Time and temperature dependent correlation functions of the 1D impenetrable electron gas

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Abstract

We consider the one-dimensional delta-interacting electron gas in the case of infinite repulsion. We use determinant representations to study the long time, large distance asymptotics of correlation functions of local fields in the gas phase. We derive differential equations which drive the correlation functions. Using a related Riemann-Hilbert problem we obtain formulae for the asymptotics of the correlation functions, which are valid at all finite temperatures. At low temperatures these formulae lead to explicit asymptotic expressions for the correlation functions, which describe power law behavior and exponential decay as functions of temperature, magnetic field and chemical potential.

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1 Introduction

We consider the electron gas with delta interaction in one space and one time dimension. It can be described by canonical Fermi fields $\psi_\alpha(x)$ with canonical equal time anti commutation relations

$$\psi_\alpha(x)\psi^\dagger_\beta(y) + \psi^\dagger_\beta(y)\psi_\alpha(x) = \delta^\alpha_\beta \delta(x-y) \quad . \tag{1}$$

The spin index α runs through two values, $\alpha = \uparrow, \downarrow$. The Hamiltonian of the model is

$$H = \int_{-\infty}^{\infty} dx \left\{ \partial_x \psi^\dagger_\alpha \partial_x \psi_\alpha + c : \left(\psi^\dagger_\alpha \psi_\alpha\right)^2 : - \mu\, \psi^\dagger_\alpha \psi_\alpha + B(\psi^\dagger \sigma^z \psi) \right\} \quad . \tag{2}$$

Here μ is the chemical potential, and B is the magnetic field. σ^z is a Pauli matrix, and c is the coupling constant. In this paper we shall consider the case of infinite repulsion $c = \infty$. We shall also limit ourselves to moderate densities, $\mu < -|B|$. We call the phase determined by this condition the electron gas phase.

The model can be solved exactly [1]. Its thermodynamics was described in [2]. The pressure

$$P = \frac{T}{2\pi} \int_{-\infty}^{\infty} dk \ln \left(1 + e^{(\mu+B-k^2)/T} + e^{(\mu-B-k^2)/T}\right) \quad . \tag{3}$$

serves as thermodynamic potential. T is the temperature. The number of up-spin particles and the number of down-spin particles are separately conserved. The densities $D_\uparrow$ of up-spin electrons and $D_\downarrow$ of down-spin electrons are obtained as

$$D_\uparrow = \frac{\partial P}{\partial(\mu - B)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \, \frac{e^{-B/T}}{2\cosh(B/T) + e^{(k^2-\mu)/T}} \quad , \tag{4}$$

$$D_\downarrow = \frac{\partial P}{\partial(\mu + B)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \, \frac{e^{B/T}}{2\cosh(B/T) + e^{(k^2-\mu)/T}} \quad . \tag{5}$$

It is obvious from these expressions that they satisfy the relation

$$D_\downarrow(B) = D_\uparrow(-B) \quad . \tag{6}$$

The density D of the gas is

$$D = D_\uparrow + D_\downarrow \quad . \tag{7}$$

Note that the density vanishes exponentially at small temperatures,

$$D \; = \; \begin{cases} \sqrt{\frac{T}{\pi}}\, e^{\mu/T} & \text{if} \quad B = 0 \quad , \\ \frac{1}{2}\sqrt{\frac{T}{\pi}}\, e^{(\mu+|B|)/T} & \phantom{\text{if}} \quad B \neq 0 \quad . \end{cases} \tag{8}$$

At very low temperatures density and pressure are related by the ideal gas law,

$$P = DT \quad . \tag{9}$$

This is one of the reasons why we call the phase determined by $\mu < -|B|$ the electron gas phase. For larger temperatures there are corrections to the ideal gas law.

In this article we study the two-point correlation functions $G^{\pm}_{\alpha}(x,t)$, $\alpha = \uparrow$, $\downarrow$, of local fields. We shall follow the approach of the book [3] to correlation functions of solvable models. The definitions of $G^{\pm}_{\alpha}(x,t)$ are given in the next section, where we remind a determinant representation for $G^{\pm}_{\alpha}(x,t)$, [4, 5], which is the starting point of our considerations. The correlation functions $G^{\pm}_{\alpha}(x,t)$ are not elementary functions. They are determined by a pair of coupled nonlinear differential partial equations [6] which is derived in section 3. In section 4 we shall formulate a corresponding Riemann-Hilbert problem [7, 8, 9]. It will fix the solution of the differential equations, which is relevant for the calculation of the correlation functions. Furthermore, the asymptotic solution of the Riemann-Hilbert problem for large x and t (x/t fixed) can be obtained by reducing it to a known form [10, 11, 12, 13, 14, 15]. This is done in section 5. Using the asymptotic solution of the Riemann-Hilbert problem and the differential equations we shall obtain the asymptotics of the correlation functions $G^{\pm}_{\alpha}(x,t)$ in terms of a contour integral in section 6. In section 7 we shall present explicit asymptotic expressions for $G^{\pm}_{\alpha}(x,t)$ at low temperatures. The asymptotic expressions for the correlation functions $G^{\pm}_{\alpha}(x,t)$ at low temperatures are products of correlation functions of free fermions and corrections caused by the interaction. The leading factors of the correction terms describe exponential decay. The next to leading factors are power law corrections. The leading factor of the asymptotics of the spin-up correlation function $G^{+}_{\uparrow}(x,t)$, for instance, is

$$G^{+}_{\uparrow}(x,t) = e^{\mathrm{i}t(\mu-B)} e^{\mathrm{i}x^2/4t} e^{-xD_{\downarrow}} \quad . \tag{10}$$

The factor $e^{\mathrm{i}t(\mu-B)}e^{\mathrm{i}x^2/4t}$ is the exponential part of the free fermionic correlation function. The factor $e^{-xD_\downarrow}$ appears due to the interaction. The explicit form of the power law correction as a function of temperature, chemical potential and magnetic field will be given in section 7 (see eqs. (133), (132), (135), (128)). The occurrence of the density of down-spin electrons in equation (10) has a clear physical interpretation. Due to the Pauli principle and the locality of the interaction, up-spin electrons are scattered only by down-spin electrons. $D_\downarrow$ appears as the reciprocal of the correlation length in eq. (10). The higher the density of down-spin electrons, the more often an up-spin electron is scattered and the smaller is the correlation length.

2 A determinant representation for correlation functions

We shall consider the following two-point correlation functions of local fields,

$$G^+_\alpha(x,t) \;=\; \frac{\mathrm{tr}\left(e^{-H/T}\,\psi_\alpha(x,t)\psi^\dagger_\alpha(0,0)\right)}{\mathrm{tr}\left(e^{-H/T}\right)} \quad , \tag{11}$$

$$G^-_\alpha(x,t) \;=\; \frac{\mathrm{tr}\left(e^{-H/T}\,\psi^\dagger_\alpha(x,t)\psi_\alpha(0,0)\right)}{\mathrm{tr}\left(e^{-H/T}\right)} \quad , \tag{12}$$

where $\alpha = \uparrow, \downarrow$. These correlation functions depend not only on space and time variables x and t, but also on the temperature T, the chemical potential μ (or the density D) and the magnetic field B. Due to the invariance of the Hamiltonian (2) under the transformation $\uparrow\rightleftharpoons\downarrow$, $B \rightleftharpoons -B$, we have the identity

$$G^\pm_\downarrow(x,t) = G^\pm_\uparrow(x,t)\Big|_{B\rightleftharpoons -B} \quad . \tag{13}$$

Hence, we can restrict our attention in the following to the correlation functions $G^\pm_\uparrow(x,t)$. These correlation functions were represented by means of determinants of integral operators [4, 5]. Now we shall remind these formulae:

$$G^+_\uparrow(x,t) \;=\; \frac{-e^{\mathrm{i}t(\mu-B)}}{2\pi}\int_{-\pi}^{\pi} d\eta\,\frac{F(\gamma,\eta)}{1-\cos(\eta)}\,b_{++}\mathrm{det}\left(\hat I + \gamma\hat V\right) \quad , \tag{14}$$

$$G_{\uparrow}^{-}(x,t) \;=\; \frac{e^{-\mathrm{i}t(\mu-B)}}{4\pi\gamma}\int_{-\pi}^{\pi} d\eta\, F(\gamma,\eta) B_{--}\det\left(\hat{I}+\gamma\hat{V}\right) \quad . \tag{15}$$

It will take us some time to define notations. First of all

$$F(\gamma,\eta) = 1 + \frac{e^{\mathrm{i}\eta}}{\gamma - e^{\mathrm{i}\eta}} + \frac{e^{-\mathrm{i}\eta}}{\gamma - e^{-\mathrm{i}\eta}} \tag{16}$$

and

$$\gamma = 1 + e^{2B/T} \quad . \tag{17}$$

$\hat{V}$ is an integral operator with a kernel $V(k,p)$. It can be represented in the "standard form" (see page 316 of the book [3])

$$\gamma V(k,p) = \frac{e_{+}(k)e_{-}(p) - e_{+}(p)e_{-}(k)}{k-p} \quad . \tag{18}$$

