The Bargmann-Wigner Formalism for Spin 2*

Valeri V. Dvoeglazov
Escuela de Física, Universidad Autónoma de Zacatecas
Apartado Postal C-580, Zacatecas 98068, ZAC., México
Internet address: valeri@cantera.reduaz.mx
URL: http://cantera.reduaz.mx/˜ valeri/valeri.htm

(January, 1998; revised: April 1998)

We proceed to derive equations for the symmetric tensor of the second rank on the basis of the Bargmann-Wigner formalism in a straightforward way. The symmetric multispinor of the fourth rank is used. It is constructed out of the Dirac 4-spinors. Due to serious problems with the interpretation of the results obtained on using the standard procedure we generalize it and obtain the spin-2 relativistic equations, which are consistent with the previous one. The importance of the 4-vector field (and its gauge part) is pointed out.

PACS number: 03.65.Pm , 04.50.+h , 11.30.Cp

The general scheme for derivation of higher-spin equations was given in [1]. A field of rest mass m and spin $j \geq \frac{1}{2}$ is represented by a completely symmetric multispinor of rank $2j$. The particular cases $j=1$ and $j=\frac{3}{2}$ were given in the textbooks, e. g., ref. [2]. The spin-2 case can also be of some interest because it is generally believed that the essential features of the gravitational field are obtained from transverse components of the $(2,0)\oplus(0,2)$ representation of the Lorentz group. Nevertheless, questions of the redandant components of the higher-spin relativistic equations are not yet understood in detail [3].

In the first section we use the commonly-accepted procedure for the derivation of higher-spin equations.

I. STANDARD FORMALISM

We begin with the equations for the 4-rank symmetric spinor:

$$\left[i\gamma^\mu\partial_\mu - m\right]_{\alpha\alpha'}\Psi_{\alpha'\beta\gamma\delta} = 0\,, \tag{1a}$$
$$\left[i\gamma^\mu\partial_\mu - m\right]_{\beta\beta'}\Psi_{\alpha\beta'\gamma\delta} = 0\,, \tag{1b}$$
$$\left[i\gamma^\mu\partial_\mu - m\right]_{\gamma\gamma'}\Psi_{\alpha\beta\gamma'\delta} = 0\,, \tag{1c}$$
$$\left[i\gamma^\mu\partial_\mu - m\right]_{\delta\delta'}\Psi_{\alpha\beta\gamma\delta'} = 0\,. \tag{1d}$$

The massless limit (if one needs) should be taken in the end of all calculations.

We proceed expanding the field function in the set of symmetric matrices (as in the spin-1 case, cf. ref. [4a]). In the beginning let us use the first two indices:[1]

---

*Submitted to "Journal of Physics A".

[1] The matrix R can be related to the CP operation in the $(1/2,0)\oplus(0,1/2)$ representation.

$$\Psi_{\{\alpha\beta\}\gamma\delta} = (\gamma_\mu R)_{\alpha\beta}\Psi^{\mu}_{\gamma\delta} + (\sigma_{\mu\nu}R)_{\alpha\beta}\Psi^{\mu\nu}_{\gamma\delta}\,. \tag{2}$$

We would like to write the corresponding equations for functions $\Psi^{\mu}_{\gamma\delta}$ and $\Psi^{\mu\nu}_{\gamma\delta}$ in the form:

$$\frac{2}{m}\partial_\mu \Psi^{\mu\nu}_{\gamma\delta} = -\Psi^{\nu}_{\gamma\delta}\,, \tag{3a}$$

$$\Psi^{\mu\nu}_{\gamma\delta} = \frac{1}{2m}\left[\partial^\mu \Psi^{\nu}_{\gamma\delta} - \partial^\nu \Psi^{\mu}_{\gamma\delta}\right]\,. \tag{3b}$$

Constraints $(1/m)\partial_\mu\Psi^{\mu}_{\gamma\delta} = 0$ and $(1/m)\epsilon^{\mu\nu}{}_{\alpha\beta}\,\partial_\mu\Psi^{\alpha\beta}_{\gamma\delta} = 0$ can be regarded as a consequence of Eqs. (3a,3b).

