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On the importance of the normalization*

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We repeat the known procedure of the derivation of the set of Proca equations. It is shown that it can be written in various forms. The importance of the normalization is point out for

the problem of the correct description of spin-1 quantized fields. Finally, the discussion of the so-called Kalb-Ramond field is presented.

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Recently, a new concept of the longitudinal magnetic field, the $\mathbf{B}^{(3)}$ field, was proposed [1]. It is based on the equation containing the cross product of two transversal modes of electromagnetism:

$$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = i B^{(0)} \mathbf{B}^{(3)\,*} \quad , \tag{1}$$

and represents itself a non-trivial generalization of the Maxwell's electromagnetic theory (see ref. [2] for the list of other generalizations). This concept advocates the mass of photon and appears to be in the contradiction with the concept of the $m \to 0$ group contraction for a photon as presented by Wigner and Inonu [3] and Kim [4].

On the other hand, in [5] it was shown that the angular momentum generators $J_{\kappa\tau}$ are equated to zero after the application of the generalized Lorentz condition $\partial_\mu F^{\mu\nu} = 0$ (and dual to that) to the quantum states, when considering the theory of quantized antisymmetric tensor fields (the representation $(1,0)\oplus(0,1)$ of the Lorentz group, according to the ordinary wisdom). The formula (9) of the cited reference tells us:[1]

$$\begin{aligned} J_{\kappa\tau} = \int d^3\mathbf{x}\, [(\partial_\mu F^{\mu\nu})(g_{0\kappa} F_{\nu\tau} - \\ g_{0\tau} F_{\nu\kappa}) - (\partial_\mu F^{\mu}{}_{\kappa}) F_{0\tau} + (\partial_\mu F^{\mu}{}_{\tau}) F_{0\kappa} + \\ + F^{\mu}{}_{\kappa}(\partial_0 F_{\tau\mu} + \partial_\mu F_{0\tau} + \partial_\tau F_{\mu 0}) - F^{\mu}{}_{\tau}(\partial_0 F_{\kappa\mu} + \partial_\mu F_{0\kappa} + \partial_\kappa F_{\mu 0})] \end{aligned} \quad .(2)$$

Therefore, the spin vector reads[2]

$$\mathbf{J}^k = \frac{1}{2}\epsilon^{ijk} J^{ij} = \epsilon^{ijk} \int d^3\mathbf{x} \left[F^{0i}(\partial_\mu F^{\mu j}) + F_\mu{}^{j}(\partial^0 F^{\mu i} + \partial^\mu F^{i0} + \partial^i F^{0\mu}) \right] \quad . \tag{3}$$

Thus, the helicity of the "photon" described by the physical fields $\mathbf{E}$ and $\mathbf{B}$ is believed [6,7] to be equated to zero (?), what is in the

strong contradiction with experimental results.[3] "The helicity – the projection of the spin onto the direction of motion – proves to be equal to zero ... even without the restriction to plane waves, the 3-vector of spin [formula (12) of the cited paper] vanishes on solutions ...", ref. [7b]. Moreover, if this construct describes the $h = 0$ fields it seems to contain internal theoretical inconsistencies because is in the contradiction with the Weinberg theorem $B - A = h$, ref. [8]. This argument was also used earlier by Evans [1,9,10]. In order

[1]While this formula

is the consequence of the particular choice of the Lagrangian of antisymmetric tensor field, nevertheless, the main conclusion is hold for other types of Lagrangians, including those which possess the Kalb-Ramond gauge invariance [6].

[2]Presenting this formula we, however, do *not* think that the question of the correct definition of the relativistic spin vector is so simple as believed before.

[3]M. Kalb and P. Ramond claimed explicitly [6c, p. 2283, the third line from below]: "thus, the massless $\phi_{\mu\nu}$ has one degree of freedom". While they call $\phi_{\mu\nu}$ as "potentials" for the field $F^{\alpha\beta\gamma} = \partial^\alpha \phi^{\beta\gamma} + \partial^\beta \phi^{\gamma\alpha} + \partial^\gamma \phi^{\alpha\beta}$, nevertheless, the physical content of the antisymmetric tesnor field of the second rank (the representation $(1,0)\oplus(0,1)$ of the Lorentz group) must be in accordance with the requirements of relativistic invariance.

to understand the nature of these contradictions one should reveal relations between the Evans-Vigier concept and the concept of the so-called Kalb-Ramond field [6]. While some insights into the problem have been already presented [10,5] in this paper we try to deepen the understanding of apparent conflicts, which were mentioned.