Here

$$e_{-}(k) \;=\; \sqrt{\frac{\gamma\vartheta(k)}{\pi}}\, e^{\tau(k)/2} \quad , \tag{19}$$

$$e_{+}(k) \;=\; \frac{1}{2}\sqrt{\frac{\gamma\vartheta(k)}{\pi}}\, e^{-\tau(k)/2}\left\{(1-\cos(\eta))e^{\tau(k)}E(k) + \sin(\eta)\right\} \quad , \tag{20}$$

$$\gamma\vartheta(k) \;=\; \frac{2\cosh(B/T)}{2\cosh(B/T) + e^{(k^2-\mu)/T}} \quad , \tag{21}$$

$$\tau(k) \;=\; \mathrm{i}k^2 t - \mathrm{i}kx \tag{22}$$

and

$$E(k) = \text{p.v.}\int_{-\infty}^{\infty} dp\, \frac{e^{-\tau(p)}}{\pi(p-k)} \quad . \tag{23}$$

The integral operator (18) closely reminds an integral operator which appears in the theory of correlation functions of the impenetrable Bose gas (cf. page 346 of [3]). The inverse of $\left(\hat{I}+\gamma\hat{V}\right)$ can be defined in the following way,

$$\left(\hat{I}+\gamma\hat{V}\right)\left(\hat{I}-\gamma\hat{R}\right) = \hat{I} \quad . \tag{24}$$

The resolvent $\hat{R}$ is an integral operator with kernel $R(k,p)$. It can be represented in the "standard form",

$$\gamma R(k,p) = \frac{f_{+}(k)f_{-}(p) - f_{+}(p)f_{-}(k)}{k-p} \quad , \tag{25}$$

where the functions $f_\pm(k)$ can be defined as solutions of the integral equations

$$f_\pm(k) + \int_{-\infty}^{\infty} dp\, \gamma V(k,p) f_\pm(p) = e_\pm(k) \quad . \tag{26}$$

This theorem is proven on page 318 of the book [3]. We should mention that both kernels $V(k,p)$ and $R(k,p)$ are symmetric,

$$V(k,p) = V(p,k) \quad , \quad R(k,p) = R(p,k) \quad . \tag{27}$$

At this point it is convenient to define potentials B_{ab} and C_{ab},

$$B_{ab} = \int_{-\infty}^{\infty} dk\, e_a(k) f_b(k) \quad , \quad C_{ab} = \int_{-\infty}^{\infty} dk\, k e_a(k) f_b(k) \quad . \tag{28}$$

Here both indices a and b run through two values, $a,b = \pm$. For example, the factor B_{--} in (15) is defined as

$$B_{--} = \int_{-\infty}^{\infty} dk\, e_-(k) f_-(k) \quad . \tag{29}$$

In order to define b_{++} we need the function

$$G(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk\, e^{-\tau(k)} = \frac{e^{-\mathrm{i}\pi/4}}{2\sqrt{\pi t}} e^{\mathrm{i}x^2/4t} \quad . \tag{30}$$

The factor b_{++} in (14) is defined as

$$b_{++} = B_{++} - (1-\cos(\eta)) G(x,t) = \int_{-\infty}^{\infty} dk\, e_+(k) f_+(k) - (1-\cos(\eta)) G(x,t) \quad . \tag{31}$$

In such a way we accomplished the description of the determinant representation of the correlation functions (14), (15). It reminds the determinant representation for correlation functions of the impenetrable Bose gas (see page 346 of the book [3]). Let us also mention that the symmetry of $V(k,p)$ leads to $B_{+-} = B_{-+}$.

Let us discuss the factor $1-\cos(\eta)$ in the denominator of (14). Let us show that there is no singularity at $\eta = 0$. Indeed, $e_+(k)$, eq. (20), is proportional to η as $\eta \to 0$. Hence $b_{++} \sim \eta^2$, and there is no singularity. Let us change the integration variable to $z = e^{\mathrm{i}\eta}$. Then

$$G^+_\uparrow(x,t) = \frac{e^{\mathrm{i}t(\mu-B)}}{\pi \mathrm{i}} \oint dz\, \frac{F(z)}{(z-1)^2} b_{++}(z) \det\left(\hat{I} + \gamma \hat{V}\right)(z) \quad , \tag{32}$$

$$G^-_\uparrow(x,t) = \frac{e^{-\mathrm{i}t(\mu-B)}}{4\pi \mathrm{i}\gamma} \oint dz\, \frac{F(z)}{z} B_{--}(z) \det\left(\hat{I} + \gamma \hat{V}\right)(z) \quad . \tag{33}$$

Here the contour of integration can be chosen as a circle of radius a little less than 1, and

$$F(z) = 1 + \frac{z}{\gamma - z} + \frac{1}{\gamma z - 1} \quad . \tag{34}$$

3 Differential equations

Starting from a determinant representation for a quantum correlation function it is possible to derive differential equations which drive this correlation function (see [6] and chapter XIV of the book [3]). In our specific case calculations similar to the ones presented on pages 345 - 353 of the book [3] lead us to the following differential equations,

$$\begin{aligned} -\,\mathrm{i}\,\frac{\partial b_{++}}{\partial t} &= \frac{\partial^2 b_{++}}{\partial x^2} + 2b_{++}^2 B_{--} \quad , \\ \mathrm{i}\,\frac{\partial B_{--}}{\partial t} &= \frac{\partial^2 B_{--}}{\partial x^2} + 2b_{++}B_{--}^2 \quad . \end{aligned} \tag{35}$$

We should notice that a change of variables,

$$x = -2\tilde{x} \quad , \quad t = 2\tilde{t} \quad , \tag{36}$$

leads to coincidence of our system of eqs. (35) with the system (5.47) on page 344 of the book. The system of equations (35) is called separated nonlinear Schrödinger equation. It is completely integrable. It has many solutions. The modern way to fix a solution of the system (35) is to formulate a corresponding Riemann-Hilbert problem (see chapter XV of the book [3]). We shall formulate this Riemann-Hilbert problem in the next section. We will be able to solve the Riemann-Hilbert problem asymptotically as $x \to \infty$ and $t \to \infty$ (x/t fixed). This will give us asymptotic expressions for b_{++} and B_{--}. The solution of the Riemann-Hilbert problem will also give us asymptotic expressions for the other potentials B_{ab} and C_{ab} (28).

Let us now explain how to express $\det(\hat{I}+\gamma\hat{V})$ in terms of these potentials. The logarithmic derivatives of the determinant can be expressed in terms of B and C,

$$\begin{aligned} \partial_x \ln\det\left(\hat{I} + \gamma\hat{V}\right) &= \mathrm{i}B_{+-} = \mathrm{i}B_{-+} \quad , \\ \partial_t \ln\det\left(\hat{I} + \gamma\hat{V}\right) &= -\mathrm{i}(C_{+-} + C_{-+} + (1 - \cos(\eta))GB_{--}) \quad . \end{aligned} \tag{37}$$

We shall also mention several identities, which will be useful for further calculations:

$$\partial_x B_{+-} = -\mathrm{i} B_{--} b_{++} \quad , \tag{38}$$

$$C_{--} = \mathrm{i}\partial_x B_{--} - B_{--} B_{+-} \quad , \tag{39}$$

$$C_{++} = -\mathrm{i}\partial_x B_{++} - 2(1-\cos(\eta)) G B_{+-} + B_{+-} B_{++} \quad , \tag{40}$$

$$C_{+-} - C_{-+} = B_{+-}^2 - B_{++} B_{--} \quad . \tag{41}$$

In the next section we shall introduce a Riemann-Hilbert problem and explain how one can express the potentials B_{ab} and C_{ab} in terms of its solution $\chi(k)$.

4 Riemann-Hilbert problem

The modern way to solve completely integrable differential equations is by means of an equivalent Riemann-Hilbert problem [7, 8, 9]. Solving a Riemann-Hilbert problem amounts to constructing a piecewise analytic matrix-function. Let us formulate the Riemann-Hilbert problem connected to the separated nonlinear Schrödinger equation (35). Let us introduce a 2×2-matrix $\chi(k)$. The matrix $\chi(k)$ should be analytic as a function of k in the upper half plane, $\mathrm{Im}(k) > 0$, and in the lower half plane, $\mathrm{Im}(k) < 0$. In general, it has different limits $\chi_+(k)$ as k approaches the real axis from above and $\chi_-(k)$ as k approaches the real axis from below. These limits are related by means of a conjugation matrix $G(k)$,

$$\chi_-(k) = \chi_+(k) G(k) \quad \text{for } \mathrm{Im}(k) = 0 \quad . \tag{42}$$

The conjugation matrix $G(k)$ is of dimension two. It is defined only on the real axis. The matrix $\chi(k)$ should also satisfy the normalization condition

$$\lim_{k \to \infty} \chi(k) = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad . \tag{43}$$

The problem is to find such a matrix $\chi(k)$. In order to relate $\chi(k)$ to the integral operator (18) we shall use a conjugation matrix $G(k)$ of the special form

$$G(k) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + 2\pi\mathrm{i} \begin{pmatrix} -e_-(k)e_+(k) & e_+^2(k) \\ -e_-^2(k) & e_+(k)e_-(k) \end{pmatrix} \quad . \tag{44}$$

The Riemann-Hilbert problem with this form of conjugation matrix was studied in detail in chapter XV of the book [3]. Let us remind some fundamental facts:

One can prove that

$$\begin{pmatrix} f_+(k) \\ f_-(k) \end{pmatrix} = \chi(k) \begin{pmatrix} e_+(k) \\ e_-(k) \end{pmatrix} \quad . \tag{45}$$

