You may find the following identity useful during the exam \( \cos 2\theta = 1 - 2\sin^2 \theta \).

1. A particle P moves along the x-axis with position function at time t being \( s(t) = t^5 - 5t^3 + 10 \).
   
   (a) What is the velocity and acceleration at \( t = 2 \)? 6 points
   
   (b) At what times is the particle stationary? 4 points
   
   (c) On which intervals is the particle moving to the right (resp. left)? 5 points
2. Find anti-derivatives for the following functions.

(a) \( \frac{e^x}{x^2 + 2x^2 + 1} \) 8 points

(b) \( \frac{x^2}{\sqrt{25-x^2}} \) 12 points
3. Answer **YES** or **NO** to the following. To get full credit you must explain your answers and show your work:

(a) Does the following limit exist:  **4 points**

\[
\lim_{x \to -1} \frac{3x^7 + 16x^3 + x + 6}{x^9 + 6x^4 + 4x^2 + 3}.
\]

(b) Does the following limit exist:  **6 points**

\[
\lim_{x \to 0} \frac{3x^2}{2 \tan^2 2x}.
\]

(c) Is the function \( f(x) = \begin{cases} 
  e^x & x < 0 \\
  2 & x = 0 \\
  e^{-x} & x > 0
\end{cases} \) is continuous at 0?  **4 points**

(d) Is the function \( h(x) = \frac{x}{x+1} \) is one-to-one.  **6 points**
4.(a) Show that the function \( f(x) = \begin{cases} 1 - x & x < 1 \\ x^2 - 3x + 2 & x \geq 1 \end{cases} \) is differentiable at 1. **10 points**

(b) Let \( f(x) = 2\sqrt{\pi - \arctan 2x} \) and \( g(x) = \frac{x^2 + 1}{x + 2} \). Compute \( (f \circ g)'(0) \). **10 points**
5.(a) Use Integration by Parts to establish the reduction formula:

\[ \int x^n e^x \, dx = x^n e^x - n \int x^{n-1} e^x \, dx. \]  

8 points

(b) Use part (a) to find \( \int x^3 e^x \, dx \).  

12 points
6. (i) Determine a partial fraction decomposition for $\frac{1}{x^4 - x^2}$. 10 points

(ii) Integrate $\int \frac{1}{e^{3x} - e^x} \, dx$. (Hint: u-substitution and recall 6(i)) 10 points
7. A kite is flying at a height of 300 feet and moving in a horizontal wind. When 500 feet of string is out the kite is pulling the string out at a rate of 20 feet/sec. What is the kite's (horizontal) velocity at this instant. 15 points
8. Find the area of the largest rectangle that can be inscribed in the ellipse

\[ \frac{x^2}{4} + \frac{y^2}{25} = 1. \quad 15 \text{ points} \]
9. (a) Sketch the region $\Omega$ bounded by the curves $x = y^2 + 3$, and $x + y = 5$ marking carefully the points where these curves meet. \textbf{8 points}

(b) Find the volume of the solid obtained by revolving the region $\Omega$ (from part (a)) about the \textbf{y-axis}. \textbf{12 points}
Questions 10, 11 and 12 refer to the function

\[ f(x) = \frac{x^2 - 5}{x + 3}. \]

10. (a) Find the intervals on which \( f \) is increasing and those on which it is decreasing. \textbf{10 points}

(b) Determine the critical values of \( f \) and the nature of any local extrema. \textbf{5 points}
11. Find the intervals on which $f$ is concave up/down and any inflection points. 10 points

12. Describe any horizontal and vertical asymptotes, $x$ and $y$ intercepts and then sketch the graph clearly marking the information on the graph. 10 points