x') = \Omega\psi(x)\ , \quad \Omega = \exp\left[\frac{i}{2}\sigma^{\mu\nu}\alpha_{\mu\nu}(x)\right]\,. \tag{16}$$

and, thus, the second "photon" was introduced. The compensating 24-component (in general) field $B_{\mu,\nu\lambda}$ reduces to the 4-vector field as follows (the notation of [26] is used

here):

$$B_{\mu,\nu\lambda} = \frac{1}{4}\epsilon_{\mu\nu\lambda\sigma}a_\sigma(x)\,. \tag{17}$$

As readily seen after the comparison of these formulas with those of refs. [10–12], the second photon is nothing more than the Ogievetskii-Polubarinov *notoph* within the normalization. Parity properties (as well as its behavior in the massless limit) are dependent not only on the explicit forms of the momentum-space field functions of the $(1/2, 1/2)$ representation, *but* also on the properties of corresponding creation/annihilation operators. Helicity properties depend on the normalization.

Finally, in my opinion, the recent theoretical concepts of *action-at-a-distance* reposed by A. E. Chubykalo *et al.*, e.g., ref. [27] and the concept of flavour-oscillation clocks governed by scalar gravitational potential [28] should find connections with the longitudinal quantum fields.

ACKNOWLEDGMENTS

This paper has been inspired by remarks of the referees of IJMPA (1994) and *Foundation of Physics* (1998), by the papers of Prof. D. V. Ahluwalia and his predecessors who worked with the theories of antisymmetric tensor fields. I am obliged to Profs. L. V. Avdeev, A. E. Chubykalo, S. Esposito, Y. S. Kim, A. F. Pashkov, S. Roy and Yu. F. Smirnov for illuminating discussions.

I am grateful to Prof. R. Santilli for his kind invitation to write the paper for the present Workshop.

I am grateful to Zacatecas University for awarding a professorship. This work has been supported in part by the Mexican Sistema Nacional de Investigadores and by the CONACyT, México under the research project 0270P-E.

---

[1] V. V. Dvoeglazov, Helv. Phys. Acta **70** (1997) 677, ibid. 686, ibid. 697.

[2] V. V. Dvoeglazov, *About the Claimed 'Longitudinal Nature' of the Antisymmetric Tensor Field After Quantization.* Preprint EFUAZ FT-95-16-REV (hep-th/9604148), Zacatecas, Jan. 1996.

[3] V. V. Dvoeglazov, *Questions in the Theory of the* $(1, 0) \oplus (0, 1)$ *Quantized Fields.* Preprint EFUAZ FT-96-31, (hep-th/9611068), Aug. 1996.

[4] V. V. Dvoeglazov, Found. Phys. Lett. **10** (1997) 383.

[5] V. V. Dvoeglazov, *On the Importance of the Normalization.* Preprint EFUAZ FT-96-39-REV (hep-th/9712036), Zacatecas,
Nov. 1997.

[6] D. V. Ahluwalia and D. J. Ernst, Mod. Phys. Lett. A**7**
(1992) 1967.

[7] D. V. Ahluwalia and D. J. Ernst, Int. J. Mod. Phys. E**2** (1993) 397; D. V. Ahluwalia, M. B. Johnson and T. Goldman, Phys. Lett. B**316** (1993) 102.

[8] D. V. Ahluwalia and M. Sawicki, Phys. Rev. D**47** (1993) 5161.

[9] Compare claims of the old papers: E. Durandin and A. Erschow, Phys. Z. Sowjet. **12** (1937) 466; F. Belinfante, Physica
6 (1939) 887.

[10] V. I. Ogievetskii and I. V. Polubarinov, Sov. J. Nucl. Phys. **4** (1967) 156.

[11] K. Hayashi, Phys. Lett. B**44** (1973) 497.

[12] M. Kalb and P. Ramond, Phys. Rev. D**9** (1974) 2273.

[13] H. C. Ohanian, Am. J. Phys. **54** (1986) 500.

[14] L. V. Avdeev and M. V. Chizhov, Phys. Lett. B**321** (1994) 212; *A queer reduction of degrees of freedom.* Preprint JINR E2-94-263 (hep-th/9407067), Dubna: JINR, 1994.

[15] D. Luriè, *Particles and Fields.* (Interscience Publishers, 1968).

[16] N. N. Bogoliubov and D. V. Shirkov, *Introduction to*
the Theory of Quantized Fields. (Moscow, Nauka, 1973).

[17] Yu. V. Novozhilov, *Introduction to Elementary Particle Theory.* (Pergamon Press, Oxford, 1975).

[18] C. Itzykson and J.-B. Zuber, *Quantum Field Theory.* (McGraw-Hill Book Co. New York, 1980).

[19] S. Weinberg, *The Quantum Theory of Fields. Vol. I. Foundations.* (Cambridge University Press, 1995), Chapter 5.

[20] V. Bargmann and E. P. Wigner, Proc. Natl. Acad. Sci. (USA) **34** (1948) 211.

[21] S. Weinberg, Phys. Rev. B**134** (1964) 882

[22] D. Han, Y. S. Kim and D. Son, Phys. Lett. **131**B (1983) 327, see also Y. S. Kim, Int. J. Mod. Phys. A**12** (1997) 71.

[23] See, for instance, N. D. Sen Gupta, Nucl. Phys. B**4** (1967) 147; A. Raspini, Fizika B**5** (1996) 159.

[24] J. P. van der Ziel, P. S. Pershan and L. D. Malmstrom, Phys. Rev. Lett. **15** (1965) 190; M. W. Evans, Physica B**182** (1992) 237; L. D. Barron, ibid. **190** (1993) 307; A. Lakhtakia, ibid. **191** (1993) 362; D. M. Grimes, ibid. **191** (1993) 367; M. Y. A. Raja *et al.*, Appl. Phys. B**64** (1997) 79 .

[25] A. C. Tam and W. Happer, Phys. Rev. Lett. **38** (1977) 278.

[26] P. C. Naik and T. Pradhan, J. Phys. A**14** (1981) 2795; T. Pradhan, R. P. Malik and P. C. Naik, Pramana J. Phys. **24** (1985) 77.

[27] A. E. Chubykalo and R. Smirnov-Rueda, Mod. Phys. Lett. A **12** (1997) 1.

[28] D. V. Ahluwalia and C. Burgard, Phys. Rev. D**57** (1998) No. 8; D. V. Ahluwalia and C. Burgard, Gen. Rel. Grav. **28** (1996) 1161; D. V. Ahluwalia, ibid. **29** (1997) 149.