The functions $f_\pm(k)$ were defined by the integral equations (26). On the other hand, the Riemann-Hilbert problem can be solved by means of these functions

$$\chi(k) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \int_{-\infty}^{\infty} \frac{dp}{p-k} \left(\begin{array}{cc} e_-(p)f_+(p) & -e_+(p)f_+(p) \\ e_-(p)f_-(p) & -e_+(p)f_-(p) \end{array} \right) \quad . \tag{46}$$

In order to express the potentials B_{ab} and C_{ab} in terms of $\chi(k)$, let us study the asymptotics of $\chi(k)$ as $k \to \infty$,

$$\chi(k) = I + \frac{\psi_1}{k} + \frac{\psi_2}{k^2} + \mathcal{O}\left(\frac{1}{k^3}\right) \quad . \tag{47}$$

Comparing (46), (47) and (28) we obtain

$$\psi_1 = \begin{pmatrix} -B_{-+} & B_{++} \\ -B_{--} & B_{+-} \end{pmatrix} \quad , \quad \psi_2 = \begin{pmatrix} -C_{-+} & C_{++} \\ -C_{--} & C_{+-} \end{pmatrix} \quad . \tag{48}$$

In such a way we have formulated the Riemann-Hilbert problem and related it to the integral operator $\hat{V}$ and to the correlation functions $G_\uparrow^+(x,t)$ and $G_\uparrow^-(x,t)$. Next we shall solve the Riemann-Hilbert problem at large space time separation $x \to \infty$, $t \to \infty$, x/t fixed. Then we shall calculate the potentials B_{ab} and C_{ab} from (47). The knowledge of the potentials will permit us to evaluate $\det(\hat{I} + \gamma\hat{V})$, see (37). This will give us an asymptotic expression for the correlation functions (14), (15). We also want to note that

$$\det G(k) = \det \chi(k) = 1 \quad . \tag{49}$$

5 Asymptotic solution of the Riemann-Hilbert problem

In order to solve the Riemann-Hilbert problem asymptotically [7, 10, 11, 12, 13, 14, 15], we shall reformulate it twice. This will bring it into the canonical form, which was solved by S. V. Manakov [10, 11].

First reformulation: We want to simplify the conjugation matrix $G(k)$. Let us define a matrix

$$\chi_0(k) = \begin{pmatrix} 1 & -(1-\cos(\eta))\int_{-\infty}^{\infty} \frac{dp\, e^{-\tau(p)}}{2\pi(p-k)} \\ 0 & 1 \end{pmatrix} \quad . \tag{50}$$

Here $\tau(p) = \mathrm{i}p^2 t - \mathrm{i}px$. The matrix $\chi_0(k)$ is analytic for $\mathrm{Im}(k) > 0$ and for $\mathrm{Im}(k) < 0$. At large k it approaches the unit matrix as a Taylor series in $1/k$. Now, instead of the unknown matrix function $\chi(k)$, let us introduce a new unknown matrix function $\tilde{\chi}(k)$,

$$\chi(k) = \tilde{\chi}(k)\chi_0(k) \quad . \tag{51}$$

Let us formulate the Riemann-Hilbert problem for $\tilde{\chi}(k)$.

(a) $\tilde{\chi}(k)$ is analytic for $\mathrm{Im}(k) > 0$ and for $\mathrm{Im}(k) < 0$.

(b) As $k \to \infty$

$$\tilde{\chi}(k) = I + \frac{\tilde{\chi}_1}{k} + \frac{\tilde{\chi}_2}{k^2} + \mathcal{O}\left(\frac{1}{k^3}\right) \quad . \tag{52}$$

(c) $\tilde{\chi}(k)$ has a discontinuity across the real axis, which is described by a conjugation matrix $\tilde{G}(k)$,

$$\tilde{\chi}_-(k) = \tilde{\chi}_+(k)\tilde{G}(k) \quad \text{for } \mathrm{Im}(k) = 0 \quad . \tag{53}$$

Let us calculate

$$\tilde{G}(k) = \begin{pmatrix} \tilde{G}_{11}(k) & \tilde{G}_{12}(k) \\ \tilde{G}_{21}(k) & \tilde{G}_{22}(k) \end{pmatrix} = \chi_0^+(k)G(k)(\chi_0^-(k))^{-1} \quad . \tag{54}$$

Substituting (44) and (50) into (54) we obtain

$$\begin{aligned}
\tilde{G}_{11}(k) &= 1 + \gamma\vartheta(k)(e^{-\mathrm{i}\eta} - 1)\,, & \tilde{G}_{12}(k) &= -\mathrm{i}(1 - \cos(\eta))e^{-\tau(k)}(1 - \gamma\vartheta(k))\,, \\
\tilde{G}_{21}(k) &= -2\mathrm{i}\gamma\vartheta(k)e^{\tau(k)} & , \tilde{G}_{22}(k) &= 1 + \gamma\vartheta(k)(e^{\mathrm{i}\eta} - 1)\,,
\end{aligned} \tag{55}$$

where

$$\gamma\vartheta(k) = \frac{2\cosh(B/T)}{2\cosh(B/T) + e^{(k^2-\mu)/T}} \quad . \tag{56}$$

Now the conjugation matrix is an elementary function of k, $\det \tilde{G}(k) = 1$ and $\det \tilde{\chi}(k) = 1$.

Using (50) and (51) we can also evaluate the coefficients of the expansion (52) of $\tilde{\chi}(k)$ for large k,

$$\tilde{\chi}_1 = \begin{pmatrix} -B_{-+} & b_{++} \\ -B_{--} & B_{+-} \end{pmatrix} \tag{57}$$

and

$$\tilde{\chi}_2 = \begin{pmatrix} -C_{-+} & C_{++} + (1 - \cos(\eta))(\mathrm{i}\partial_x G + G B_{+-}) \\ -C_{--} & C_{+-} + (1 - \cos(\eta)) G B_{--} \end{pmatrix} \quad . \tag{58}$$

Let us remind that

$$G(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, e^{-\tau(k)} = \frac{e^{-\mathrm{i}\pi/4}}{2\sqrt{\pi t}} e^{\mathrm{i}x^2/4t} \quad . \tag{59}$$

In order to prepare for the second step let us introduce a function $\alpha(k)$,

$$\alpha(k) = \exp\left\{ \frac{\mathrm{i}}{2\pi}\int_{-\infty}^{\infty} \frac{dp}{p-k} \ln\left(1 + \gamma\vartheta(p)(e^{-\mathrm{i}\eta} - 1)\right)\right\} \quad . \tag{60}$$

This function solves the scalar Riemann-Hilbert problem

$$\alpha_-(k) = \alpha_+(k)\left(1 + \gamma\vartheta(k)(e^{-\mathrm{i}\eta} - 1)\right) \quad . \tag{61}$$

The conjugation function here is the element $\tilde{G}_{11}(k)$ of the conjugation matrix $\tilde{G}(k)$, eq. (55). Later we shall need the coefficients α_1, α_2 of the expansion of $\ln(\alpha(k))$ for large k,

$$\ln(\alpha(k)) = \frac{\alpha_1}{k} + \frac{\alpha_2}{k^2} + \mathcal{O}\left(\frac{1}{k^3}\right) \quad . \tag{62}$$

Second reformulation: We want to bring the Riemann-Hilbert problem to the canonical form. Instead of $\tilde{\chi}(k)$ let us define another 2×2-matrix

$$\Phi(k) = \tilde{\chi}(k) e^{-\sigma^z \ln \alpha(k)} \quad . \tag{63}$$

This matrix function is analytic for $\mathrm{Im}(k) > 0$ and $\mathrm{Im}(k) < 0$. It approaches the unit matrix at large k,

$$\Phi(k) = I + \frac{\Phi_1}{k} + \frac{\Phi_2}{k^2} + \mathcal{O}\left(\frac{1}{k^3}\right) \quad . \tag{64}$$

It has a discontinuity on the real axis, which can be described by a conjugation matrix $G_\Phi(k)$,

$$\Phi_-(k) = \Phi_+(k) G_\Phi(k) \quad . \tag{65}$$

Here

$$G_\Phi(k) = e^{\sigma^z \ln \alpha_+(k)} \tilde{G}(k) e^{-\sigma^z \ln \alpha_-(k)} \quad . \tag{66}$$

The matrix elements of $G_\Phi(k)$ are

$$\begin{aligned} (G_\Phi)_{11} &= 1 \quad , & (G_\Phi)_{12} &= \alpha_+(k)\alpha_-(k)\tilde{G}_{12}(k) \quad , \\ (G_\Phi)_{21} &= \frac{\tilde{G}_{21}(k)}{\alpha_-(k)\alpha_+(k)} \quad , & (G_\Phi)_{22} &= \tilde{G}_{11}(k)\tilde{G}_{22}(k) \quad . \end{aligned} \tag{67}$$

We also note that

$$\det G_\Phi(k) = \det \Phi(k) = 1 \quad . \tag{68}$$

Let us introduce the notations

$$p(k) = -\mathrm{i}(1-\cos(\eta))(1-\gamma\vartheta(k))\alpha_+(k)\alpha_-(k) \quad , \tag{69}$$

$$q(k) = \frac{-2\mathrm{i}\gamma\vartheta(k)}{\alpha_-(k)\alpha_+(k)} \quad . \tag{70}$$

Then we can write the conjugation matrix $G_\Phi(k)$ in the canonical form

$$G_\Phi(k) = \begin{pmatrix} 1 & p(k)e^{-\tau(k)} \\ q(k)e^{\tau(k)} & 1 + p(k)q(k) \end{pmatrix} \quad . \tag{71}$$

Here $\tau(k) = \mathrm{i}k^2 t - \mathrm{i}kx$. In the following the stationary point

$$k_0 = \frac{x}{2t} \tag{72}$$

of the phase $\tau(k)$, $\partial_k \tau(k_0) = 0$, will play an important role.