Next, we present the vector-spinor and tensor-spinor functions as

$$\Psi^{\mu}_{\{\gamma\delta\}} = (\gamma^\kappa R)_{\gamma\delta}G_\kappa{}^{\mu} + (\sigma^{\kappa\tau}R)_{\gamma\delta}F_{\kappa\tau}{}^{\mu}\,, \tag{4a}$$

$$\Psi^{\mu\nu}_{\{\gamma\delta\}} = (\gamma^\kappa R)_{\gamma\delta}T_\kappa{}^{\mu\nu} + (\sigma^{\kappa\tau}R)_{\gamma\delta}R_{\kappa\tau}{}^{\mu\nu}\,, \tag{4b}$$

i. e., using the symmetric matrix coefficients in indices γ and δ. Hence, the total function is

R$)_{\gamma\delta}F_{\kappa\tau}{}^{\mu} +$

$+\, (\sigma_{\mu\nu}R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}T_\kappa{}^{\mu\nu} + (\sigma_{\mu\nu}R)_{\alpha\beta}(\sigma^{\kappa\tau}R)_{\gamma\delta}R_{\kappa\tau}{}^{\mu\nu}\,;$

(5)

and the resulting tensor equations are:

$$\frac{2}{m}\partial_\mu T_\kappa{}^{\mu\nu} = -G_\kappa{}^{\nu}\,, \tag{6a}$$

$$\frac{2}{m}\partial_\mu R_{\kappa\tau}{}^{\mu\nu} = -F_{\kappa\tau}{}^{\nu}\,, \tag{6b}$$

$$T_\kappa{}^{\mu\nu} = \frac{1}{2m}\left[\partial^\mu G_\kappa{}^{\nu} - \partial^\nu G_\kappa{}^{\mu}\right]\,, \tag{6c}$$

$$R_{\kappa\tau}{}^{\mu\nu} = \frac{1}{2m}\left[\partial^\mu F_{\kappa\tau}{}^{\nu} - \partial^\nu F_{\kappa\tau}{}^{\mu}\right]\,. \tag{6d}$$

The constraints are re-written to

$$\frac{1}{m}\partial_\mu G_\kappa{}^{\mu} = 0\,, \quad \frac{1}{m}\partial_\mu F_{\kappa\tau}{}^{\mu} = 0\,, \tag{7a}$$

$$\frac{1}{m}\epsilon_{\alpha\beta\nu\mu}\partial^\alpha T_\kappa{}^{\beta\nu} = 0\,, \quad \frac{1}{m}\epsilon_{\alpha\beta\nu\mu}\partial^\alpha R_{\kappa\tau}{}^{\beta\nu} = 0\,. \tag{7b}$$

However, we need to make symmetrization over these two sets of indices $\{\alpha\beta\}$ and $\{\gamma\delta\}$. The total symmetry can be ensured if one contracts the function $\Psi_{\{\alpha\beta\}\{\gamma\delta\}}$ with *antisymmetric* matrices $R^{-1}_{\beta\gamma}$, $(R^{-1}\gamma^5)_{\beta\gamma}$ and $(R^{-1}\gamma^5\gamma^\lambda)_{\beta\gamma}$ and equate all these contractions to zero (similar to the $j = 3/2$ case considered in ref. [2, p. 44]. We obtain additional constraints on the tensor field functions:

