1) (30) Let \( f(x, y) = x^3 + y^3 + x^2y^2 \).
   a) (5 points) Find \( f_x, f_y \)
   b) (10 points) Find all critical points
   c) (5 points) Find \( M, \det(M), \text{tr}(M) \)
   d) (10 points) Use c) to classify the critical points if possible.

2) (45 points) Let \( z = y^2 + x^2; \ P = P(0, 1) \)
   a) (10 points) Sketch the surface; locate \( P, f(P) \) on the graph
      Your sketch should take up a quarter to a third of your page.
   b) (5 points) Write this as an implicit surface, \( F(x, y, z) = 0 \)
   c) (10 points) Find an implicit tangent plane at \( P \); first find \( \vec{n}, \vec{r}_0 \), then
      the equation for the plane.
   d) (10 points) Find a parametric representation \( \vec{r}(t) = (x(t), y(t), z(t)) \)
      for the trace \( z = f(0, y) \). Also find a \( t_0 \) such that at \( t_0 \), the parametric
      curve goes through \( P, f(P) \).
   e) (5 points) Find \( \vec{r}'(t) \) and \( \vec{r}'(t_0) \).
   f) (5 points) Show that the tangent vector \( \vec{r}'(t_0) \) lies in the tangent plane
      in c).

3) (25 points) Let \( z = f(x, y) = (x - y^2)^2; \ P = P(1, 0) \)
   a) (5 points) Find the level curve \( z = f(P) \), that passes through \( P \).
      Sketch it in the \( xy \) plane and locate \( P \) on the curve.
   b) (5 points) Find \( \nabla f, \nabla f(P) \).
   c) (5 points) Sketch \( \nabla f(P) \) on the graph in a), with its tail at \( P \).
   d) (5 points) Find a parametric representation \( \vec{r}(t) \), that lies on the
      curve in a), and goes through the point \( P \). Find a \( t_0 \) with \( \vec{r}(t_0) = P \).
   e) (5 points) Compute \( \vec{r}'(t_0) \) and compute that \( \nabla f(P) \) is perpendicular
      to \( \vec{r}'(t_0) \).
1) Let \( z = f(x, y) = \frac{y^3}{3} + \frac{x^3}{3} - x^2y \). Find all critical points of \( f \) and use the second derivative test to classify them.

1)(30 points) Let \( z = f(x, y) = x^3 - xy + y^3 \). Find all critical points of \( f \), and use the second derivative test to classify them.

1)(30 points) Let \( z = f(x, y) = \frac{1}{3}y^3 + x^4 - x^2y \). Find all critical points of \( f \).

1)(30 points) Let \( z = f(x, y) = x^2 + y^2 - x^2y \). Find all critical points of \( f \) and use the second derivative test to classify them, if possible.

5)(50 points) Let \( z = f(x, y) = x^2 + y^2 \).
Let \( P = P(\frac{1}{2}, \frac{1}{2}) \)

a) Sketch the surface; locate the point \( P \).

b) Find \( \nabla f \); \( \nabla f(P) \).

c) Sketch \( \nabla f(P) \) on the graph in b). If you were to follow the gradient from \( P \), you would trace out a path on the surface. ON A NEW GRAPH OF THE SURFACE, sketch that path.

d) Why does \( \nabla f(P) \) point in the direction it does?

e) In the \( xy \) plane, sketch the level curve \( f(x, y) = f(P) \)

f) On the sketch in e), draw the gradient of \( f \), with its tail at \( P \).