1. Find the line through \((3,1,-2)\) that intersects and is perpendicular to the line 
\[x = -1 + t, \quad y = -2 + t, \quad z = -1 + t.\]

The line through \((3,1,-2)\) is described by \((3,1,-2) + s(a, b, c)\).

The other line is described by \((1,−2,−1) + t(1,1,1)\).

The condition that the two lines are perpendicular is \((a, b, c) \cdot (1,1,1) = 0\) 
\[a + b + c = 0.\]

There is ambiguity in the choice of \(s\) and \((a, b, c)\), so we choose them
so that \((3,1,-2) + (a, b, c)\) is the intersection point of the two lines.

That is, we need to find \(t\) so that \((3,1,-2) + (a, b, c) = (1,−2,−1) + t(1,1,1)\),
or \((a, b, c) = (-4,−3,1) + t(1,1,1)\).

If we add the coordinates they sum to zero: \(-4−3+1+3t = 0 \Rightarrow t = 2\).

Thus, \(a = -2\), \(b = -1\), \(c = 3\). The intersection point is \((1,0,1)\).

The line joining \((3,1,-2)\) and \((1,0,1)\) is \(u(3,1,-2) + (1−u)(1,0,1)\).

2. Compute the tangent plane of \(z = x^2 + y^4 + e^{xy}\) at the point \((1,0,2)\).

The tangent plane is given by
\[z = f(x_0, y_0) + \frac{df}{dx}(x_0, y_0)(x-x_0) + \frac{df}{dy}(x_0, y_0)(y-y_0)\]

At \((x_0, y_0, f(x_0, y_0))\) \(f(x, y) = x^2 + y^4 + e^{xy}\),

\[\frac{df}{dx}(1,0) = 2 \quad \frac{df}{dy}(1,0) = 1\]

So, the tangent plane is
\[z = 2 + 2(x-1) + 1(y-0) = 2x + y\]

Could also use gradients to find the tangent plane to a level surface.
3. Calculate a unit normal to the surface \( \cos(xy) = e^z - 2 \) at \((1, \pi, 0)\).

Let \( f(x, y, z) = e^z - \cos(xy) \). The surface is the level surface for \( f(x, y, z) = 2 \).

The gradient of \( f \) is normal to the level surface.

\[
\nabla f = (y \sin(xy), x \sin(xy), e^z) \quad \nabla f(1, \pi, 0) = (0, 0, 1), \text{ which is a unit vector.}
\]

4. Find and classify the critical points of \( f(x, y) = x^2 - y^2 - xy \).

The critical points satisfy \( \nabla f = 0 = (2x - y, -2y - x) \).

That is critical points satisfy the two equations

\( y = 2x \), \( y = -\frac{1}{2} x \). The only point on both lines is the origin \((0, 0)\).

Apply the second derivative test.

\[
\frac{\partial^2 f}{\partial x^2} = 2, \quad \frac{\partial^2 f}{\partial y \partial x} = -1, \quad \frac{\partial^2 f}{\partial y^2} = -2.
\]

\[
D = \left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial y \partial x}\right)^2 = (2)(-2) - (-1)^2 = -5 < 0
\]

so the critical point \((0, 0)\) is a saddle point.
5. Determine the absolute minimum and maximum values of \( f(x, y) = x^2 + 3xy + y^2 \) on the unit disk \( D = \{(x, y) : x^2 + y^2 \leq 1\} \).

First look for local extrema, using first derivative test.
\[ \nabla f = (2x + 3y, 3x + 2y) = (0, 0). \] The only solution is the origin \( x = y = 0 \).

Apply the second derivative test:
\[ D = \left( \frac{\partial^2 f}{\partial x^2} \right) \left( \frac{\partial^2 f}{\partial y^2} \right) - \left( \frac{\partial^2 f}{\partial x \partial y} \right)^2 = (2)(2) - (3)^2 = -5 \leq 0 \]

Therefore \((0,0)\) is a saddle point, and the maxima and minima occur on the boundary.

We can parameterize the boundary as \((\cos \theta, \sin \theta)\) as \(0\leq \theta \leq 2\pi\).

\[ h(\theta) = f(\cos \theta, \sin \theta) = \cos^2 \theta + 3 \cos \theta \sin \theta + \sin^2 \theta = 1 + 3 \sin \theta \cos \theta \]

For \(0 \leq \theta \leq 2\pi\), this function has maxima at \( \frac{\pi}{4}, \frac{5\pi}{4} \) with value \( \frac{5}{2} \) and minima at \( \frac{3\pi}{4}, \frac{7\pi}{4} \) with value \( -\frac{1}{2} \).