$$G_{\mu}{}^{\mu}=0\,,\quad G_{[\kappa\,\mu]}=0\,,\quad G^{\kappa\mu}=\frac{1}{2}g^{\kappa\mu}G_{\nu}{}^{\nu}\,, \tag{8a}$$
$$F_{\kappa\mu}{}^{\mu}=F_{\mu\kappa}{}^{\mu}=0\,,\quad \epsilon^{\kappa\tau\mu\nu}F_{\kappa\tau,\mu}=0\,, \tag{8b}$$
$$T^{\mu}{}_{\mu\kappa}=T^{\mu}{}_{\kappa\mu}=0\,,\quad \epsilon^{\kappa\tau\mu\nu}T_{\kappa,\tau\mu}=0\,, \tag{8c}$$
$$F^{\kappa\tau,\mu}=T^{\mu,\kappa\tau}\,,\quad \epsilon^{\kappa\tau\mu\lambda}(F_{\kappa\tau,\mu}+T_{\kappa,\tau\mu})=0\,, \tag{8d}$$
$$R_{\kappa\nu}{}^{\mu\nu}=R_{\nu\kappa}{}^{\mu\nu}=R_{\kappa\nu}{}^{\nu\mu}=R_{\nu\kappa}{}^{\nu\mu}=R_{\mu\nu}{}^{\mu\nu}=0\,, \tag{8e}$$
$$\epsilon^{\mu\nu\alpha\beta}(g_{\beta\kappa}R_{\mu\tau,\nu\alpha}-g_{\beta\tau}R_{\nu\alpha,\mu\kappa})=0\quad \epsilon^{\kappa\tau\mu\nu}R_{\kappa\tau,\mu\nu}=0\,. \tag{8f}$$

Thus, we encountered with the known difficulty of the theory for spin-2 particles in the Minkowski space. We explicitly showed that all field functions become to be equal to zero. Such a situation cannot be considered as a satisfactory one (because it does not give us any physical information) and can be corrected in several ways.[2]

II. GENERALIZED FORMALISM

We shall modify the formalism in the spirit of ref. [4b]. The field function (2) is now presented as

$$\Psi_{\{\alpha\beta\}\gamma\delta}=\alpha_1(\gamma_\mu R)_{\alpha\beta}\Psi^{\mu}_{\gamma\delta}+\alpha_2(\sigma_{\mu\nu}R)_{\alpha\beta}\Psi^{\mu\nu}_{\gamma\delta}+\alpha_3(\gamma^5\sigma_{\mu\nu}R)_{\alpha\beta}\widetilde{\Psi}^{\mu\nu}_{\gamma\delta}\,, \tag{9}$$

with

$$\Psi^{\mu}_{\{\gamma\delta\}}=\beta_1(\gamma^\kappa R)_{\gamma\delta}G_{\kappa}{}^{\mu}+\beta_2(\sigma^{\kappa\tau}R)_{\gamma\delta}F_{\kappa\tau}{}^{\mu}+\beta_3(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{F}_{\kappa\tau}{}^{\mu}\,, \tag{10a}$$
$$\Psi^{\mu\nu}_{\{\gamma\delta\}}=\beta_4(\gamma^\kappa R)_{\gamma\delta}T_{\kappa}{}^{\mu\nu}+\beta_5(\sigma^{\kappa\tau}R)_{\gamma\delta}R_{\kappa\tau}{}^{\mu\nu}+\beta_6(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{R}_{\kappa\tau}{}^{\mu\nu}\,, \tag{10b}$$
$$\widetilde{\Psi}^{\mu\nu}_{\{\gamma\delta\}}=\beta_7(\gamma^\kappa R)_{\gamma\delta}\widetilde{T}_{\kappa}{}^{\mu\nu}+\beta_8(\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{D}_{\kappa\tau}{}^{\mu\nu}+\beta_9(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}D_{\kappa\tau}{}^{\mu\nu}\,. \tag{10c}$$

Hence, the function $\Psi_{\{\alpha\beta\}\{\gamma\delta\}}$ can be expressed as a sum of nine terms:

$$\begin{aligned}\Psi_{\{\alpha\beta\}\{\gamma\delta\}}&=\alpha_1\beta_1(\gamma_\mu R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}G_{\kappa}{}^{\mu}+\alpha_1\beta_2(\gamma_\mu R)_{\alpha\beta}(\sigma^{\kappa\tau}R)_{\gamma\delta}F_{\kappa\tau}{}^{\mu}+\\&+\alpha_1\beta_3(\gamma_\mu R)_{\alpha\beta}(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{F}_{\kappa\tau}{}^{\mu}++\alpha_2\beta_4(\sigma_{\mu\nu}R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}T_{\kappa}{}^{\mu\nu}+\\&+\alpha_2\beta_5(\sigma_{\mu\nu}R)_{\alpha\beta}(\sigma^{\kappa\tau}R)_{\gamma\delta}R_{\kappa\tau}{}^{\mu\nu}+\alpha_2\beta_6(\sigma_{\mu\nu}R)_{\alpha\beta}(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{R}_{\kappa\tau}{}^{\mu\nu}+\\&+\alpha_3\beta_7(\gamma^5\sigma_{\mu\nu}R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}\widetilde{T}_{\kappa}{}^{\mu\nu}+\alpha_3\beta_8(\gamma^5\sigma_{\mu\nu}R)_{\alpha\beta}(\sigma^{\kappa\tau}R)_{\gamma\delta}\widetilde{D}_{\kappa\tau}{}^{\mu\nu}+\\&+\alpha_3\beta_9(\gamma^5\sigma_{\mu\nu}R)_{\alpha\beta}(\gamma^5\sigma^{\kappa\tau}R)_{\gamma\delta}D_{\kappa\tau}{}^{\mu\nu}\,.\end{aligned} \tag{11}$$