Let us express the potentials B_{ab} in terms of Φ_1, eq. (64),

$$\Phi_1 = \begin{pmatrix} -B_{+-} - \alpha_1 & b_{++} \\ -B_{--} & B_{+-} + \alpha_1 \end{pmatrix} \quad . \tag{73}$$

Here

$$\alpha_1 = \frac{1}{2\pi \mathrm{i}} \int_{-\infty}^{\infty} dp \, \ln \left(1 + \gamma \vartheta(p)(e^{-\mathrm{i}\eta} - 1)\right) \tag{74}$$

is the first coefficient of the expansion (62) of $\ln(\alpha(k))$ for large k. Using (37) and (73) we obtain

$$\partial_x \ln \det \left(\hat{I} + \gamma \hat{V}\right) = \mathrm{i}B_{+-} = \mathrm{i}B_{-+} = -\mathrm{i}((\Phi_1)_{11} + \alpha_1) \quad . \tag{75}$$

The time derivative of $\ln \det \left(\hat{I} + \gamma \hat{V}\right)$ can be expressed in terms of Φ_2, eq. (64),

$$\begin{aligned} \partial_t \ln \det \left(\hat{I} + \gamma \hat{V}\right) &= -\mathrm{i}(C_{+-} + C_{-+} + (1 - \cos(\eta))GB_{--}) \\ &= \mathrm{i}\left\{\mathrm{tr}(\sigma^z \Phi_2) + 2\alpha_2\right\} \quad , \end{aligned} \tag{76}$$

where α_2 is the second coefficient in the expansion (62),

$$\alpha_2 = \frac{1}{2\pi \mathrm{i}} \int_{-\infty}^{\infty} dp \, p \ln \left(1 + \gamma \vartheta(p)(e^{-\mathrm{i}\eta} - 1)\right) \quad . \tag{77}$$

This is the end of the second reformulation.

The asymptotic solution of the Riemann-Hilbert problem with conjugation matrix $G_\Phi(k)$, eq. (71), is known. It is given by Manakov's Ansatz (see page 452 of the book [3]). In order to describe Manakov's Ansatz we first solve a scalar Riemann-Hilbert problem for a function $\delta(k)$,

$$\delta_+(k) = \delta_-(k) \left(1 + p(k)q(k)\theta(k_0 - k)\right) \quad . \tag{78}$$

Here θ is the Heavyside step function,

$$\theta(k) = \begin{cases} 1 & \text{if} \quad k > 0 \\ 0 & \phantom{\text{if}} \quad k < 0 \end{cases} \quad . \tag{79}$$

The solution is

$$\delta(k) = \exp\left\{\frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{k_0}\frac{du}{u-k}\ln(1+p(u)q(u))\right\} \quad . \tag{80}$$

Now we can define functions $I^p(k)$ and $I^q(k)$,

$$I^p(k) = \frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{\infty}\frac{du}{u-k}\,\delta_+(u)\delta_-(u)p(u)e^{-\tau(u)} \quad , \tag{81}$$

$$I^q(k) = \frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{\infty}\frac{du}{u-k}\,\frac{q(u)e^{\tau(u)}}{\delta_+(u)\delta_-(u)} \quad . \tag{82}$$

Using these functions we can write the solution of the Riemann-Hilbert problem (65), (71) as

$$\Phi(k) = \begin{pmatrix} 1 & -I^p(k) \\ -I^q(k) & 1 \end{pmatrix} e^{\sigma^z \ln\delta(k)} \quad . \tag{83}$$

This solution is an asymptotic solution which is valid only as $x\to\infty$, $t\to\infty$ for fixed ratio x/t.

6 Asymptotics of the correlation functions

We solved the Riemann-Hilbert problem. Now we can calculate the logarithmic derivatives of the determinant,

$$\partial_x \ln\det\left(\hat{I}+\gamma\hat{V}\right) = \mathrm{i}B_{+-} = \mathrm{i}B_{-+} = -\mathrm{i}((\Phi_1)_{11}+\alpha_1) \quad , \tag{84}$$

$$\partial_t \ln\det\left(\hat{I}+\gamma\hat{V}\right) = \mathrm{i}\left\{\mathrm{tr}(\sigma^z\Phi_2)+2\alpha_2\right\} \quad . \tag{85}$$

In order to obtain Φ_1 and Φ_2 we have to decompose our asymptotic solution (83) into a Taylor series in $1/k$ (64). Then

$$-B_{+-} = \alpha_1 + \delta_1 \quad . \tag{86}$$

For α_1 see (74). In order to define δ_1 let us recall that

$$\ln(\delta(k)) = \frac{1}{2\pi\mathrm{i}}\int_{-\infty}^{k_0}\frac{dp}{p-k}\ln\left(1-2(1-\cos(\eta))\gamma\vartheta(p)(1-\gamma\vartheta(p))\right) \quad , \tag{87}$$

$k_0 = x/2t$. Here we used (69) and (80). At large k, $\ln(\delta(k))$ can be decomposed into a Taylor series in $1/k$,

$$\ln(\delta(k)) = \frac{\delta_1}{k} + \frac{\delta_2}{k^2} + \mathcal{O}\left(\frac{1}{k^3}\right) \quad . \tag{88}$$

So

$$\delta_1 = -\frac{1}{2\pi \mathrm{i}} \int_{-\infty}^{k_0} dp \ln\left(1 - 2(1-\cos(\eta))\gamma\vartheta(p)(1-\gamma\vartheta(p))\right) \quad , \tag{89}$$

$$\delta_2 = -\frac{1}{2\pi \mathrm{i}} \int_{-\infty}^{k_0} dp\, p \ln\left(1 - 2(1-\cos(\eta))\gamma\vartheta(p)(1-\gamma\vartheta(p))\right) \quad . \tag{90}$$

Substitution of α_1, eq. (74), and δ_1 into (84) gives

$$B_{+-} = -\frac{1}{2\pi \mathrm{i}} \int_{-\infty}^{\infty} dp\, \mathrm{sign}(p-k_0) \ln\left(1 + \gamma\vartheta(p)\left(e^{-\mathrm{i}\eta \mathrm{sign}(p-k_0)} - 1\right)\right) \quad , \tag{91}$$

where $k_0 = x/2t$ and $\mathrm{sign}(k)$ is the sign function,

$$\mathrm{sign}(k) = \begin{cases} 1 & \text{if} \quad k > 0 \\ -1 & \phantom{\text{if}} \quad k < 0 \end{cases} \quad . \tag{92}$$

Using(84) we obtain

$$\partial_x \ln\det(\hat{I} + \gamma\hat{V}) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} dp\, \mathrm{sign}(p-k_0) \ln\left(1 + \gamma\vartheta(p)\left(e^{-\mathrm{i}\eta \mathrm{sign}(p-k_0)} - 1\right)\right) \quad . \tag{93}$$

We shall calculate the time derivative in a similar way,

$$\mathrm{tr}(\sigma^z \Phi_2) = 2\delta_2 \quad , \tag{94}$$

see (83) and (90). From (85) we have

$$\partial_t \ln\det(\hat{I} + \gamma\hat{V}) = 2\mathrm{i}(\delta_2 + \alpha_2) \quad , \tag{95}$$

see (77) and (90). Finally,

$$\partial_t \ln\det(\hat{I} + \gamma\hat{V}) = \frac{1}{\pi} \int_{-\infty}^{\infty} dp\, p\, \mathrm{sign}(p-k_0) \ln\left(1 + \gamma\vartheta(p)\left(e^{-\mathrm{i}\eta \mathrm{sign}(p-k_0)} - 1\right)\right) \quad . \tag{96}$$

Now we have calculated space (93) and time derivative of $\ln\det(\hat{I}+\gamma\hat{V})$. Let us integrate,

$$\ln\det(\hat{I}+\gamma\hat{V}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dp\,|x-2pt|\ln\left(1+\gamma\vartheta(p)\left(e^{-\mathrm{i}\eta\,\mathrm{sign}(p-k_0)}-1\right)\right) \quad . \tag{97}$$

This is the leading term of the asymptotics. Let us remind that

$$\gamma\vartheta(k) = \frac{2\cosh(B/T)}{2\cosh(B/T)+e^{(k^2-\mu)/T}} \quad . \tag{98}$$

In order to evaluate the correlation functions (14), (15) we also need b_{++} and B_{--}.

