Ideas for Class Projects
M375T: Multivariable Analysis
March 25, 2013

Here are a few ideas to get you started on projects. You are encouraged to seek out other ideas as well. I am very open to suggestions. Many of these descriptions indicate much more ground than you need explore for your project. The idea is to narrow down to something more specific as you get to know your topic.

You will probably have more fun with the project if you team up with a classmate. See the Project Description for more information.

Calculus of variations

The calculus of variations is ordinary calculus in a function space. We saw a basic example—minimizing the length of a curve—and another example in the homework. There are beautiful classical problems (Johann Bernoulli’ brachistochrone problem, for example). One formulation of classical mechanics, called the principle of least action, uses the calculus of variations as well. It is fundamental in theories of fields and strings (as opposed to particles). As a project you can explore some of the basic existence theorems of the subject, say for functions of one variable, and also pursue some of these problems.

Calculus with differential forms on manifolds

This is appropriate for those of you taking Differential Topology concurrently. Differential forms are the best computational tool for calculus, and there are a handful of important formulas that you can explore in some generality. They relate Cartan’s $d$ operator, Lie derivative, interior product, and integration. Applications abound.

Jordan measure theory

In Analysis I you developed the Riemann integral for functions of one variable. We will not treat the higher (finite) dimensional version in lecture, but you can do so in a project. Working independently of sources and by analogy with the one variable case you may discover what the main issues are to develop a multivariable integration theory. You can then try to overcome them and eventually use sources after you have struggled yourself. An important theorem here is the change of variables formula.

Geometry of Minkowski spacetime

I have some old notes of John Milnor on this topic, and you might have fun learning from those. You can explore the Poincaré group, physical notions of mechanics in special relativity, electromagnetism and Maxwell’s equations, etc. There is much to explore, for example reformulating what he
does there using calculus with differential forms. You can also explore variational principles (see Calculus of variations above).

**The Laplace equation**

This is one of the basic classical differential equations. The others are the heat and wave equations. These are all *partial* differential equations, as opposed to *ordinary* differential equations: they involve partial derivatives of functions of several variables. There are several mathematical directions to explore related to the Laplace equation: basic solutions in flat space, formulation in terms of differential forms and generalization to Riemannian manifolds, applications, etc.

**More about ordinary differential equations**

We proved a very general existence and uniqueness theorem in lecture, and you can explore special cases. You might, for example, ask for conditions under which global solutions (in time) exist. You can also construct the *flow* generated by a vector field. In another direction, you can explicitly solve *linear* ordinary differential equations and investigate their solutions. You can also explore special second order equations: Sturm-Liouville theory.