---

[2]The reader can compare our results of this Section with those of G. Marques and D. Spehler, Mod. Phys. Lett. A13 (1998) 553-569.

I became aware about their consideration from Dr. D. V. Ahluwalia (personal communications, May 5, 1998) after completing the first version of this paper. I consider their discussion of the standard formalism in the Sections I and II, as insufficient.

The corresponding dynamical equations are given by the set[3]

$$\frac{2\alpha_2\beta_4}{m}\partial_\nu T_\kappa{}^{\mu\nu} + \frac{i\alpha_3\beta_7}{m}\epsilon^{\mu\nu\alpha\beta}\partial_\nu \widetilde{T}_{\kappa,\alpha\beta} = \alpha_1\beta_1 G_\kappa{}^{\mu}\,; \tag{12a}$$

$$\frac{2\alpha_2\beta_5}{m}\partial_\nu R_{\kappa\tau}{}^{\mu\nu} + \frac{i\alpha_2\beta_6}{m}\epsilon_{\alpha\beta\kappa\tau}\partial_\nu \widetilde{R}^{\alpha\beta,}$$

$\mu\nu + \frac{i\alpha_3\beta_8}{m}\epsilon^{\mu\nu\alpha\beta}\partial_\nu \widetilde{D}_{\kappa\tau,\alpha\beta} -$

-$\frac{\alpha_3\beta_9}{2\epsilon^{\mu\nu\alpha\beta}\epsilon_{\lambda\delta\kappa\tau}D^{\lambda\delta}{}_{\alpha\beta} = \alpha_1\beta_2 F_{\kappa\tau}{}^{\mu} + \frac{i\alpha_1\beta_3}{2}\epsilon_{\alpha\beta\kappa\tau}\widetilde{F}^{\alpha\beta,\mu}}$;(12b)

$2\alpha_2\beta_4 T_\kappa{}^{\mu\nu} + i\alpha_3\beta_7\epsilon^{\alpha\beta\mu\nu}\widetilde{T}_{\kappa,\alpha\beta} = \frac{\alpha_1\beta_1}{m}(\partial^\mu G_\kappa{}^{\nu} - \partial^\nu G_\kappa{}^{\mu})$; (12c)

$2\alpha_2\beta_5 R_{\kappa\tau}{}^{\mu\nu} + i\alpha_3\beta_8\epsilon^{\alpha\beta\mu\nu}\widetilde{D}_{\kappa\tau,\alpha\beta} + i\alpha_2\beta_6\epsilon_{\alpha\beta\kappa\tau}\widetilde{R}^{\alpha\beta,\mu\nu} - \frac{\alpha_3\beta_9}{2}\epsilon^{\alpha\beta\mu\nu}\epsilon_{\lambda\delta\kappa\tau}D^{\lambda\delta}{}_{\alpha\beta}$

$=$

$= \frac{\alpha_1\beta_2}{m(\partial^\mu F_{\kappa\tau}{}^{\nu} - \partial^\nu F_{\kappa\tau}{}^{\mu}) + \frac{i\alpha_1\beta_3}{2m}}$

$\epsilon_{\alpha\beta\kappa\tau}(\partial^\mu \widetilde{F}^{\alpha\beta,\nu} - \partial^\nu \widetilde{F}^{\alpha\beta,\mu})$.(12d)