$$b_{++} = (\Phi_1)_{12} \quad , \quad B_{--} = -(\Phi_1)_{21} \quad , \tag{99}$$

see (73). Using the explicit estimate for the error (see page 453 of the book [3]) of the asymptotic solution $\Phi(k)$, eq. (83), of the Riemann-Hilbert problem we find

$$|b_{++}| \sim \frac{1}{\sqrt{t}} \quad , \quad |B_{--}| \sim \frac{1}{\sqrt{t}} \quad , \tag{100}$$

which shows that b_{++} and B_{--} decay with time. In order to get a more precise estimate of the asymptotics, we shall turn to the differential equations (35). We shall use some information from the book [3]. So we shall rewrite the differential equations (35) in the variables $\tilde{x}=-x/2$, $\tilde{t}=t/2$. Then $k_0=-\tilde{x}/2\tilde{t}$, and

$$\begin{aligned} -\mathrm{i}\,\frac{\partial b_{++}}{\partial\tilde{t}} &= \frac{1}{2}\,\frac{\partial^2 b_{++}}{\partial\tilde{x}^2} + 4b_{++}^2 B_{--} \quad , \\ \mathrm{i}\,\frac{\partial B_{--}}{\partial\tilde{t}} &= \frac{1}{2}\,\frac{\partial^2 B_{--}}{\partial\tilde{x}^2} + 4b_{++}B_{--}^2 \quad . \end{aligned} \tag{101}$$

The asymptotics of the decaying solutions of this system is well known [16, 17, 18]. The leading terms are

$$\begin{aligned} b_{++} &= \tilde{u}_0\,\tilde{t}^{-1/2-\mathrm{i}\nu} e^{\mathrm{i}\tilde{x}^2/2\tilde{t}} \quad , \\ B_{--} &= \tilde{v}_0\,\tilde{t}^{-1/2+\mathrm{i}\nu} e^{-\mathrm{i}\tilde{x}^2/2\tilde{t}} \quad . \end{aligned} \tag{102}$$

Here $\tilde{u}_0$ and $\tilde{v}_0$ depend only on k_0, T, μ and B. Substitution of these asymptotic solutions into the differential equations (101) gives

$$\nu = -4\tilde{u}_0\tilde{v}_0 \quad . \tag{103}$$

In order to calculate ν we use the identity (38),

$$\frac{\partial B_{+-}}{\partial \tilde{x}} = 2\mathrm{i}B_{--}b_{++} \quad . \tag{104}$$

Let us substitute (91) here,

$$\frac{\partial B_{+-}}{\partial \tilde{x}} = \frac{\mathrm{i}}{4\pi\tilde{t}}\ln\left\{1-2(1-\cos(\eta))\gamma\vartheta(k_0)(1-\gamma\vartheta(k_0))\right\} \quad . \tag{105}$$

From (102) and (103) we have

$$2\mathrm{i}B_{--}b_{++} = \frac{2\mathrm{i}\tilde{u}_0\tilde{v}_0}{\tilde{t}} = \frac{-\mathrm{i}\nu}{2\tilde{t}} \quad . \tag{106}$$

Using (104), (105) we get

$$\nu = -\frac{1}{2\pi}\ln\left\{1-2(1-\cos(\eta))\gamma\vartheta(k_0)(1-\gamma\vartheta(k_0))\right\} \quad . \tag{107}$$

So we have calculated the asymptotics of b_{++} and B_{--}, see (102). We can use the differential equations (101) even more. We can improve the asymptotic expression (97) for $\ln\det(\hat{I}+\gamma\hat{V})$. Following the steps from pages 455-457 of the book [3] we obtain

$$\ln\det(\hat{I}+\gamma\hat{V}) = \\ \frac{1}{2\pi}\int_{-\infty}^{\infty} dp\,|x-2pt|\ln\left(1+\gamma\vartheta(p)\left(e^{-\mathrm{i}\eta\mathrm{sign}(p-k_0)}-1\right)\right) + \frac{\nu^2}{2}\ln t\,. \tag{108}$$

I order to get the correlation functions (14), (15) let us calculate

$$b_{++}\det(\hat{I}+\gamma\hat{V}) = u_0 t^{-\frac{1}{2}(1+\mathrm{i}\nu)^2}\exp(\mathrm{i}x^2/4t) \\ \cdot\exp\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty} dp\,|x-2pt|\ln\left(1+\gamma\vartheta(p)\left(e^{-\mathrm{i}\eta\mathrm{sign}(p-k_0)}-1\right)\right)\right\}\ , \tag{109}$$

$$B_{--}\det(\hat{I}+\gamma\hat{V}) = v_0 t^{-\frac{1}{2}(1-\mathrm{i}\nu)^2}\exp(-\mathrm{i}x^2/4t) \\ \cdot\exp\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty} dp\,|x-2pt|\ln\left(1+\gamma\vartheta(p)\left(e^{-\mathrm{i}\eta\mathrm{sign}(p-k_0)}-1\right)\right)\right\}\ . \tag{110}$$

We did not calculate the coefficients u_0 and v_0 in these expressions. We calculated the leading exponential factor and power (of time) corrections. Further down, while substituting we shall not pay attention to numerical coefficients. Let us get all notations together, and let us present our asymptotic expressions for the correlation functions (11), (12),

$$G^{+}_{\uparrow}(x,t) = e^{\mathrm{i}t(\mu-B)+ix^2/4t} \oint \frac{dz}{(z-1)^2}\, u_0 F(z) t^{-\frac{1}{2}(1+\mathrm{i}\nu(z))^2} e^{t\varphi(z)} \quad , \tag{111}$$

$$G^{-}_{\uparrow}(x,t) = e^{-\mathrm{i}t(\mu-B)-ix^2/4t} \oint \frac{dz}{z}\, v_0 F(z) t^{-\frac{1}{2}(1-\mathrm{i}\nu(z))^2} e^{t\varphi(z)} \quad . \tag{112}$$

Here the contour of integration is a circle of radius a little less than 1.

$$F(z) = 1 + \frac{z}{\gamma - z} + \frac{1}{\gamma z - 1} \quad , \tag{113}$$

$$\nu(z) = -\frac{1}{2\pi} \ln\left\{1 - (2 - z - z^{-1})\gamma\vartheta(k_0)(1-\gamma\vartheta(k_0))\right\} \quad , \tag{114}$$

$$t\varphi(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dp\, |x - 2pt| \ln\Big(1 + \gamma\vartheta(p)\left(z^{-\mathrm{sign}(p-k_0)} - 1\right)\Big) \quad , \tag{115}$$

$$\gamma = 1 + e^{2B/T}\ ,\ k_0 = x/2t\ ,\ \gamma\vartheta(k) = \frac{2\cosh(B/T)}{2\cosh(B/T) + e^{(k^2-\mu)/T}} \ . \tag{116}$$

At the end of section 2 we showed that b_{++} cancels the second order pole at $z=1$. This means that u_0 will cancel the second order pole at $z=1$ in (111). The remaining contour integrals in (111), (112) can be evaluated by the method of steepest descent (see next section). Eqs. (111), (112) are integral formulae for the large time, long distance asymptotics of correlation functions of local fields for the delta-interacting electron gas. These formulae are valid for arbitrary finite temperatures. They are the main result of the present article. In the next section we shall simplify our asymptotic expressions (111), (112) at low temperatures. Let us note that $\varphi(z)$ can be written as

$$\varphi(z) = \frac{1}{\pi}\int_{-\infty}^{\infty} dp\, |p - k_0| \ln\left\{\frac{e^{(k^2+B-\mu)/T} + \gamma z^{-\mathrm{sign}(p-k_0)}}{e^{(k^2+B-\mu)/T} + \gamma}\right\} \quad . \tag{117}$$

7 Low temperature asymptotics

We obtained the asymptotic formulae (111), (112). The remaining contour integral should be taken by means of the steepest descent (saddle point) method. Let us consider here only the leading asymptotic factor. Then we can rewrite the simplified expression for the asymptotics as

$$G^+_\uparrow(x,t) = e^{\mathrm{i}t(\mu-B)} e^{\mathrm{i}x^2/4t}\,\Omega_+ \quad , \tag{118}$$

$$G^-_\uparrow(x,t) = e^{-\mathrm{i}t(\mu-B)} e^{-\mathrm{i}x^2/4t}\,\Omega_- \quad , \tag{119}$$

$$\Omega_\pm = \oint dz\, F(z) t^{-\frac{1}{2}(1\pm \mathrm{i}\nu(z))^2} e^{t\varphi(z)} \quad . \tag{120}$$

The saddle point equation $\partial\varphi/\partial z = 0$ can be represented in the form

$$\int_0^\infty \frac{dk\,k}{1+\frac{1}{\gamma z} e^{((k-k_0)^2+B-\mu)/T}} = \int_0^\infty \frac{dk\,k}{1+\frac{z}{\gamma} e^{((k+k_0)^2+B-\mu)/T}} \quad , \tag{121}$$

$k_0 = x/2t$, $\gamma = 1 + e^{2B/T}$. We prove in appendix A that there is only one solution of this equation in the interval $0 < z \leq 1$. Considering the pure time direction, $k_0 = 0$, we find the solution

$$z = 1 \quad , \tag{122}$$

which leads to

$$\Omega_\pm \sim t^{-1} \quad . \tag{123}$$

This result is valid for all finite temperatures.

Let us consider the other asymptotic regions, $k_0 > 0$, $x \to \infty$ and $t \to \infty$. In these regions the saddle point equation (121) can be solved explicitly only for small temperatures. There are two solutions $z_\pm = \pm z_c$,

$$z_c = \frac{T^{3/4}}{2\pi^{1/4} k_0^{3/2}}\, e^{-k_0^2/2T} \quad . \tag{124}$$

If we deform the contour of integration from a circle to a line through z_+ parallel to the imaginary axis, we pass the saddle point in the right direction (see appendix A).

