Q4 Practice

1) Let \( z = 1 - x^2 - y^2; \) \( P = P\left(0, \frac{1}{2}\right) \)
   a)(10 points) Sketch the surface; locate \( P, f(P) \) on the graph
      Your sketch should take up a quarter to a third of your page.
   b)(5 points) Write this as an implicit surface, \( F(x, y, z) = 0 \)
   c)(10 points) Find an implicit tangent plane at \( P \); you’ll need \( \vec{\mathbf{n}}, \vec{r}_0 \)
   d)(10 points) Find a parametric representation \( \vec{r}(t) = (x(t), y(t), z(t)) \)
      for the trace \( z = f(0, y) \). Also find a \( t_0 \) such that at \( t_0 \), the parametric
      curve goes through \( P, f(P) \).
   e)(5 points) Find \( \vec{r}'(t) \) and \( \vec{r}'(t_0) \).
   f)(5 points) Show that the tangent vector \( \vec{r}'(t_0) \) lies in the tangent plane
      in c).

Q5 Practice

Let \( z = f(x, y) = (y - \sin x)^2; \) \( P = P(0, 1) \)
   a)(5 points) Find the level curve \( z = f(P) \), that passes through \( P \).
      Sketch it in the \( xy \) plane and locate \( P \) on the curve.
   b)(5 points) Find \( \nabla f; \nabla f(P) \).
   c)(5 points) Sketch \( \nabla f(P) \) on the graph in a), with its tail at \( P \).
   d)(5 points) Find a parametric representation \( \vec{r}(t) \), that lies on the
      curve in a), and goes through the point \( P \). Find a \( t_0 \) with \( \vec{r}(t_0) = P \).
   e)(5 points) Compute \( \vec{r}'(t_0) \) and compute that \( \nabla f(P) \) is perpendicular
      to \( \vec{r}'(t_0) \).