Essential constraints are:

$$\alpha_1\beta_1 G^\mu{}_\mu = 0\,, \quad \alpha_1\beta_1 G_{[\kappa\mu]} = 0\,; \tag{13a}$$

$$2i\alpha_1\beta_2 F_{\alpha\mu}{}^{\mu} + \alpha_1\beta_3\epsilon^{\kappa\tau\mu}{}_{\alpha}\widetilde{F}_{\kappa\tau,\mu} = 0\,; \tag{13b}$$

$$2i\alpha_1\beta_3 \widetilde{F}_{\alpha\mu}{}^{\mu} + \alpha_1\beta_2\epsilon^{\kappa\tau\mu}{}_{\alpha}F_{\kappa\tau,\mu} = 0\,; \tag{13c}$$

$$2i\alpha_2\beta_4 T^\mu{}_{\mu\alpha} - \alpha_3\beta_7\epsilon^{\kappa\tau\mu}{}_{\alpha}\widetilde{T}_{\kappa,\tau\mu} = 0\,; \tag{13d}$$

$$2i\alpha_3\beta_7 \widetilde{T}^\mu{}_{\mu\alpha} - \alpha_2\beta_4\epsilon^{\kappa\tau\mu}{}_{\alpha}T_{\kappa,\tau\mu} = 0\,; \tag{13e}$$

$$i\epsilon^{\mu\nu\kappa\tau}\left[\alpha_2\beta_6\widetilde{R}_{\kappa\tau,\mu\nu} + \alpha_3\beta_8\widetilde{D}_{\kappa\tau,\mu\nu}\right] + 2\alpha_2\beta_5 R^{\mu\nu}{}_{\mu\nu} + 2\alpha_3\beta_9 D^{\mu\nu}{}_{\mu\nu} = 0\,; \tag{13f}$$

$$i\epsilon^{\mu\nu\kappa\tau}\left[\alpha_2\beta_5 R_{\kappa\tau,\mu\nu} + \alpha_3\beta_9 D_{\kappa\tau,\mu\nu}\right] + 2\alpha_2\beta_6 \widetilde{R}^{\mu\nu}{}_{\mu\nu} + 2\alpha_3\beta_8 \widetilde{D}^{\mu\nu}{}_{\mu\nu} = 0\,; \tag{13g}$$

$$2i\alpha_2\beta_5 R_{\beta\mu}{}^{\mu\alpha} + 2i\alpha_3\beta_9 D_{\beta\mu}{}^{\mu\alpha} + \alpha_2\beta_6\epsilon^{\nu\alpha}{}_{\lambda\beta}\widetilde{R}^{\lambda\mu}{}_{\mu\nu} + \alpha_3\beta_8\epsilon^{\nu\alpha}{}_{\lambda\beta}\widetilde{D}^{\lambda\mu}{}_{\mu\nu} = 0\,; \tag{13h}$$

$$2i\alpha_1\beta_2 F^{\lambda\mu}{}_{\mu} - 2i\alpha_2\beta_4 T_\mu{}^{\mu\lambda} + \alpha_1\beta_3\epsilon^{\kappa\tau\mu\lambda}\widetilde{F}_{\kappa\tau,\mu} + \alpha_3\beta_7\epsilon^{\kappa\tau\mu\lambda}\widetilde{T}_{\kappa,\tau\mu} = 0\,; \tag{13i}$$

$2i\alpha_1\beta_3 \widetilde{F}^{\lambda\mu}{}_{\mu} - 2i\alpha_3$

$\beta_7\widetilde{T}_\mu{}^{\mu\lambda} + \alpha_1\beta_2\epsilon^{\kappa\tau\mu\lambda}F_{\kappa\tau,\mu} + \alpha_2\beta_4\epsilon^{\kappa\tau\mu\lambda}T_{\kappa,\tau\mu} = 0$; (13j)

---

[3]All indices in this formula are already pure vectorial and have nothing to do with previous notation. The coefficients α_i and β_i may, in general, carry some dimension.

