Graduate Course Description

Course Title: Hamiltonian Systems

Course and unique no.: 393C (57680)

Instructor: R. de la Llave  Semester: Spring 05

Time and Place: TTh 9:30-11:00

Brief description: Hamiltonian mechanics is the mathematical description of many physical phenomena, including mechanics, optics, and calculus of variations.

The goal of the course will be to present the field and to reach some of the active areas of research. One of the goals is to provide an introduction to KAM theory.

The students will be encouraged to carry out some projects, which could be either rigorous mathematics or computational depending on the interests of the students.

We will plan to cover (depending on the interest of the students)

• Mathematical formulations of mechanics.
  - Newtonian formulation.
  - Lagrangian formulation.
  - Hamiltonian formulation.
  - The Hamilton-Jacobi equations.

• Examples
  - Kepler problem.
  - 3 and N body problem.
  - Spherical pendulum.
  - Oscillators.
  - Integrable systems (Toda Lattice, Calogero system).
  - Numerical algorithms for eigenvalues as Hamiltonian systems.
  - Geodesic flows.
  - Hydrodynamics as a Hamiltonian system (formally)

• Transformation theory
  - Normal forms.
  - Generating functions.
  - Lie series.
  - Deformation theory.

• Perturbation theory.
  - Perturbation theory for periodic orbits.
  - Lindstedt series for quasiperiodic orbits.
  - Poincaré’s proof of stable manifold theorem.
  - Canonical perturbation theory.
  - Numerical implementation of Lindstedt series.
Numerical calculation of invariant manifolds with Poincaré method.

- KAM theorem on persistence of smooth tori.
  - The Kolmogorov proof.
  - The Arnol’d proof.
  - Proofs based on Hamilton-Jacobi equations.

- Variational theory.
  - Lagrangian submanifolds and their intersections.
  - Variational theory for periodic orbits.
  - Variational methods for quasi-periodic orbits.
  - Morse theory of geodesics.
  - Weak KAM theory.

- Other topics to be discussed if there is time or to be assigned as projects.
  - Aubry-Mather theory.
  - Renormalization description of breakdown of tori.
  - Systems of coupled oscillators.
  - Infinite dimensional Hamiltonian systems. (PDE’s, systems from Stat. Mech.)
  - Adiabatic invariants.
  - Invariant manifolds. (Theory and computation)

**Prerequisites:**

**Textbook(s):**

- W. Thirring A. course in Mathematical Physics Vol I (Recommended)
- V. I. Arnol’d Mathematical Methods of Classical Mechanics (Recommended)
  (An expanded version is in ftp.ma.utexas.edu/pub/papers/llave/tutorial.pdf)

The instructor will make available several programs and computational tools.

**Consent of instructor required:** NO

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