We believe in the power of the group-theoretical methods in the analyses of the physical behaviour of different-type classical (and quantum) fields. We also believe that the Dirac equation can be applied to some particular quantum states of the spin 1/2. Finally, we believe that the spin-0 and spin-1 particles can be constructed by taking the direct product of the spin-1/2 field functions [11]. So, on the basis of these postulates let us firstly repeat the Bargmann-Wigner procedure of obtaining the equations for bosons of spin 0 and 1. The set of basic equations for $j=0$ and $j=1$ are written, e.g., ref. [12]

$$\left[i\gamma^\mu \partial_\mu - m\right]_{\alpha\beta} \Psi_{\beta\gamma}(x) = 0 \quad , \tag{4a}$$
$$\left[i\gamma^\mu \partial_\mu - m\right]_{\gamma\beta} \Psi_{\alpha\beta}(x) = 0 \quad . \tag{4b}$$

We expand the 4×4 matrix wave function into the antisymmetric and symmetric parts

$$\Psi_{[\alpha\beta]} = R_{\alpha\beta}\phi + \gamma^5_{\alpha\delta} R_{\delta\beta}\widetilde{\phi} + \gamma^5_{\alpha\delta}\gamma^\mu_{\delta\tau} R_{\tau\beta}\widetilde{}$$

A_μ , (5a)

$\Psi_{\{\alpha\beta\}} = \gamma^\mu_{\alpha\delta} R_{\delta\beta} A_\mu + \sigma^{\mu\nu}_{\alpha\delta} R_{\delta\beta} F_{\mu\nu}$, (5b)

where $R = CP$ has the properties (which are necessary to make expansions (5a,5b) to be possible in such a form)

$$R^T = -R \quad , \quad R^\dagger = R = R^{-1} \quad , \tag{6a}$$
$$R^{-1}\gamma^5 R = (\gamma^5)^T \quad , \tag{6b}$$
$$R^{-1}\gamma^\mu R = -(\gamma^\mu)^T \quad , \tag{6c}$$
$$R^{-1}\sigma^{\mu\nu} R = -(\sigma^{\mu\nu})^T \quad . \tag{6d}$$

The explicit form of this matrix can be chosen:

$$R = \begin{pmatrix} i\Theta & 0 \\ 0 & -i\Theta \end{pmatrix} \quad , \quad \Theta = -i\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} , \tag{7}$$

provided that γ^μ matrices are in the Weyl representation. The equations (4a,4b) lead to the Kemmer set of the $j=0$ equations:

$$m\phi = 0 \quad , \tag{8a}$$
$$m\widetilde{\phi} = -i\partial_\mu \widetilde{A}^\mu \quad , \tag{8b}$$
$$m\widetilde{A}^\mu = -i\partial^\mu \widetilde{\phi} \quad , \tag{8c}$$

and to the Proca set of the equations for the $j = 1$ case:[4,5]

$$\partial_\alpha F^{\alpha\mu} + \frac{m}{2} A^\mu = 0 \quad , \tag{10a}$$

$$2mF^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad , \tag{10b}$$

In the meantime, in the textbooks, the latter set is usually written as (*e.g.*, ref. [14, p.135])

$$\partial_\alpha F^{\alpha\mu} + m^2 A^\mu = 0 \quad , \tag{11a}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad , \tag{11b}$$

The set (11a,11b) is obtained from (10a,10b) after the normalization change $A_\mu \to 2mA_\mu$ or $F_{\mu\nu} \to \frac{1}{2m} F_{\mu\nu}$. Of course, one can investigate other sets of equations with different normalization of the $F_{\mu\nu}$ and A_μ fields. Are all these sets of equations equivalent? As we shall see, to answer this question is not trivial. Papers [15] argued that the physical normalization is such that in the massless-limit zero-momentum field functions should vanish in the momentum representation (there are no massless particles at rest). Next, we advocate the following approach: the massless limit can and must be taken in the end of all calculations only, *i. e.*, for physical quantities.

Let us proceed further. In order to be able to answer the question about the behaviour of the spin operator $\mathbf{J}^i = \frac{1}{2}\epsilon^{ijk} J^{jk}$ in the massless limit one should know the behaviour of the fields $F_{\mu\nu}$ and/or A_μ in the massless limit. We want to analyze the first set (10a,10b).

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[4] We could use another symmetric matrix $\gamma^5 \sigma^{\mu\nu} R$ in the expansion of the symmetric spinor of the second rank. In this case the equations would read

$$i\partial_\alpha \widetilde{F}^{\alpha\mu} + \frac{m}{2} B^\mu = 0 \quad , \tag{9a}$$

$$2\text{im}\, \widetilde{F}^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu \quad . \tag{9b}$$

in which the dual tensor $\widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$ presents, because we used that in the Weyl representation $\gamma^5 \sigma^{\mu\nu} = \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}$; B^μ is the corresponding vector potential. The equation for the antisymmetric tensor field (which can be obtained from this set) does not change its form (cf. [2]) but we see some "renormalization" of the field functions. In general, it is permitted to choose various relative phase factors in the expansion of the symmetric wave function (5b). We would have additional phase factors in equations relating the physical fields and the 4-vector potentials. They can be absorbed by the redefinition of the potentials/fields (the choice of normalization). The above discussion shows that the dual tensor of the second rank can also be epxanded in potentials, as opposed to the opinion of the referee (JPA) of my previous paper.

[5] Recently, after completing this work the paper [13] was brought to our attention. It deals with the redundant components in the $j = 3/2$ spin case. If the claims of that paper are correct we would have to change a verbal terminology which we use to describe the above equations.