$\alpha_1\beta_1(2G^\lambda{}_\alpha - g^\lambda{}_\alpha G^\mu{}_\mu) - 2\alpha_2\beta_5(2R^{\lambda\mu}{}_{\mu\alpha}$
$+2\mathrm{R}_{\alpha\mu}{}^{\mu\lambda} + g^\lambda{}_\alpha R^{\mu\nu}{}_{\mu\nu}) +$
$+\ 2\alpha_3\beta_9(2D^{\lambda\mu}{}_{\mu\alpha} + 2D_{\alpha\mu}{}^{\mu\lambda} + g^\lambda{}_\alpha D^{\mu\nu}{}_{\mu\nu}) + 2i\alpha_3\beta_8(\epsilon_{\kappa\alpha}{}^{\mu\nu}\widetilde{D}^{\kappa\lambda}{}_{\mu\nu} - \epsilon^{\kappa\tau\mu\lambda}\widetilde{D}_{\kappa\tau,\mu\alpha}) -$
$-\ 2\mathrm{i}\alpha_2\beta_6(\epsilon_{\kappa\alpha}{}^{\mu\nu}\widetilde{R}^{\kappa\lambda}{}_{\mu\nu} - \epsilon^{\kappa\tau\mu\lambda}\widetilde{R}_{\kappa\tau,\mu\alpha}) = 0\,;$ (13k)

$2\alpha_3\beta_8(2\widetilde{D}^{\lambda\mu}{}_{\mu\alpha} + 2\widetilde{D}_{\alpha\mu}{}^{\mu\lambda} + g^\lambda{}_\alpha\widetilde{D}^{\mu\nu}{}_{\mu\nu}) - 2\alpha_2\beta_6(2\widetilde{R}^{\lambda\mu}{}_{\mu\alpha} + 2\widetilde{R}_{\alpha\mu}{}^{\mu\lambda} +$
$+\ \mathrm{g}^\lambda{}_\alpha\widetilde{R}^{\mu\nu}{}_{\mu\nu}) + +2i\alpha_3\beta_9(\epsilon_{\kappa\alpha}{}^{\mu\nu}D^{\kappa\lambda}{}_{\mu\nu} - \epsilon^{\kappa\tau\mu\lambda}D_{\kappa\tau,\mu\alpha}) -$
$-\ 2\mathrm{i}\alpha_2\beta_5(\epsilon_{\kappa\alpha}{}^{\mu\nu}R^{\kappa\lambda}{}_{\mu\nu} - \epsilon^{\kappa\tau\mu\lambda}R_{\kappa\tau,\mu\alpha}) = 0\,;$ (13l)

$\alpha_1\beta_2(F^{\alpha\beta,\lambda} - 2F^{\beta\lambda,\alpha} + F^{\beta\mu}{}_\mu g^{\lambda\alpha} - F^{\alpha\mu}{}_\mu$
$\mathrm{g}^{\lambda\beta}) -$
$-\alpha_2\beta_4(T^{\lambda,\alpha\beta} - 2T^{\beta,\lambda\alpha} + T_\mu{}^{\mu\alpha}g^{\lambda\beta} - T_\mu{}^{\mu\beta}g^{\lambda\alpha}) +$
$+\mathrm{i}\,\overline{2\alpha_1\beta_3(\epsilon^{\kappa\tau\alpha\beta}\widetilde{F}_{\kappa\tau}{}^\lambda + 2\epsilon^{\lambda\kappa\alpha\beta}\widetilde{F}_{\kappa\mu}{}^\mu + 2\epsilon^{\mu\kappa\alpha\beta}\widetilde{F}^\lambda{}_{\kappa,\mu}) -}$
$-\ \mathrm{i}\,\overline{2\alpha_3\beta_7(\epsilon^{\mu\nu\alpha\beta}\widetilde{T}^\lambda{}_{\mu\nu} + 2\epsilon^{\nu\lambda\alpha\beta}\widetilde{}}$
$\mathrm{T}^\mu{}_{\mu\nu} + 2\epsilon^{\mu\kappa\alpha\beta}\widetilde{T}_{\kappa,\mu}{}^\lambda) = 0\,.$(13m)

They are the results of contractions of the field function (**??**)

with three antisymmetric matrices, as above. Furthermore, one should recover the relations (8a-8f) in the particular case when $\alpha_3 = \beta_3 = \beta_6 = \beta_9 = 0$ and $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = \beta_4 = \beta_5 = \beta_7 = \beta_8 = 1$.