In appendix A we obtain the following low temperature expansion for the phase $t\varphi(z)$,

$$t\varphi(z) = -2k_0 Dt \left\{ \left(1 - \frac{1}{z}\right) z_c^2 + 1 - z \right\} \quad . \tag{125}$$

Here D is the density (8),

$$D = \begin{cases} \sqrt{\frac{T}{\pi}}\, e^{\mu/T} & \text{if} \quad B = 0 \quad , \\ \frac{1}{2}\sqrt{\frac{T}{\pi}}\, e^{(\mu+|B|)/T} & \phantom{\text{if}} \quad B \neq 0 \quad . \end{cases} \tag{126}$$

Eq. (125) shows that at low temperatures the relevant parameter for the calculation of the asymptotics of $\Omega_\pm$ is $2k_0 Dt = xD$ rather than t. The parameter xD has a simple interpretation. It is the average number of particles in the interval $[0, x]$. We have to distinguish two limiting cases.

(a) $xD \to 0$, the number of electrons in the interval $[0, x]$ vanishes. In this regime the interaction of the electrons is negligible. An electron propagates freely from 0 to x. $\Omega_\pm$ cannot be calculated by the method of steepest descent. We have to use (111), (112) to calculate the asymptotics of $G_\uparrow^+(x,t)$ and $G_\uparrow^-(x,t)$. Now $t\varphi(z)$ and $\nu(z)$ tend to zero on the contour of integration. Thus $G_\uparrow^\pm(x,t) \sim t^{-\frac{1}{2}} e^{\pm \mathrm{i}t(\mu-B) \pm \mathrm{i}x^2/4t}$, which, as expected, is the same as for free fermions. The proportionality factor is different for $G_\uparrow^+(x,t)$ and $G_\uparrow^-(x,t)$. To calculate it from (111), (112) we would have to know u_0 and v_0. By comparison with free fermions (see below) we expect it to be a number for $G_\uparrow^+(x,t)$ and proportional to $e^{(\mu-B-k_0^2)/T}$ for $G_\uparrow^-(x,t)$.

(b) $xD \gg 1$, the average number of electrons in the interval $[0, x]$ is large. Now the interaction becomes important, and we can use the method of steepest descent to calculate $\Omega_\pm$. This is the most interesting case. In the following we will investigate it in detail.

The contribution of the saddle point to $\Omega_\pm$, eq. (120), is

$$\Omega_\pm \sim t^{-(1 \pm \mathrm{i}\nu(z_+))}\, e^{-xD_\downarrow} \quad . \tag{127}$$

where

$$\nu(z_+) = -\frac{2D_\downarrow k_0^{3/2} e^{-k_0^2/2T}}{\pi^{1/4} T^{5/4}} \quad , \tag{128}$$

$$D_\downarrow = \tfrac{1}{2}\sqrt{\tfrac{T}{\pi}}\, e^{(\mu+B)/T} \quad . \tag{129}$$

$D_\downarrow$ is the low temperature expression for the density of down-spin electrons, eq. (5).

We further have to take into account the pole of the function F at $z = \gamma^{-1}$. While deforming the contour, we could have crossed it. It turns out that the pole contributes to $\Omega_\pm$, when the magnetic field B is below a critical positive value

$$B < B_c = k_0^2/4 \tag{130}$$

(see appendix A). The pole contribution always dominates the contribution of the saddle point. It is obtained as

$$\Omega_\pm \sim t^{-\left(\frac{1}{2}\pm \mathrm{i}\nu(\gamma^{-1})\right)} e^{-xD_\downarrow} \quad , \tag{131}$$

$$\nu(\gamma^{-1}) = -\frac{e^{(3B+\mu-k_0^2)/T}}{2\pi} \quad . \tag{132}$$

Finally, we have the following low temperature asymptotic expressions for the leading factors of the correlation functions. For

$B < B_c$:

$$G_\uparrow^+(x,t) = t^{-\frac{1}{2}-\mathrm{i}\nu(\gamma^{-1})} e^{\mathrm{i}t(\mu-B)} e^{\mathrm{i}x^2/4t} e^{-xD_\downarrow} \quad , \tag{133}$$

$$G_\uparrow^-(x,t) = t^{-\frac{1}{2}+\mathrm{i}\nu(\gamma^{-1})} e^{-\mathrm{i}t(\mu-B)} e^{-\mathrm{i}x^2/4t} e^{-xD_\downarrow} \quad , \tag{134}$$

where $\nu(\gamma^{-1})$ is given by eq. (132), and for

$B > B_c$:

$$G_\uparrow^+(x,t) = t^{-1-\mathrm{i}\nu(z_+)} e^{\mathrm{i}t(\mu-B)} e^{\mathrm{i}x^2/4t} e^{-xD_\downarrow} \quad , \tag{135}$$

$$G_\uparrow^-(x,t) = t^{-1+\mathrm{i}\nu(z_+)} e^{-\mathrm{i}t(\mu-B)} e^{-\mathrm{i}x^2/4t} e^{-xD_\downarrow} \quad , \tag{136}$$

where $\nu(z_+)$ is given by eq. (128). Recall that we have assumed the average number of electrons in the interval $[0,x]$ to be large, $xD \gg 1$. The corresponding expressions for $G_\downarrow^\pm(x,t)$ follow from equation (13).

The leading low temperature expressions for D and $D_\downarrow$ agree for positive magnetic field. Hence, above the critical field B_c the correlation functions depend on the magnetic field only through the trivial factors $e^{\pm \mathrm{i}t(\mu-B)}$. The system is saturated. Another way of looking at the condition (130), which would be appropriate for experiments with fixed magnetic field B, is the following. For positive magnetic field there are two different asymptotic regions. One is the space like region, where $k_0^2 > 4B$, the other one is the time like region, where $k_0^2 < 4B$. Correlation functions of local fields have asymptotics (133), (134) in the space like region and asymptotics (135), (136) in the time like region. For non-positive magnetic field there is no distinction between space like and time like region.

It is instructive to compare the asymptotic expressions (133) - (136) for the correlation functions with the corresponding asymptotics for free fermions. The two-point correlation functions for free fermions have the low temperature asymptotics

$$\frac{\mathrm{tr}\left(e^{-H_0/T}\,\psi_\uparrow(x,t)\psi_\uparrow^\dagger(0,0)\right)}{\mathrm{tr}\left(e^{-H_0/T}\right)} = C_+\,t^{-\frac{1}{2}}e^{\mathrm{i}t(\mu-B)}e^{\mathrm{i}x^2/4t} \quad , \tag{137}$$

$$\frac{\mathrm{tr}\left(e^{-H_0/T}\,\psi_\uparrow^\dagger(x,t)\psi_\uparrow(0,0)\right)}{\mathrm{tr}\left(e^{-H_0/T}\right)} = C_-\,t^{-\frac{1}{2}}e^{-\mathrm{i}t(\mu-B)}e^{-\mathrm{i}x^2/4t} \quad , \tag{138}$$

as $x \to \infty$, $t \to \infty$. Here H_0 is the free Hamiltonian ((2) with $c = 0$). The leading low temperature contributions to the constants in (137) and (138) are $C_+ = e^{-\mathrm{i}\frac{\pi}{4}}/2\sqrt{\pi}$ and $C_- = e^{(\mu-B-k_0^2)/T}e^{\mathrm{i}\frac{\pi}{4}}/2\sqrt{\pi}$. Comparing the free fermionic expressions with (133), (134) or (135), (136), respectively, we can interpret the factors $t^{\pm\mathrm{i}\nu(\gamma^{-1})}e^{-xD_\downarrow}$ and $t^{-\frac{1}{2}\pm\mathrm{i}\nu(z_+)}e^{-xD_\downarrow}$ as low temperature corrections, which appear due to the interaction. The occurrence of the density of down-spin electrons in the exponential factors in eqs. (133) - (136) has a natural interpretation. Correlations decay due to interaction. Because of the Pauli principle and the locality of the interaction in our specific model, eq. (2), up-spin electrons interact only with down-spin electrons. Therefore the correlation length is expected to be a decreasing function of the density of down-spin electrons, which diverges as the density of down spin electrons goes to zero. We see from (133) - (136) that the low temperature expression for the correlation length is just $1/D_\downarrow$ and thus meets these expectations.

In the limit $B \to -\infty$, $\mu \to -\infty$, $\mu - B$ fixed there are no $\downarrow$-spin electrons left in the system, $D_\downarrow \to 0$, $D_\uparrow \to D$. This is the free fermion limit. In the free fermion limit $B < B_c$, and the asymptotics of $G_\uparrow^+(x,t)$ and $G_\uparrow^-(x,t)$ are given by eqs. (133), (134), which then turn into the expressions (137), (138) for free fermions.

So we have explicitly evaluated the asymptotics of correlation functions of local fields $G^{\pm}_{\uparrow}(x,t)$, (11), (12). Our main results are formulae (111), (112). In this section we considered the saddle point calculation of the remaining contour integral, and we discussed the physical meaning of our asymptotics in the low temperature regime.

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Appendix A Steepest descent calculation of the leading asymptotic factors

In this appendix we discuss how to calculate the leading asymptotic contribution to the integrals in (111), (112) by means of the method of steepest descent. The leading contributions determine the exponential decay and the power law behavior of the correlation functions (11), (12). We will neglect all sub-leading factors. Then we are left with the calculation of the contour integral (120),

$$\Omega_{\pm} = \oint dz\, F(z) t^{-\frac{1}{2}(1\pm i\nu(z))^2} e^{t\varphi(z)} \quad . \tag{A.1}$$

The contour of integration is a circle of radius a little less than 1.

$$F(z) = 1 + \frac{z}{\gamma - z} + \frac{1}{\gamma z - 1} \quad , \tag{A.2}$$

$$\nu(z) = -\frac{1}{2\pi} \ln\left\{1 - (2 - z - z^{-1})\gamma\vartheta(k_0)(1 - \gamma\vartheta(k_0))\right\} \quad , \tag{A.3}$$

$$\varphi(z) = \frac{1}{\pi}\int_{-\infty}^{\infty} dp\, |p - k_0| \ln\left\{\frac{e^{(k^2+B-\mu)/T} + \gamma z^{-\mathrm{sign}(p-k_0)}}{e^{(k^2+B-\mu)/T} + \gamma}\right\} \quad . \tag{A.4}$$

The integrand in eq. (A.1) is analytic in the complex plane except for two essential singularities at $z = 0$, $z = \infty$ and the two simple poles of $F(z)$ at

$z = \gamma^{\pm 1}$. We may therefore deform the contour of integration as long as we never cross $z = 0$ and take into account the contributions from the residua, if we cross $z = \gamma$ or $z = \gamma^{-1}$.

Let us comment on the somewhat unusual fact that t in eq. (A.1) appears not only in the exponent. Calculating an integral of the form (A.1) by the method of steepest descent amounts to deforming the contour of integration in such a way that we are left with a Laplace type integral. Let us therefore consider the large t asymptotics of the integral

$$\int_{-\infty}^{\infty} dx\, t^{g(x)} e^{-tf(x)} = \int_{-\infty}^{\infty} dx\, e^{-t(f(x) - \varepsilon g(x))} \quad . \tag{A.5}$$

Here $\varepsilon = \ln(t)/t$ becomes small as $t \to \infty$. We assume that $f(x)$ has a unique minimum x_0. Then, first calculating the large t asymptotics of the right hand side of (A.5) by Laplace's method and expanding for small ε afterwards, we obtain the leading asymptotics

$$\int_{-\infty}^{\infty} dx\, t^{g(x)} e^{-tf(x)} = \sqrt{\tfrac{2\pi}{f''(x_0)}}\, t^{g(x_0) - \frac{1}{2}} e^{-tf(x_0)} \quad . \tag{A.6}$$

This means that the factor $t^{-\frac{1}{2}(1 \pm \mathrm{i}\nu(z))^2}$ in equation (A.1) can be treated the same way as a t-independent function. Thus, to leading order, a saddle point z_c contributes to $\Omega_\pm$ as

$$\Omega_\pm \sim t^{-\frac{1}{2} - \frac{1}{2}(1 \pm \mathrm{i}\nu(z_c))^2} e^{-t\varphi(z_c)} \quad . \tag{A.7}$$

The saddle point equation $\partial\varphi/\partial z = 0$ can be represented in the form

$$\int_0^\infty \frac{dk\, k}{1 + \frac{1}{\gamma z} e^{((k-k_0)^2 + B - \mu)/T}} = \int_0^\infty \frac{dk\, k}{1 + \frac{z}{\gamma} e^{((k+k_0)^2 + B - \mu)/T}} \quad , \tag{A.8}$$

$k_0 = x/2t$, $\gamma = 1 + e^{2B/T}$. This is still a transcendental equation, which cannot be solved explicitly for z except at low or high temperatures.

Before turning to the case of low temperatures, let us show that (A.8) restricted to the positive real axis, $z > 0$, has a unique solution, which is located in the interval $0 < z \le 1$. Let us define two functions

$$J_\pm(x) = \int_0^\infty \frac{dk\, k}{1 + x e^{(k \pm k_0)^2/T}} \quad . \tag{A.9}$$

We will further use the abbreviation

$$\Lambda = \frac{e^{(B-\mu)/T}}{\gamma} \quad . \tag{A.10}$$

Note that $\Lambda > 0$. Using the above conventions the saddle point equation (A.8) reads

$$f(z) = \frac{J_+(\Lambda z)}{J_-(\Lambda/z)} = 1 \quad . \tag{A.11}$$

Now

$$f'(z) = \Lambda \left\{ J'_+(\Lambda z) J_-(\Lambda/z) + (1/z^2) J_+(\Lambda z) J'_-(\Lambda/z) \right\} \Big/ J^2_-(\Lambda/z) \quad , \tag{A.12}$$

where the primes denote derivatives with respect to the arguments. Since

$$J'_\pm(x) = -\int_0^\infty dk \, \frac{k e^{(k\pm k_0)^2/T}}{(1 + x e^{(k\pm k_0)^2/T})^2} \; < \; 0 \tag{A.13}$$

and $J_\pm(x) > 0$ for all positive x, we conclude that $f(z)$ is monotonically decreasing for positive z.

We can decompose $J_-(x)$ into

$$J_-(x) = 2k_0\sqrt{T} I(x) + J_+(x) \quad , \tag{A.14}$$

where

$$I(x) = \int_0^\infty dk \, \frac{1}{1 + x e^{k^2}} \quad . \tag{A.15}$$

It follows that

$$f(1) = \frac{J_+(\Lambda)}{2k_0\sqrt{T} I(\Lambda) + J_+(\Lambda)} \leq 1 \quad , \tag{A.16}$$

since $I(x), J_+(x) > 0$ for all positive x. Thus the saddle point equation (A.8) has no positive real solution z with $z > 1$. We also learn from (A.16) that $z = 1$ solves the saddle point equation for $k_0 = 0$.

Let us work out the behavior of $f(z)$ for $z \to 0+$ and B, μ, T and k_0 fixed. We have the following asymptotic expansions of $I(x)$ and $J_+(x)$ for large positive x,

$$I(x) \;=\; \frac{\sqrt{\pi}}{2} \frac{1}{x} + \mathcal{O}\left(\frac{1}{x^2}\right) \quad , \tag{A.17}$$

$$J_+(x) \;=\; \frac{1}{x} \int_0^\infty dk \, k e^{-(k+k_0)^2/T} + \mathcal{O}\left(\frac{1}{x^2}\right) \quad . \tag{A.18}$$

Hence

$$J_-(\Lambda/z) = (z/\Lambda)\left(k_0\sqrt{\pi T} + \int_0^\infty dk\, k e^{-(k+k_0)^2/T}\right) + \mathcal{O}(z^2) \quad . \tag{A.19}$$

We can further show that

$$\lim_{x\to 0+}\left\{J_+(x) - \tfrac{1}{2}\left(\sqrt{-T\ln(x)} - k_0\right)^2\right\} = 0 \quad , \tag{A.20}$$

which implies that for small positive z

$$J_+(z) = -T\ln(z)/2 + \mathcal{O}\left(\sqrt{-\ln(z)}\right) \quad . \tag{A.21}$$

Then

$$f(z) = \frac{-\ln(z)}{z}\,\frac{T\Lambda}{2}\left(k_0\sqrt{\pi T} + \int_0^\infty dk\, k e^{-(k+k_0)^2/T}\right)^{-1} + \mathcal{O}\left(\frac{\sqrt{-\ln(z)}}{z}\right) \quad . \tag{A.22}$$

Eq. (A.22) shows that $\lim_{z\to 0+} f(z) = +\infty$. Since $f(1) \le 1$, eq. (A.16), we conclude that the saddle point equation (A.8) always has a real positive solution in the interval $0 < z \le 1$.

At low temperatures we can solve the saddle point equation (A.8) explicitly. Recall that we are dealing with the gas phase $|B| + \mu < 0$. In this phase

$$\Lambda^{-1} = (1 + e^{-2|B|/T})e^{(|B|+\mu)/T} \tag{A.23}$$

becomes a small parameter as T goes to zero. We will solve the saddle point equation (A.8) selfconsistently. Let us assume that

$$\Lambda^{-1} e^{-k_0^2/T} \ll |z| \ll \Lambda \quad . \tag{A.24}$$

We further assume that $k_0 > 0$, since the case $k_0 = 0$ can be treated for all temperatures (cf. eq. (A.16)). Define

$$I_\pm = \int_0^\infty dk\, k e^{-(k\pm k_0)^2/T} \quad . \tag{A.25}$$

(A.24) implies that $|\Lambda/z| \gg 1$, and thus to leading order

$$J_-(\Lambda/z) = \frac{zI_-}{\Lambda} \quad . \tag{A.26}$$

On the other hand $|\Lambda z| e^{k_0^2/T} \gg 1$, which implies that

$$J_+(\Lambda z) = \frac{I_+}{\Lambda z} \quad . \tag{A.27}$$

Inserting (A.26) and (A.27) into the saddle point equation (A.11) we obtain two solutions $z_\pm = \pm z_c$,

$$z_c = \sqrt{I_+/I_-} \quad . \tag{A.28}$$

The low temperature asymptotics of I_+ and I_- are easily calculated. We find

$$I_+ = \left(\frac{T}{2k_0}\right)^2 e^{-k_0^2/T} \quad , \tag{A.29}$$

$$I_- = k_0\sqrt{\pi T} + I_+ \approx k_0\sqrt{\pi T} \quad . \tag{A.30}$$

It follows that

$$z_c = \frac{T^{3/4}}{2\pi^{1/4} k_0^{3/2}} e^{-k_0^2/2T} \quad . \tag{A.31}$$

Note that this solution is consistent with (A.24).