As a discussion we note that in such a framework we already have physical content because only certain combinations of field functions would be equal to zero. In general, the fields $F_{\kappa\tau}{}^\mu$, $\widetilde{F}_{\kappa\tau}{}^\mu$, $T_\kappa{}^{\mu\nu}$, $\widetilde{T}_\kappa{}^{\mu\nu}$, and $R_{\kappa\tau}{}^{\mu\nu}$, $\widetilde{R}_{\kappa\tau}{}^{\mu\nu}$, $D_{\kappa\tau}{}^{\mu\nu}$, $\widetilde{D}_{\kappa\tau}{}^{\mu\nu}$ can correspond to different physical states and the equations above describe oscillations one state to another.

Furthermore, from the set of equations (12a-**??**) one obtains the *second*-order equation for symmetric traceless tensor of the second rank ($\alpha_1 \neq 0$, $\beta_1 \neq 0$):

$$\frac{1}{m^2}\left[\partial_\nu\partial^\mu G_\kappa{}^\nu - \partial_\nu\partial^\nu G_\kappa{}^\mu\right] = G_\kappa{}^\mu\,. \tag{14}$$

After the contraction in indices κ and μ this equation is reduced to the set

$$\partial_\mu G^\mu{}_\kappa = F_\kappa \tag{15a}$$
$$\frac{1}{m^2}\partial_\kappa F^\kappa = 0\,, \tag{15b}$$

i. e., to the equations connecting the analogue of the energy-momentum tensor and the analogue of the 4-vector potential. As we showed in our recent work [4] the longitudinal potential is perfectly suitable for construction of electromagnetism (see also recent works on the notoph and notivarg concept [5]). Moreover, according to the Weinberg

theorem [6] for massless particles it is the gauge part of the 4-vector potential which is the physical field. The case, when the longitudinal potential is equated to zero, can be considered as a particular case only. This case may be relevant to some physical situation but hardly to be considered as a fundamental one.

Further investigations may provide additional foundations to “surprising” similarities of gravitational and electromagnetic equations in the low-velocity limit, refs. [7,8].

ACKNOWLEDGMENTS

I acknowledge discussions with Profs. D. V. Ahluwalia, A. E. Chubykalo, S. Esposito, Y. S. Kim, A. F. Pashkov, G. Quznetzov and R. Santilli.

I am grateful to Zacatecas University, México, for the professorship. This work has been partly supported by the Mexican Sistema Nacional de Investigadores, by the CONACyT, México under the research project 0270P-E.

---

[1] V. Bargmann and E. P. Wigner, Proc. Nat. Acad. Sci. (USA) **34** (1948) 211.

[2] D. Lurié, *Particles and Fields.* (Interscience Publisher. New York, 1968), Chapter 1.

[3] M. Kirchbach, Mod. Phys. Lett. A**12** (1997) 2373.

[4] V. V. Dvoeglazov, *On the Importance of the Normalization.* Preprint EFUAZ FT-96-39-REV (hep-th/9712036), Nov. 1997;
Photon-Notoph Equations. Preprint EFUAZ FT-97-53-REV (physics/9804010), April 1998.

[5] J. Rembieliński and W. Tybor, Acta Phys. Polon. B**22** (1991) 439; ibid. 447; M. Bakalarska and W. Tybor, *On Notivarg Propagator.* Preprint hep-th/9801065; *On the Deser-Siegel-Townsend Notivarg.* Preprint hep-ph/9801216, Łódź, Poland.

[6] S. Weinberg, Phys. Rev. **133**B (1964) 1318; ibid **134**B 882.

[7] S. Weinberg, *Gravitation and Cosmology.* (John Wiley & Sons, New York, 1972).

[8] O. D. Jefimenko, *Causality, Electromagnetic Induction and Gravitation.* (Electret Sci. Co., Star City, 1992); T. Chang, Galilean Electrodynamics **3** (1992) 36.