In order to determine the directions of steepest descent let us calculate $\varphi''(z_\pm)$. First we rewrite $\varphi(z)$, eq. (A.4), as

$$\varphi(z) = \frac{1}{\pi}\int_0^\infty dk\, k \ln\left\{\frac{1+(z\Lambda)^{-1}e^{-(k+k_0)^2/T}}{1+\Lambda^{-1}e^{-(k+k_0)^2/T}}\;\frac{1+z\Lambda^{-1}e^{-(k-k_0)^2/T}}{1+\Lambda^{-1}e^{-(k-k_0)^2/T}}\right\} . \tag{A.32}$$

For all z, which satisfy (A.24), we obtain the low temperature expansion

$$\begin{aligned}\varphi(z) &= \frac{1}{\pi\Lambda}\left\{\left(\frac{1}{z}-1\right)I_+ + (z-1)I_-\right\} \\ &= -2k_0 D\left\{\left(1-\frac{1}{z}\right)z_c^2 + 1 - z\right\} \quad ,\end{aligned} \tag{A.33}$$

where D is the density, eq. (8). Thus

$$\varphi''(z_\pm) = \pm\frac{4k_0 D}{z_c} \quad , \tag{A.34}$$

which means that $\varphi''(z_+) > 0$, $\varphi''(z_-) < 0$. Therefore the path of steepest descent through z_- is part of the real axis, whereas the path of steepest

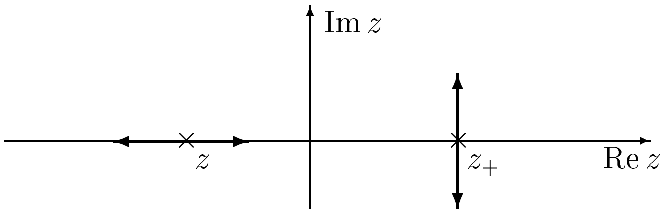

Figure 1: Directions of steepest descent

descent through z_+ is perpendicular to the real axis. The directions of steepest descent are depicted in figure 1. We conclude from the figure that the relevant saddle point for the calculation of our integral (A.1) is z_+.

Now the saddle point contribution to the integral (A.1) follows from eq. (A.33). We obtain

$$\varphi(z_+) = -2k_0 D(z_+ - 1)^2 \approx -2k_0 D \quad . \tag{A.35}$$

The exponent $\nu(z)$, which determines the power law behavior of the correlation functions, has the following low temperature expansion,

$$\nu(z) = -\tfrac{1}{2\pi} \ln(1 - (2 - z - z^{-1})\Lambda^{-1} e^{-k_0^2/T}) \quad . \tag{A.36}$$

Since z_c satisfies (A.24) and $z_c \ll 1$, we obtain

$$\nu(z_+) = -\frac{e^{-k_0^2/T}}{2\pi z_c \Lambda} = -\frac{2Dk_0^{3/2} e^{-k_0^2/2T}}{\pi^{1/4} T^{5/4}} \quad . \tag{A.37}$$

Eqs. (A.35) and (A.37) determine the saddle point contribution to $\Omega_\pm$,

$$\Omega_\pm \sim t^{-(1 \pm \mathrm{i}\nu(z_+))}\, e^{-xD} \quad . \tag{A.38}$$

Here we suppressed $\nu^2(z_+)$, which is small compared to 1.

The function $F(z)$, eq. (A.2), has two simple poles at $z = \gamma^{\pm 1}$, which may contribute to the integral (A.1), if they are crossed in the process of deformation of the path of integration. Let us investigate this question in the low temperature limit. The cases $B > 0$, $B = 0$ and $B < 0$ have to be studied separately.

(a) $B > 0$: In this case $\gamma = e^{2B/T}$, $\gamma^{-1} = e^{-2B/T}$. Since z_+ becomes small at low temperatures, we may have crossed the pole at γ^{-1}. The condition for having no contribution from the pole is $e^{-2B/T} < z_+$, or, using (A.31), $k_0^2 < 4B$. Let us assume that

$$k_0^2 > 4B \quad . \tag{A.39}$$

Then $z = \gamma^{-1}$ satisfies (A.24), and we can use the low temperature expansion (A.33) to obtain the contribution of the pole,

$$\begin{aligned} \varphi(\gamma^{-1}) &= -\,2k_0 D((1-\gamma)z_c^2 + 1 - \gamma^{-1}) \\ &\approx -\,2k_0 D(1-\gamma z_c^2) \approx -\,2k_0 D \quad . \end{aligned} \tag{A.40}$$

Here we used (A.39) once more to estimate that $\gamma z_c^2 \ll 1$. We see that the contribution of the pole to the exponential decay is exactly the same as the contribution of the saddle point. This is no longer true for the power law. (A.39) implies that $\gamma \Lambda^{-1} e^{-k_0^2/T} \ll 1$. Using this fact and the low temperature expansion (A.36) we obtain

$$\nu(\gamma^{-1}) = -\frac{e^{(3B+\mu-k_0^2)/T}}{2\pi} \quad . \tag{A.41}$$

(b) $B = 0$: In this case $\gamma = 2$, $\gamma^{-1} = \frac{1}{2}$ and $z_+ < \gamma^{-1}$. The pole at γ^{-1} contributes to the integral (A.1). $z = \gamma^{-1} = \frac{1}{2}$ satisfies (A.24). Thus

$$\varphi(\gamma^{-1}) = -\,k_0 D(1 - 2z_c^2) \approx -\,k_0 D \quad . \tag{A.42}$$

Comparing this expression with the saddle point contribution (A.35) we see that the pole now yields the leading contribution to the exponential decay. For $\nu(\gamma^{-1})$ we find

$$\nu(\gamma^{-1}) = -\frac{e^{(\mu-k_0^2)/T}}{2\pi} \quad . \tag{A.43}$$

(c) $B < 0$: Now both poles are very close to 1, $\gamma = 1 + e^{2B/T}$, $\gamma^{-1} = 1 - e^{2B/T} + e^{4B/T}$. The pole at γ^{-1} contributes to the integral, and $z = \gamma^{-1}$ satisfies (A.24). Using again (A.33) we obtain

$$\varphi(\gamma^{-1}) = -\,2k_0 D e^{2B/T}(1 - z_c^2) \approx -2k_0 D e^{2B/T} \quad . \tag{A.44}$$

Since $B < 0$, this gives again the leading exponentially decaying contribution to $\Omega_\pm$. $\nu(\gamma^{-1})$ is obtained from (A.36),

$$\nu(\gamma^{-1}) = -\tfrac{1}{2\pi}\ln(1 + e^{4B/T}\Lambda^{-1}e^{-k_0^2/T}) \approx -\frac{e^{(3B+\mu-k_0^2)/T}}{2\pi} \quad . \tag{A.45}$$

Using the explicit formulae for the density D, eq. (8), we obtain a uniform description of the cases (a), (b) and (c),

$$\begin{aligned} \Omega_\pm &= \oint dz\, F(z)\, t^{-\frac{1}{2}(1\pm \mathrm{i}\nu(z))^2} e^{t\varphi(z)} \\ &\sim t^{-\left(\frac{1}{2}\pm \mathrm{i}\nu(\gamma^{-1})\right)} \exp\left\{-\tfrac{x}{2}\sqrt{\tfrac{T}{\pi}}e^{(\mu+B)/T}\right\} \quad , \end{aligned} \tag{A.46}$$

$$\nu(\gamma^{-1}) = -\frac{e^{(3B+\mu-k_0^2)/T}}{2\pi} \quad . \tag{A.47}$$

For a physical interpretation of eq. (A.46) we look at the formula (5) for the density of down-spin electrons. Its low temperature expansion is

$$D_\downarrow = \tfrac{1}{2}\sqrt{\tfrac{T}{\pi}}e^{(\mu+B)/T} \quad . \tag{A.48}$$

Inserting this expression into (A.46) we arrive at eq. (131) of the main text. Furthermore, comparing (A.48) and (8) we see that we can identify $D_\downarrow$ with D, when the magnetic field is positive.Hence we may replace D by $D_\downarrow$ in eqs. (A.37) and (A.38), which implies eqs. (127) and (128) in section 7.

We still have to discuss the range of validity of the above low temperature calculations. The low temperature expression for the saddle point z_c is sufficiently precise, as long as z_c satisfies (A.24). Yet, there is another condition to be satisfied. Note that the phase $\varphi(z)$ in low temperature approximation, eq. (A.33), is multiplied by a factor which vanishes as the temperature approaches zero. We have

$$t\varphi(z) = -2k_0 Dt\left\{\left(1-\frac{1}{z}\right)z_c^2 + 1 - z\right\} \quad . \tag{A.49}$$

Thus the large parameter, which we need for the saddle point approximation to be valid, is $xD = 2k_0Dt$ rather than t.

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